The Great Pyramid: Star-Fixed When?

The Ancients’ Grandest Eclipse Cycle?

1325 Years
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**News Notes: Greater Pyramid Misses Old Kingdom’s Polestar**

In this issue, Hugh Thurston explains (§11) the curious 2 new-famous 2-star “precession” pyramid-orientation method. Reacting to such theories, DIO has noted (Nature 412:699; 2001/8/16) that, by the simplest 2-star or 1-star theories, the star 10i Draconis was central2 to orienting Khufu’s Great Pyramid. (See articles cited on back cover here.) Though it was ancient Egypt’s best polestar, 10i Dra has evidently been lighthoro-mentioned throughout pyramids’ by-now-Great Pyramid of ever-accreting literature on Khufu’s tomb.

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1 How did stars with r asc α permitting just enough azimuthal speed (to explain pyramid orientations’ “trend”) get chosen? (Anciently or modern?) After all, had Haack’s stars been solstitial (α ≈ 90° or 6°), the speed would’ve been negligible (§1 fn 5). (I.e., Alphard [α Hydra] would’ve been useful for E-W orienting as Haack’s Acrac [β Sco], and [at α ≈ 5°/6] would’ve stayed virtually put [in declination δ] over centuries of precession. Other near-equator stars [with various α]: ι Peg, β Crv, η Oph.) Same constancy applies if the Spence star-pair’s α had been equinoctial (e.g. α Dra & β Boo). Note confirmability/Lack: Spence stopped afar for a star-pair whose line’s speed fit Spence 2000 Fig.4 line a’s pyramid-based 0.28/yr slope (Haack 1984 p.512 made it 0.33/yr), later altered by her (to become the 0.31/yr Mizar-Kochab slope which DIO’s Nature paper established), needlessly, since DIO showed that 2 stars [11α & 10i Dra], each merely 1° from the 2627 BC celestial N.pole, fit [better than her own stars] her original 0.28/yr slope. Note: Spence 2001 Fig.1 exhibits only 1 line, not the [2000 Fig.4] original’s two for slope-comparison [theoretical vs empirical]. What’s a fit? Next considered yet another alternative (Archaeoastronomy 26 [JHA 23] S1-S20 [2001] Figs.3f): γ & δ UMA, whose extended line was crossed by the pole in mid-26th cy BC. His line-speed (barred by Spence’s empirical slopes): 40°cy in azimuth. (Actually less. Since annual azimuthal speed dA/dt = sin ε sin α / cos γ = 0.39 sin α [compare §1 fn 5], no speeds top 39°cy without high p.m.)[b] By unfortunate contrast to Spence (and to R&P), his stars’ midpoint is nearly 4 times farther from observational than the stars are from each other, which leverages observational error disadvantageously.

2 As against the several two-star “precessional” methods (details: §1), DR prefers the one-star DIO alternate theory that the Great Pyramid was oriented via bisecting 10i Dra’s right circumpolar semi-arc (1° radius) during 1 winter-solstice evening, when it was virtually symmetric in azimuth. (Impossible then for 11α Dra [Thuban], though Spence [Nature 412:699-700 (2001/8/16) p.700] can’t see why.) Suggestion (Rawlins&Pickering 2001 p.699): could S.Haack-K.Spence’s allegedly precessional orientational “trend” merely reflect the rise&fall of Old Kingdom surveying science, which obviously peaked in the time of Khufu, whose pyramid’s sophistication is nicely consistent with such a theory? (By contrast: according to precessional theories [incl. R&P’s], it is purely coincidental that the smallest orientation-error occurred for Khufu’s pyramid.) Spence explains away pyramid-orientation's apparently random signs by assuming the lined-up stars were precisely inverted for the 2 discrepant cases among her 8 pyramids. (It has been speculated that using Mizar&Kochab was a ritual. Hmm. Was it permissible for Egyptian rituals to be performed in reverse? And at different times of the year?) But does this just serve to explain-away weakness in the precession approach? (For its main strength, see §1 fn 2.) Note: [a] A 6-2 coin-split flip is asymmetric at less than 5-to-2 odds (twotailed), hardly significant. [b] Within ordmag 1°, the Khafe pyramid’s orientation equals Khufu’s, in magnitude and sign, suggesting co-orientation (p.3 fn 4). But the Spence theory, faced with the need to explain the orientations of these two best-oriented pyramids (several decades apart) by using an other-pyramids-based (§1 Fig.4) speed of about 3/decade, cannot straightforwardly be reconciled with such a small orientation-difference; so she must save the situation by bringing in special assumption-ex-machina: [a] her Mizar-Kochab line had swept past the pole during the years between Khufu&Khafe; so (as absolutely required, to compensate for [a]) [b] her method was used in opposite ways (M-atop-K for Khufu vs K-atop-M for Khafe). Thus, two conveniently-cancelling 0.1-sized errors are adduced to convert the actual difference (ordmag 1°) into the required theoretical difference (ordmag 10°).
H  Sparse-ReMotes vs Truckload-Beamers [Note added 2003/12/30]

By automatically rejecting the discoveries of the present paper — as well as Rawlins 2002B & Rawlins 2002H — purportedly on the basis that there are no remote 13th century astronomical records directly surviving, our glowingy self-satisfied Mufffiosi invite the following ghastly mote-beam (Matt. 7.3-5, Luke 6.41-42) observation, which DR put forcefully to the world of top Babylonianists (2003/6/22), the latest University of Notre Dame biennial history-of-astronomy conference: while a blank in 13th century BC records is perfectly understandable (given the rarity10 of extremely early astronomical observations, as exemplified by the uniqueness of the even-earlier Venus tables of Ammizaduga),11 no such excuse is at all possible to explain away the total absence (in extant Seleukid-era Babylonian cuneiform records) of any explanation of how “Babylonian” astronomical parameters & tables were arrived at, for this period from which (unlike the 13th century BC) a truckload of astronomical-math cuneiform texts do survive. Such critical explanatory ancient texts we have in detail from our slim Greek astronomical-math heritage, where (by contrast to Babylonian) we occasionally can even discern theory-founding empirical methods in action (see especially Jones 1999A [or DIO 9.1 p.2]) and can very precisely trace tabular parameters’ empirical bases: e.g., Rawlins 2003F eq.31 & Rawlins 2002A eqs.6-13. Our utter blank in parallel Babylonian material is completely inconsistent with DR’s long-loathed position (Rawlins 1991W §§H3-H4 & fn 73) that Babylonian astronomy is derivative; but it is embarrassingly inconsistent with the sacred-central Mufa tenet that Babylonians were the true originators of serious ancient mathematical astronomy.

References

D.Rawlins 2002A. DIO 11.1 §1.
D.Rawlins 2002B. DIO 11.1 §2.
D.Rawlins 2002H. DIO 11.1 §3.
Gerald Toomer 1984, Ed. Ptolemy’s Almagest, NYC.
1 On the Orientation of Early Egyptian Pyramids
by Hugh Thurston1

A Introduction

A1 Eight early Egyptian pyramids are oriented amazingly accurately; their eastern and western sides are less than degree from a true north-to-south line (§A3). If we arrange them in order of date, we find that with two exceptions their orientations drift slowly but steadily clockwise.

A2 This suggests that the pyramids were aligned by a method vulnerable to precession. The two exceptions, the pyramids of Khafre and Sahure, are out by the amount expected but in the opposite direction, suggesting that the method was reversible2 and was used in reverse for these two pyramids.

A3 If we reverse the sign of the deviation for the two exceptions cited in §A2 (clockwise deviations here are positive), we have the following data, listing pyramids in chronological order:

<table>
<thead>
<tr>
<th>Site</th>
<th>Pharaoh</th>
<th>east side</th>
<th>west side</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meidum</td>
<td>Sneferu</td>
<td>−20°.6</td>
<td>−18°.1</td>
</tr>
<tr>
<td>Dahshur, south</td>
<td>Sneferu</td>
<td>−17°.3</td>
<td>−11°.8</td>
</tr>
<tr>
<td>Dahshur, north</td>
<td>Sneferu</td>
<td>−8°.7</td>
<td></td>
</tr>
<tr>
<td>Giza</td>
<td>Khufu</td>
<td>−3°.4</td>
<td>−2°.8</td>
</tr>
<tr>
<td>Giza</td>
<td>Khafre</td>
<td>+6°.0</td>
<td>+6°.0</td>
</tr>
<tr>
<td>Abusir</td>
<td>Sahure</td>
<td>+23°</td>
<td></td>
</tr>
<tr>
<td>Abusir</td>
<td>Neferikare</td>
<td>+30°</td>
<td></td>
</tr>
</tbody>
</table>

The deviations of the east and west sides are quoted from a paper [1] by Kate Spence, who used a number of sources, primarily the work of J. Dorner — especially his 1981 dissertation [2].

B Prior Suggested Methods: Haack, Spence, Belmonte

B1 Steven Haack [3] suggested that the Egyptians searched for a star that appeared to rise precisely due east and then aligned their pyramid’s north and south sides on it.

1 Professor Emeritus of Mathematics, University of British Columbia. Address: Unit 3 12951, 17th Avenue, South Surrey, BC, Canada V4A-8T7; phone 604-531-8716. See Thurston’s other writings on the pyramids in his Early Astronomy (Springer 1994) and in Griffith Observer 2001 September. All footnotes here are by DR. Dio expresses thanks to Peter Dorman (Oral Inst, Univ Chicago) for advice (he regards it as not firmly established that ancient Egyptian cattle-counts were biennial) and to Michaela Rossini (U.Limsbruck) for transmitting J.Dorner’s thesis.

2 In spite of alternate possibilities, etc., noted elsewhere here (p.2 fn 2 and p.3 fn 2), DR must in fairness take space to emphasize the prime strength of Spence’s theory (& Haack’s earlier one): for her eight-pyramid sample (where she has dropped one of Haack’s data [Zoser]; added another [Dahshur-Red]; & altered others [Sahure & Dahshur-Bent], presumably for the better), the temporal trend of misorientations’ absolute magnitudes is monotonic (in either direction from her theory’s null-error time) — which is consistent with the precession theory’s explanation. (Though, some of the standard deviations estimated by Spence [2000 Table 1] are comparable to [and in some cases exceed] the associated differences which establish this monotonicity.)

For the information of our readers, we here supply the dates of gaps for all four of our long delicate cycles; these gaps3 indeed appear about every 6 centuries (§F4):

<table>
<thead>
<tr>
<th>Yr</th>
<th>Date</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>690</td>
<td>788/11/12</td>
<td>386/4/15</td>
</tr>
<tr>
<td>795</td>
<td>683/9/23</td>
<td>9/19</td>
</tr>
<tr>
<td>1010</td>
<td>830/4/10</td>
<td>4/19</td>
</tr>
<tr>
<td>1325</td>
<td>262/6/12</td>
<td>7/15</td>
</tr>
</tbody>
</table>


G Reflections

I do not know with certainty whether others have previously explored all of the peculiarities and the variety of interacting periodicities & pulsations which we have here revealed, in connection with the peculiar class: delicate vast eclipse cycles. (The lengthy study van den Bergh 1955 anticipated none of our new results.) But I hope that these will be of interest both to astronomers and to historians — and that the latter will be assisted (by the foregoing scientific findings) in future analyses of ancient astronomers’ methods.

Footnotes:

[Note added 2008. Dio 11.1 and Dio 13.1 have found solutions to all three previously unsolved ancient lunar period-relations, adding five eclipse-pairs. The condition that all ten eclipses be above the horizon for at least some portion of the umbral phase relates to ΔT researches. (Of the 5 older eclipses, 4 were near the horizon, possibly helping later astronomers know their hour.) ΔT for the 13th century BC has heretofore been exclusively based on an extraplate leap. (Across a 1/2 millennium gulf between then & the earliest extant eclipse records, data elicited and analysed by dedicated experts, for whom Dio has high admiration.) By contrast, Dio’s triple-solution, a “mere” computational speculation, represents a mathematical leap, to an isolated, non-extrapolated 13th century BC snapshot. As noted, our method is Greek-standard-attested (and easily explains all 3 period-relations’ integrality & high accuracy); but if firm incompatibilities between the two leaps develop, it will be up to Dio to snipely publish the other side and up to others to choose.)

which, again, could cause an overestimate of the synodic month's length (after dividing the pair's separation \(d\) by 16385) by about a part in 2 million.

**F Long Delicate Eclipse Cycles’ Patterns**

In recent years, DIO has examined four huge delicate eclipse cycles (citations at \[\text{F5}\]): 690°, 795°, 1010°, & 1325°. In doing so, we have discovered certain features common to all four, and these have physical as well as historical interest.

**F1** Several saros-strings (ss) are always simultaneously active, and long delicate cycles are woven of ss ends (grafting-eclipses). Cycle-pairs' lunar anomalies \(\nu\) are usually spaced about 120° apart. When ss successively appear, disappear, & are replaced by new ss, delicate-cycle eclipse-pairs' anomalies tend (except during occasional transition periods) to be either very nearby or about 120° distant — an effect resembling a cinema of a variably-diffuse equilateral triangle.

**F2** For any given epoch, the \(\nu\) at which a cycle's eclipse-pairs occur are near\(^7\) the three points of this slowly precessing equilateral triangle, which we have already dubbed the “PBT” (after analysing it in Rawlins 1996C). Within each ss, the successive anomalies flow in retrograde at a mean speed of a little under 3°/ss or nearly a degree/6\(^2\).

**F3** However, as each ss is replaced by succeeding ones (in the PBT cinema), the anomalistic triangle moves not backward at \(\text{F2}\)'s degree-per-6° pace — but rather forward at about a degree per 17\(^0\), a speed controlled by the lunar anomalistic precession of the highly accurate 441° Hipparcian draconic equation (Almajest 4.2; Rawlins 1996C §F10 & eq.19 [or Rawlins 2002H eqs.1&3]).

**F4** We have mentioned previously (§B1, Rawlins 1996C §F, Rawlins 2002B §B3) that our delicate long cycles are so fragile that (for all four) whole decades or even centuries will pass without a single umbral pair occurring. That is, during such a temporal gap, if an eclipse has a prior cycle-mate, it’s penumbral & thus effectively invisible. For each of our four cycles, the mean periodicity of the occurrence of these gaps is roughly 6 centuries, an effect influenced by the near-integrality of a nest of secular lunisolar returns of about that length (inter-related by the saros & the 65° cycle): 6667\(^0\) (539\(^0\)), 7248\(^0\) (586\(^0\)), 7471\(^0\) (604\(^0\)), 8052° (651\(^0\)). E.g.,

\[
7248^0 = 7765^0 - 52^0 = 7865^0 \div 2 + 0^0 = 586^0 - 4^0 = 214037^0 \times 18^0 (11)
\]

**F5** Of course, as noted in §F4, the period of disappearance (of umbral eclipses having earlier umbral matches, according to a cycle) does not occur with discrete suddenness. During fertile (i.e., non-gap) periods, the rate of pair-occurrence rises and reaches a maximum at about the halfway-point between gaps — and the average length of the intervals between the pairs themselves also reaches an extremum there. (And in the time just before&after a gap, the number of umbral pairs noticeably ebbs in comparison to the maximal density occurring halfway between gaps.) These extremal effects will combine to bias-correct any estimate of monthlength based upon naive averages of data (as remarked at §E and Rawlins 2002B §B8), since pair-intervals' sizes reach the opposite extremum during a gap (whose intervals would of course be missing from the average's input, by the gap's very definition).

\(\text{eq.3-mate eclipse will almost always have been visible (i.e., above-the-horizon) in Babylon, one hour to the east. This is one of the reasons why visible eq.3 pairs were more frequent than visible eq.2 pairs.}\)

\(\text{The density of eclipse-occurrence is greatest for the anomaly-point (of the equilateral PBT's three points) which is nearest perigee, and it is least (indeed often virtually or exactly nil) for that which is nearest apogee.}\)

\(\text{Precessing ss-Bound anomalistic-Triangle (Rawlins 1996C §F). (Some cycles [e.g., 690°] exhibit more than one PBT.) The PBT's advancement/gap is c.1/10 of the zodiac (a rate which increases very slightly over the centuries), so that after roughly 2 millennia, the three PBT points are advanced near enough 1/3 of a circle that the PBT is approximately as it was.}\)

**B2** The azimuth where the star appeared to rise would rotate clockwise. The azimuth where it appeared to set would rotate in the opposite direction at the same rate, and Haack suggested using this to orient Khafre’s pyramid.

**B3** However, the Egyptians cannot have done this.\(^4\) As I saw on a recent visit (having been forewarned by Kate Spence), the plateau at Giza slopes upward to the west and when the Egyptians levelled the ground for Khafre’s pyramid they left a cliff some 10 metres high a little way to the west. This would form a high local horizon and cause a star that rises due east to set much too far south (of due west) to account for the slight deviation of the pyramid.

**B4** Haack suggested that the ancient surveyors used the star \(\alpha:\) Arietis for the two pyramids at Abusir, \(\beta: Scorp\) for the others.\(^5\)

**C Suggested Methods: Spence**

**C1** Kate Spence suggested that the Egyptians thought that the pole was directly between the stars Mizar and Kochab, and indeed in 2467 BC it was \([1]\). For Mizar above Kochab, the azimuth would drift clockwise, so most pyramids were aligned with Mizar above Kochab. For the two exceptions, Kochab was above Mizar. Both drifts were at 0.31 per year \([4]\).

**C2** 2467 BC is very late in, or later than, the dates given by various authorities for the fourth dynasty. Dates for the start of the dynasty vary from 2640 BC (quoted in D.Arnold, Building in Egypt [1991] as from "R.Krauss, 1985") to 2575 BC (Baines & Malek, \([5]\)). Dates for the end of the dynasty vary from 2504 BC (the earliest accepted by Beckerath \([6]\)) to 2454 BC (the latest accepted by him).

**D Suggested Methods: R&P**

**D1** Dennis Rawlins and Keith Pickering suggested \([4]\) that the Egyptians thought that the pole was equally far from Thuban and 10\(i\) Draconis,\(^6\) and indeed in 2627 BC it was. This would imply that when these stars were aligned horizontally, the point midway between them was due north.

**D2** Most pyramids would be aligned with the two stars above the pole; the two exceptions, under the pole. This alignment would drift at 0.274 per year. The suggested date 2627 BC is very early in, or earlier than, the dates for the fourth dynasty given in §C2.

**E Observations**

**E1** According to Haack’s method, the Egyptians simply watched\(^7\) for a star that rose due east or set due west and they aligned each pyramid on the rising or setting of this star. The observations had to be made at the time of year when the star rose or set at night.

\(\text{See [4]'s last paragraph.}\)

\(\text{These stars had two different speeds in azimuth: 0°.34 per year for \(\alpha: Arietis\), -0°.39 per year for \(\beta: Scorpii\). Both values for azimuthal speed provided here are for 2600 BC, when \(\alpha\) was 332° for Hamal (\(\alpha: Ari\)) and 180° for Acracb (\(\beta: Scor\)). (During the period of our interest, Hamal’s speed increased merely 0°.0044 per cycle, while Acracb’s was virtually constant.) In general: } dA/dt = \pm\frac{24\times0.39}{0.39+0.332}\text{ (C for due E or W at geographical latitude, }\gamma = 30°\text{ and (for mid-3rd millennium BC) annual precession }p = 0°.82\text{ and obliquity }\iota = 24°\text{. If }dA/dt\text{ is positive the rising and setting points rotate northward; otherwise, southward.}\)

\(\text{The star 10\(i\) Dra is of variable magnitude (4.1/2-to-5). In almost exactly 2800 BC, adjacent Thuban became the most prominent pole star in history (brighter than 4th magnitude): less than 0°.1 from the exact pole. But, starting in 2627 BC, no brighter star was nearer the pole than 10\(i\) Dra, for the next 11 centuries. (And no brighter-than-5th-magn star nearer for the next 9 \(\text{X}\). A direct method would be: just looking for stars that appeared to set 180° from where they rose.}\)
E2 For the other two methods the Egyptians needed to find the pole: not only its direction but also its altitude (which is 29°58′−59′N true\(^8\)) at Giza, a little less at the other sites. If they didn’t know where the pole was, they would not know that it was collinear with Mizar and Kochab or equidistant from Thuban and 10i Draconis. These two methods are most easily carried out by using a plumb-line. Stand at a corner\(^9\) of the pyramid-to-be and watch the two chosen stars. When they are aligned, vertically or horizontally as the case may be, they are north. Set up a plumb-line north of the position of observation, high enough to cover the stars and very nearly touching the ground.

E3 A little earlier the next night, go to the same spot, taking with you a small ring fixed to the top of a tripod formed by three sticks lashed together. (If you want to test the practicability of this for yourself, you will find that a camera tripod serves very well.)

E4 For the vertical alignment method, watch one of the stars through the ring, moving the tripod to a position where the plumb-line covers the star. Move the tripod sideways to keep the star covered (the movement needed will be quite slow) and when the plumb-line covers the other star, drop a plumb-line from the ring. The line between the two plumb-lines gives the orientation sought.

E5 The horizontal alignment method is similar. Keep the plumb-line covering the midpoint between the two stars and drop the plumb-line when they are judged to be horizontally aligned. Because the two stars are close together (merely 1°1/2 apart), a midpoint judged by eye will not be far out, and because the midpoint itself is so near the pole (barely 3/5 of a degree away), a slight misjudgement in the time of horizontal alignment will not move the midpoint very far sideways.

F Precedents

There is precedent for using two stars collinear with the pole to find north: today we (or at least Boy Scouts & Girl Guides) use the Pointers. And there is a precedent for using a vertical alignment: Polynesian and Micronesian navigators knew that when the Southern Cross was upright it was pretty well due south.\(^10\) There is no known precedent for either of the other methods.

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\(^8\)The apparent (not true) altitude of the pole is 30°00′ (1/12 of a circle) as seen from all three Giza pyramids. (Latitudes: Khufu 29°58′.7 N, Khafre 29°58′.5 N, Menkaure 29°58′.3 N.) Whether this is accidental is discussed in D.Rawlins, “Ancient Geodesy: Achievement & Corruption” (Vistas in Astronomy 29:255-268 [1985] pp.255-256).


\(^10\)If based upon verticality of Cruc’s mast (α&γ Cru), the Pacific peoples’ S.Cross-method would have provided exact south just before 1000 BC; or after 3000 BC, if horizontality of the crossbar (β&ε Cru) was used instead. The UMa Pointers’ line hasn’t pierced the pole since the mid-13th century AD, but the Pointers are still popular anyway — primarily for finding Polaris (α UMi), a star which provided north by being about 1° from the true pole, as α Dra & 10i Dra were, back in the 27th century BC. (The gyroscopically-precessing celestial pole was 71′ from Polaris in 1908 [when the Scouts were founded]; now, 43′ away; in 2102, the pole will pass within 28′ of Polaris and then recede from it.)
D But What of the 795 Year Cycle?

D1 So does the case for eq.3 now overturn Rawlins 1996C’s belief (esp. §E) that eq.2 underlay eq.1? It very well may. However, one’s view of this question depends in part upon whether one accepts that eq.1 had to be new for Ptolemy to cite it. (Not the firmest basis of argumentation.) After all, it is possible (if not probable) that eq.1 arose from both eqs.2 & 3.

D2 Though obviously we now have a strong case that eq.1 originated in Ptolemy’s era, the foregoing is not a rigid bar to Hipparchos having known of eq.1, too. Several eq.2-eclipse-pairs were available to him; and, for using eq.2, he would require eclipse data only back in the 10th (not 13th) century BC. (In eq.2’s favor, see also Rawlins 2002B §G & Rawlins 1996C §115 item [d]). Further, paralleling Ptolemy’s coincidence (§C1): the only Hipparchos partial lunar eclipse (−140/127) happens to have a prior eq.2-mate (−935/326) — though (see fn 5), if current estimates of Earth-acceleration are correct, it ended before moonrise in Babylon (while partly visible ordmag 10° of longitude to the east thereof). (No Hipparchos eclipse has a previous eq.3-match, which is consistent with our general theory that classical-era astronomers were not using data older than the 13th century BC.) So all 3 extant small eclipses of Hipparchos & Ptolemy are connectible to the 3277° cycle (eq.1), an impressively big-stretch coincidence, given the eclipse-pairs’ infrequency.

E Implications

E1 In toto, the foregoing obviously favors eq.3 as the source of eq.1, but it also requires Greek access to such early Babylonian data that the classic-conservatives (who’ve long ruled ancient-astronomy-history’s petrified landscape) may find eq.3 even less palatable a progenitor than eq.2. But, if one responds receptively to the pro-eq.3 evidence of §B, the implications are more unsettling and important than those of Rawlins 1996C’s theories (which assumed eq.2-ancestry). [See below at §H.]

E2 As noted at §B2, eq.1 appears to be based upon eclipse-pairs ending in the 2nd century AD — and (according to eq.3) with separation 1325°. Thus, since eq.1 could not have been later than c.165 AD (§B1), simple subtraction tells us that eq.1 was based upon eclipse-pairs whose early eq.3-mates were no later than:

\[165 - 1325° = -1160\]  

— an era far earlier than the ‘til-now conventionally-assumed limit (747 BC: §A1) for eclipse records that could have survived into classical antiquity. We’ve already seen (§C1) that Ptolemy’s small eclipses both have eq.3-matches that occurred shortly before eq.6’s date: Babylon-visible eclipses in −1200 & −1189. Moreover, several other eclipses that were recent to Ptolemy were matched by Babylon-visible eclipses 16385° earlier. (Note: all 16385°-cycle eclipse pairs are from ss whose Meeus & Suckle 1992 numbers differ by 14.) We will list a few such pairs, also appending the two Ptolemy-eclipse matches:

\[
\begin{align*}
-1247/7/22 & & 78/4/16 \\
-1236/6/20 & & 89/3/15 \\
-1207/5/31 & & 118/2/23 \\
-1200/7/11 & & 125/4/5 \\
-1189/6/12 & & 136/3/6 \\
\end{align*}
\]

E3 Adding to Rawlins 2002B & Rawlins 2002H, the foregoing considerations represent the 3rd piece of DIO-products evidence indicating that the astronomers of classical antiquity had access to 13th century BC (§C1) Babylonian eclipse records.

E4 Note two revealing implications here: [A] The possibility considered in Rawlins 2002H (§D4 vs fn 14), that Hipparchos may have had special access to 13th century BC Babylonian data, is hardly compatible with the indication here (§E2) that astronomers of Ptolemy’s time may have used a different set of eclipses (presumably from a publicly

Hugh Thurston  Pyramid Orientation  2003 December  DIO 13.1 §1

G The Intervals of Time Between the Pyramids

G1 Except for Sneferu’s 2nd & 3rd pyramids, the dates at which the pyramids were started will be (fn 12) within a year or two of the lengths of the pharaohs’ reigns; a pharaoh will obviously start his pyramid early in his reign.

G2 The lengths of the reigns are fundamental to Egyptian chronology. Early chronology depends on a Sothic date in the 12th dynasty, which pins the 7th year of Senosret III at 1872 BC. (Most Egyptologists regard Sothic dating as valid. Those who don’t have no anchor point.)

G3 Dates before Senosret III depend entirely on estimating the lengths of individual reigns and working back step by step.

G4 The fourth and fifth dynasties, together with the sixth constitute the Old Kingdom. Then came a period of chaos known as the first intermediate period (FIP), after which the eleventh dynasty started the Middle Kingdom.

G5 Our main source for the lengths of the reigns is the Turin canon, a papyrus document which unfortunately is far from complete. It can be supplemented by a late compilation in Greek by Manetho, by the Palermo stone, and by inscriptions at Abydos & Saqqara.

G6 We can put a lower limit on the length of a reign if we find a record of an event such as the 24th cattle-count in the reign of Sneferu. Cattle-counts probably took place every 2 years; if so, Sneferu must have reigned for at least 48 years. (See R.Stadelmann, “Beiträge zur Geschichte des Alten Reiches,” MDAIK 43:229-240 [1986].)

G7 The records are complete enough to take us back from Senosret III to the beginning of his dynasty, the twelfth (Baines & Malek describe this date as “known with precision”, [5], page 36), and with fair confidence to the beginning of the eleventh.

G8 We then have to deal with the FIP, about which there is practically no information. Cyril Aldred [8] makes it 149 years. Beckerath gives it a maximum of 50 years and a minimum of zero [6].

G9 Obviously for the Old Kingdom we should not put much faith in absolute dates, however, we can look at the lengths of the reigns. In the Turin canon, most of the names of the pharaohs are missing; but all except two can be supplied from tablets at Saqqara and Abydos. The two in parentheses below are from Manetho: we do not know the Egyptian forms of these names. This gives the table below.

<table>
<thead>
<tr>
<th>Pharaoh</th>
<th>Length of Reign</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sneferu</td>
<td>24°</td>
</tr>
<tr>
<td>Khufu</td>
<td>18°</td>
</tr>
<tr>
<td>Djedefre</td>
<td>8°</td>
</tr>
<tr>
<td>Khafre</td>
<td>20°</td>
</tr>
<tr>
<td>(Bicheris)</td>
<td>[1]°</td>
</tr>
<tr>
<td>Menkaure</td>
<td>[18]°</td>
</tr>
<tr>
<td>Shespeskaf</td>
<td>4°</td>
</tr>
<tr>
<td>(Thamphthis)</td>
<td>2°</td>
</tr>
</tbody>
</table>

G10 Manetho had two pharaohs named Suphis (presumably Khufu and Khafre) with reigns of 63 and 66 years. Beckerath assumed that he obtained the 63 by adding 40 to the correct number, to agree with a remark by Herodotus that the pharaohs who built the enormous pyramids reigned over 60 years. Beckerath assumed that Manetho did the same for Khafre, and that therefore his missing digit is 6. He altered Sneferu’s reign to 35 years. (We saw in §G6 that this is still too low.) At one point Beckerath stated that Menkaure’s reign was 18, 28, or 38 years, presumably because there was not room on the missing fragment for more than three of the hieroglyphs for 10. He settled for 28 in his final list.
He allotted 7 years to Bicheris from the 7 allotted to Sebercheres by Manetho (who allotted 22 to Bicheris). He altered Djedefre to 9 and Shepseskaf to 5.

**G11** This, together with Beckerath’s dates for the fifth dynasty gives column Bk in the table below. For comparison, columns B&K and CAH give the corresponding intervals from Baines & Malek [5] and the Cambridge Ancient History [9], respectively. Columns Sp and R&P give the time in years to produce the changes in orientation of the eastern sides of successive pyramids, as shown in §A3, using the methods of Spence (Sp) and Rawlins & Pickering (R&P), respectively.

<table>
<thead>
<tr>
<th>Bk</th>
<th>B&amp;M</th>
<th>CAH</th>
<th>Sp</th>
<th>R&amp;P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snerfu to Khufu</td>
<td>35°</td>
<td>24°</td>
<td>24°</td>
<td>55°</td>
</tr>
<tr>
<td>Khufu to Khafre</td>
<td>32°</td>
<td>31°</td>
<td>31°</td>
<td>30°</td>
</tr>
<tr>
<td>Khafre to Menkaure</td>
<td>33°</td>
<td>30°</td>
<td>25°</td>
<td>21°</td>
</tr>
<tr>
<td>Menkaure to Sahure</td>
<td>43°</td>
<td>32°</td>
<td>41°</td>
<td>34°</td>
</tr>
<tr>
<td>Sahure to Neferirk.</td>
<td>13°</td>
<td>12°</td>
<td>14°</td>
<td>22°</td>
</tr>
</tbody>
</table>

When reading this table bear in mind that the best historical estimate for Snerfu-to-Khufu is none of those listed. It is 48 (§G6).

**G12** Spence’s method[^1] makes Khufu’s reign start about 2480 BC. R&P make it about 2638 BC. Haack made the fourth dynasty start about 2640 BC.

**G13** I don’t know what chronologists will make of this. So far they seem to have ignored it. But at least it confirms that they were right to reject Manetho’s long reigns for the pharaohs who built the Giza pyramids and backs Stadelmann’s case [cited §G6] that estimates for Snerfu’s reign cited in §G11 are substantially too short.

### Appendix: Some Mathematics

**H1** Spence’s paper [1] gave rise to some trigonometry which, though not relevant to Egyptology, is of interest in itself. And it sparked the interest of Rawlins & Pickering.

**H2** Let us denote by $\phi$ the angular distance between the celestial pole (as seen from Giza) and the plane through Giza, Mizar, & Kochab at instants when these stars are aligned vertically. Spence interpreted $\phi$ as the deviation from north given by the vertical line. It is not.

**H3** Figure 1 shows the situation. G is Giza. GP is in the direction of the pole. GN is horizontal, and PN is vertical, so GN points horizontally north.

**H4** How do we find the angle between GP and the vertical plane through Giza and the stars? Answer: drop a perpendicular PQ to this plane; then the angle PGP is the angle required (namely $\phi$).

[^1]: The misorientations of Khufu’s sides (Domer 1981 p.77), all W: N 2°28’’, S 2°31’’, W 2°47’’, E 3°26’’. (Note: parallel Khufu W&E sides’ azimuths would disagree by 4° from Earth-sphericity; not negligible if we display 0.1 precision.) Taking the W side (above: p.3) as the closest approximation to the ancient surveyor’s original orientation-error, R&P’s Thuban-10i Dra method produces 2636 BC for the Khufu pyramid’s start. [Proceeding as in fn 12: –2626 – 2°47’’.274 = –2626 – 2637 BC.] The R&P dates for Khafre are nearer conventional ones than Spence’s (as noted in [4]).

[^2]: Taking the mean 3.1 W misorientation (fn 11 or §A3) of the Great Pyramid’s W&E sides, one can divide by Spence’s original (flawed: §H2) Mizar-Kochab 0°.28 speed ([11] Fig.4 line a’s slope) and subtract this from –2466 (2467 BC), the date of null error (when the pole was exactly on the Mizar-Kochab line): –2466 – 3.1°0.28 = –2477 = 2478 BC. She further corrected for a presumed 2° gap ([G1]) between the pharaoh’s ascension and his pyramid’s start, to find 2480 BC for the ascension date. (Repeating her calculation with correct slope [0°.31/yr] yields 2479 BC, instead: it’s a tiny difference in this case, but date-corrections for some of the other pyramids are ordmag 10°.)

### B Which Cycle Was Ptolemy’s Final Lunar Equation Based Upon?

**B1** There is a remarkable feature common to all four of §A4’s long&delicate cycles (690°, 795°, 1010°, 1325°); each exhibits gaps during which no eclipse-pairs occur. (Note: such unfragile relations as the key 345° cycle [§F7] have no gaps at all.) We will discuss details later (§F4). But the immediate connexion is this: the 795° cycle’s classical-era gap [zero eclipses with mates 795° earlier] extends from –36 to +254, while Ptolemy’s writing of the PlanHyp (containing our sole equation [§A1] of eq.1) was about 160-170 AD. By contrast, for our newly-proposed 1325° cycle: the classical periods when no eclipses occur (which have prior umbral eq.3-mates) are (§F6) –262/1/26 to –193/5/11 and +331/3/10 to +393/5/12.

**B2** Now, it is generally believed (and Rawlins 2002B §L bolsters the conventional view) that PlanHyp (source of eq.1) improves the theories of the Almajest by using data from Ptolemy’s own time. If this theory is on the right track, it starts us in the direction of favoring eq.3 as eq.1’s (prime) basis — for the obvious reason that no eq.2-separated eclipse-pairs ended during Ptolemy’s career, or indeed for well over 100° before he was even born (§B1). By contrast, eq.3-pairs repeatedly occurred (§E2) during his century.

### C Ptolemy’s Era Connectable to 3277 Month Cycle After All

**C1** And it gets only better when we check in detail. An eclipse which has a mate (another umbral eclipse) 16385° distant must be a partial eclipse of magnitude less than 10 digits. Of Ptolemy’s four eclipses (Almajest 4.6&9), the only ones of sub-10-digit magnitude are those of 125/4/5 & 136/3/6. By a striking coincidence, each has a mate 16385° vertically. Spence interpreted this? We will evaluate the odds in two ways.

**C2** Approach 1: During the years 125–141, seven sub-10-digit eclipses were visible in Babylon: –1200/7/11 & –1189/6/12, respectively. How a priori-unlikely is this? We will evaluate the odds in two ways.

**C3** Approach 2: During the years 125–141, seven sub-10-digit eclipses were visible in Egypt, only two of which had previous eq.3-mates. The probability $p_1$, that these four eclipses should include the only two that had prior eq.3-mates, is:

$$p_1 = C_2^2/C_{26}^2 = 6/120 = 1/20 = 5\%$$

(4)

**C4** Either way, the odds (though not huge) are statistically significant, and this in a pure probability case — i.e., where the gauging of degree-of-significance is not muddied by the nonGaussian vagaries which typically attend observational error.

2 Vast Eclipse Cycles: Stabilities & Gaps

Delicate Huge Eclipse Cycles’ Six-Century Pulsations

Ancients’ Longest Period-Relation? — 1325 Years

Ptolemy Now Connectable to His 3277 Month Cycle

Greek Use of 13th Century BC Data: Yet Another Hint

by Dennis Rawlins

A The 3277 Month Cycle & Our Expanding Temporal Horizon

A1 DIO 6 §1 §E investigated Ptolemy’s last lunar equation (Planetary Hypotheses 1.1.6 or Rawlins 1996C [www.dioi.org] eq.10):

\[ 3277^a = 3512^v \]  

(Superscripts here & below: u = synodic months, v = anomalistic months, w = draconitic months; g = anomalistic years, y = tropical [Metonic] years, y = sidereal years, K = Kallippic years; d = days, h = hours, m = timeminutes. Degree-remainders merely signify 360ths.)

A2 In Rawlins 1996C (eq.11), we found that tripling eq.1 produced an eclipse cycle:

\[ 9831^u = 10536^v = 10668^w 1/2 + 22^o = 795^g - 65^o = 290315^d 07^h \]  

It was there discovered that this would require eclipse data from no later than the 9th century BC, 84 years before Nabonassar 1 (the long overconfidently assumed 747 BC limit for Babylonian astronomical observations later used by the Greeks). But DR did not then go beyond, since the very idea of Greek access to pre-1000 BC data seemed just too outré.

A3 But since DIO 11.1 (Rawlins 2002B & Rawlins 2002H), we have consistent indications (esp. Rawlins 2002H §§C9&D1) of Greek use of 13th century BC eclipses. So, on 2003/1/26, while walking along Baltimore’s University Parkway, the DIO 11-triggered afterthought (finally . . .) arrived: I’d never checked multiples of eq.1 beyond three; so I swiftly tried out higher ones — and immediately (17:20 EST) found that five times eq.1 hands us the following eclipse-cycle:

\[ 16385^u = 17560^v = 17781^w - 23^o = 1325^g - 109^o = 483858^d 19^h \]

1 See Rawlins 1996C fn 13.

2 See Rawlins 2002B fn 7.

3 In reality, eq.3’s 2nd term was 17560 + 4^o. (See eq.10.) Eq.3 adds yet another eclipse-cycle to DIO’s collection thereof (fully listed at [F7]), which we have reconstructed just out of simple multiples of anciently-attested lunar period-relations. Rawlins 2002H §E3 (or, better, fn 17’s large parenthesis) found it improbable — at a moderately significant level — that so many eclipse-cycles could thus arise merely by accident. However, the fact that eq.1 leads us to more than one eclipse cycle (eqs.2&3) does not increase these odds, since the computation of each cycle’s likelihood (of having possible-ancestor-eclipse-cycles) was found by asking merely for the probability of non-zero potential ancestors.

Figure 1: Showing the relation between $\theta$ and $\phi$. [See §H3.]
H5 Drop a perpendicular QL to the horizontal plane through Giza. Then GL is the ground-level orientation given by the stars. The angle LGN is the deviation from north; let us call it $\theta$.

H6 Because PQ = LN while PG and QG are greater than NG and LG, the angle PGQ is clearly smaller than the angle NGL. That is to say, $\phi < \theta$. By elementary trigonometry, we can find the exact relation between $\theta$ and $\phi$. The angle PGN is the altitude of the pole, which is Giza’s latitude $\gamma$. Then (in Figure 1):

$$\sin \phi = \frac{QP}{GP} = \frac{LN}{GP} = \frac{GN}{GP} = \sin \theta \cos \gamma$$  \hspace{1cm} (1)

H7 An indirect calculation can be done on the celestial sphere (e.g., Figure 2), which is a mathematical fiction devised by the ancient Greeks who had no device like the vector for dealing with directions. It is a large imaginary sphere with its centre at the centre of the Earth. Any direction in space is represented by the point on the sphere in that direction, and angles are represented by arcs of great circles on the sphere. The Greeks could then use spherical trigonometry in their calculations.

H8 Figure 2 illustrates the celestial sphere centred at Giza. G is Giza. GZ is vertically upwards. P is the celestial north pole. K and M are Kochab and Mizar when they are aligned vertically, so the great circle through them goes through Z. Both L and N are on the horizontal plane through Giza. PQ is an arc of the great circle through P perpendicular to plane ZMKL. Then $\phi$ is represented by the arc PQ and this in fact is how Spence quoted it. $\theta$ is the angle LZN.

H9 If we denote the interior angles of the spherical triangle ZQP by $Z$, $Q$, and $P$, and the sides by $z$, $q$, and $p$, we have (by the law of sines): $\sin z/\sin Z = \sin q/\sin Q$. But here $Z = \theta$, $z = \phi$, $Q = 90^\circ$, and $q = 90^\circ - \gamma$. So we have:

$$\sin \phi = \sin \theta \cos \gamma$$  \hspace{1cm} (2)

just as in eq.1.

H10 Since $\phi$ and $\theta$ are tiny, the horizontal arc $\phi'$ from P to ZMKL is very close to the arc PQ.

$$\phi' = \theta \cos \gamma$$  \hspace{1cm} (3)

Rawlins & Pickering used $\phi'$ instead of $\phi$. The difference, if $\theta$ and $\phi$ are small, is negligible. (In case you are interested, if $\gamma = 29^\circ 59'$ and $\theta = 12'$, then the exact formula eq.1 gives $\phi = 0^\circ.1732341$, while the simpler relation eq.3 gives $\phi' = 0^\circ.1732342$.)

H11 As a result, the intervals between pyramids used by Spence [1] have to be divided by $\sec \gamma$ (which varies from 1.155 at Giza to 1.149 at Meidum).

References