‡2 Evidence of an Ecliptical Coordinate Basis in the *Commentary* of Hipparchos

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A Hipparchos' Spherical-trig Slip

A1 The only surviving complete work of the Greek astronomer Hipparchos (2nd century BC) is his *Commentary on Aratos and Eudoxos,* which contains several hundred partial positions of stars. These positions are expressed in a manner that is quite different from modern forms. For example, Hipparchos will state that a star rises at the same time as a certain degree of the ecliptic rises, or that a star sets at the same time as a certain degree of the ecliptic rises. Positions of this type are collectively referred to as "phenomena." The most common phenomena are those where Hipparchos states that a star culminates (i.e., transits the meridian) at the same time as a certain degree of the ecliptic culminates. There are over 200 stars described this way in the *Commentary* — many of them more than once, and frequently with different (conflicting[2]) results. These simultaneous culmination phenomena he called "mid-heaven" phenomena; here, we call these data "polar longitudes."

A2 There is now ample evidence that the Ancient Star Catalog (ASC) preserved in the *Almagest* of Claudius Ptolemy (2nd century AD) was in fact plagiarized[3] from Hipparchos, after adding 2°40' to the longitudes for precession. But contra these evidences, some believe that because the *Commentary* (unlike the ASC) contains no star positions in the standard ecliptical reference frame, that Hipparchos did not take ecliptical positions — and therefore could not have been the true author of the ASC. So if it can be shown that Hipparchos did in fact take ecliptical positions of stars, the case for his authorship of the ASC becomes even stronger.

A3 Not much is known about how Hipparchos obtained the data in the *Commentary*. It has been frequently assumed, on the basis of comments by Ptolemy, that Hipparchos used a celestial globe to chart his star positions, and may also have used the globe to perform spherical coordinate transformations. However, if Hipparchos used an armillary astrolabe to obtain star positions (and even this has been disputed by Neugebauer), a much simpler method is available. After sighting the star through the pinnules of the astrolabe, Hipparchos simply rotates the astrolabe (around its equatorial axis) until the pinnules are in line with the astrolabe's horizon ring. The astrolabe now is in a position that represents the sky at the time of the star's rising (or setting), and Hipparchos can simply read the position of the ecliptic ring against the eastern horizon ring to get the degree of the ecliptic that rises at the same time as the star. Since the ecliptic ring is not directly adjacent to the horizon ring, however, the result necessarily will be rougher than the star's ecliptical position; and indeed, the positions in the *Commentary* are recorded only to the nearest half-degree, about three times less precise than the positions in the ASC.

A4 But there is another possibility: if Hipparchos observed and recorded the stars ecliptically, then the positions in the *Commentary* could have been derived by using spherical trigonometry. This is especially true of the polar longitudes, because while the spherical

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[2] This despite those apologists for Ptolemy who claim that "standard ancient practice" forbade publishing multiple discordant data for a given phenomenon.

[3] Newton 1977, 211-256; Rawlins 1982; Rawlins 1994.

trig conversions for rising and setting phenomena are cumbersome (and differ by latitude), the spherical trig required for polar longitudes is fairly straightforward.

A5 Obviously, a polar longitude is simply a different way of expressing the right ascension α of a star. Converting ecliptically-observed coordinates into a polar longitude is a two-step process: first Hipparchos computes the star's α from its ecliptical latitude and longitude. Then he finds the degree of the ecliptic that has the same α. This second step was probably done tabularly, and need not have been done at the same time as the first step.

A6 Although the suggestion of spherical-trig conversions may seem speculative, there is evidence in the *Commentary* that this is exactly how the polar longitudes were derived, at least in some cases. This is because sometimes, in making these conversions, Hipparchos slipped up. In the second step of the process, Hipparchos is supposed to find the star's α in a table, and read across to find the degree of the ecliptic with the same α. But occasionally, instead using the star's right ascension α for the lookup, he would use the star's ecliptical longitude λ by mistake. Since α and λ usually have similar values, this error is not immediately obvious.

A7 The clearest case in which this error occurred is that of 32 Cygni, which is listed in the ASC at latitude $\beta = 64°30'$, longitude $\lambda = 302°40'$. After subtracting 2°40' for Ptolemy's incorrect precession, Hipparchos' observed longitude becomes $\lambda = 300°$. Hipparchos converted to equatorial coordinates to get $\alpha = 287°$, a value which he notes in his catalog. (Or, following the intelligent suggestion[4] of Grasshoff 1990, this was more likely a working proto-catalog from which both the *Commentary* and the ASC were derived.) But at this point the α is inadvertently dropped. Looking under the wrong column, Hipparchos used 300° (rather than the correct 287°) to look up the degree of the ecliptic needed for the mid-heaven phenomenon, and arrived at 298°, the exact value that appears in the *Commentary*. If Hipparchos had not slipped in step 2, he would have arrived at (after rounding) 285°30' for the polar longitude of 32 Cygni. The resulting twelve degree error is one of the largest of all polar longitudes in the *Commentary*.

A8 Finding all such errors in the *Commentary* turns out to be rather easy. First, we convert the ASC back to the epoch of Hipparchos by subtracting 2°40' from the longitudes. We then use these positions to compute α for each star with a polar longitude in the *Commentary*[5] (using the Hipparchan obliquity of 23°40'), and determine the "correct" polar longitude. We compare our computed polar longitude to the values given in the *Commentary* to determine the true error T. Next, we recompute the polar longitudes assuming Hipparchos actually made the λ-for-α slip described above. We compare our recomputed polar longitudes to the values given in the *Commentary* to determine the "slip-up" error S. If Hipparchos did not make a mistake, error T will be small and error S will be large; but if Hipparchos made the λ-for-α mistake, T will be large and S will be small. Therefore, dividing T by S will produce a very small number if the mistake was not made (which is usually the case), but a large number if the mistake was made. We plot these values in Figure 1 for all 273 polar longitude data in the *Commentary*, representing 203 stars. For convenience, we use the ecliptical longitude as the x-axis, and plot the absolute values of T/S on the y-axis.

A9 While most of the data hug the x-axis, a few discordant stars leap to attention. It is now easy to see that there may be several cases in which Hipparchos made this mistake. To determine if these are just statistical flukes, we need to find the probability that T/S is greater than a given value V. First we find the mean μ and standard deviation σ of all T and all S in the population. These are: for T, $\mu = -0.05, \sigma = 4.95$; and for S, $\mu = 3.72, \sigma = 23.72$. Next, assuming that T and S are normally distributed, we find the probability that T is **outside** of a given range $\pm t$, and multiply by the probability that S

[4] Grasshoff went out on a limb suggesting that Hipparchos' catalog was ecliptical in nature, so it is satisfying to see his courage vindicated here — a sentiment shared by the author (KP) and publisher (DR) of the present paper.

[5] Grasshoff 1990, Appendix C.

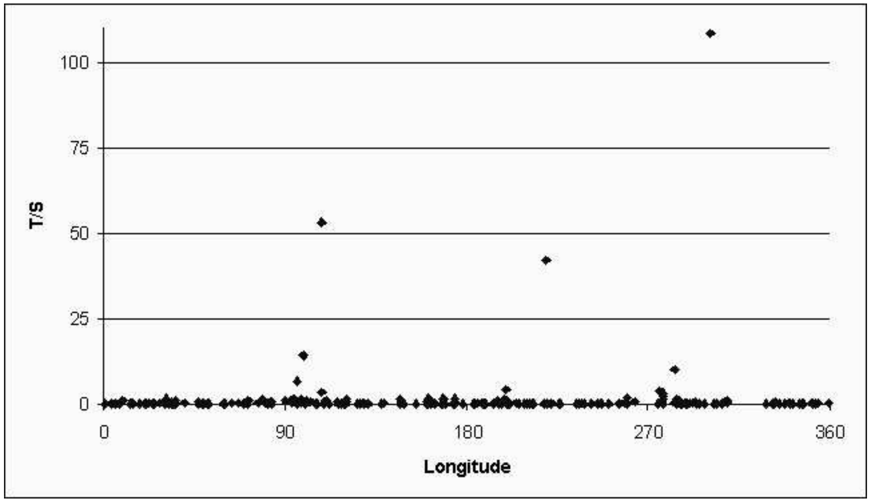

Figure 1: Parameter *T/S* plotted by longitude.

is **inside** the range $\pm t/V$. (As it turns out, in this case the probability is maximized when $t = 3.72$, regardless of V.)

A10 We find that in a population of 273, we would expect to find 1.1 cases where $T/S > 14$ (probability = 4.0×10^{-3}); but the *Commentary* actually has four. And we expect only 0.36 stars where $T/S > 42$ (probability = 1.3×10^{-3}); the *Commentary* has three. This makes it quite likely that these three cases are not random. These cases are 32 Cygni, λ Leonis, and ζ Ophiuchi; the details of each are given in Table 1 below.

A11 It is important to note that there are two requirements for this type of error to occur. First, Hipparchos must have had the ecliptical positions recorded in his proto-catalog. And second, Hipparchos must have converted these positions using spherical trig; this type of error is simply not possible using a globe or an astrolabe as an analog computer.

Table 1. λ-for-α error candidates.

Name	λ	β	α	Apl	Spl	Cpl	T	S	T/S
32 Cyg	300	64.5	287.0	285.7	297.9	298	12.3	0.1	108.3
ζ Oph	219.5	11.83	220.8	223.3	222.0	222	1.3	0.0	42.2
λ Leo	108.5	7.5	111.3	109.6	107.0	107	2.6	0.0	53.2

λ = longitude from the *Almagest* – 2°40'; β = latitude from the *Almagest*; α = RA computed from λ and β, using 23°40' obliquity; Apl = polar longitude computed from α; Spl = polar longitude assuming the slip occurred; Cpl = polar longitude given in *Commentary*; T = absolute value of Apl – Cpl; S = absolute value of Spl – Cpl.

B Theta Geminorum

B1 Another interesting proof of Hipparchos' use of ecliptical coordinates (and of Ptolemy's theft) is the case of θ Gem. This star was observed by Hipparchos with ecliptical coordinates using an ecliptical astrolabe. Hipparchos' originally observed coordinates were very likely $\lambda = 71°$, $\beta = 10°$, a position within a degree of correct in both coordinates, which is not atypical. (The fact that both of these are integers is the dead-giveaway for ecliptical observation.)

B2 But soon after this number was written down, a scribal error was made by Hipparchos or one of his assistants. In ancient Greek, the numeral 1 (uncial alpha) closely resembles the numeral 4 (uncial delta), and this is the most common[6] number confusion in many ancient Greek astronomical texts. So while the longitude 71° was observed at the astrolabe, and was written on an observational note, the number 74° was recorded in the proto-catalog, a huge three-degree error. Since this is several standard deviations above the mean Hipparchan error, it is a firm indication of a scribal slip.

B3 Hipparchos proceeds as before: converting his (incorrect) ecliptical coordinates to right ascension, he gets 71°.2; then he converts a to polar longitude, getting 72°.7, which is also three degrees too high because of the scribal error. He rounds[7] this to the nearest integer as 73°, the number that appears in the *Commentary*.

B4 Then near the end of his career, Hipparchos again read the erroneous 74° position from his proto-catalog and published it in his final star catalog. Three centuries later, Ptolemy adopted Hipparchos' longitude of 74°, added 2°40′, and arrived at 76°40′, the value he published in the *Almagest*. If Ptolemy had really observed θ Gem himself, he could not have gotten a result this bad — not even with repeated observation, the common excuse of Ptolemy's defenders regarding his planetary fabrications. And in Ptolemy's case, no write-1-read-4 scribal slip can be invoked, since his longitude contains neither a 1 nor a 4.

B5 We also know in this case that the polar longitude cannot have been the original observation, in the following way. Suppose that the polar longitude was the original number and the ecliptical coordinates were derived from it. In that case, starting with a polar longitude of 73°, Hipparchos would have gotten a Right Ascension of 71°30′. Combined with the latitude 10°, this would have given an ecliptical longitude of 74°15′ (after rounding to the nearest fraction in the ASC). Ptolemy would have added 2°40′ to this, getting 76°55′, which he would have rounded to 77°: a full 1/3 degree higher than the longitude (76°40′) that actually appears in the *Almagest*.

B6 So the polar longitude can be derived from the ecliptical coordinates, but the reverse is not true: the ecliptical coordinates cannot be derived from the polar longitude. Therefore the ecliptical coordinates must be original, and the polar longitude (which appears in the *Commentary*) must be derived from them.

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[6] Peters & Knobel 1915, 9.

[7] Note that if Hipparchos had rounded to the nearest half- degree, as often occurs in the *Commentary*, he would have gotten a polar longitude of 72°30′. Why he sometimes rounds to halves and sometimes to whole numbers remains a mystery.