Aristarchos & the “Babylonian” System B Month

The Empirical and Calendaric Bases of Ancient Astronomy’s Prime Parameter

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A Derivation from the 345 Year Cycle & Aristarchos’ Great Year

A1 Greek civilization achieved technological superiority over Babylon — even conquering & permanently occupying the city (fn 2 [f]) — ordnag a century before our earliest records of the famous and highly accurate (§A3 [b]) System B “Babylonian” month:

$$M_A = 29^d31^{h}05''08''00'' = 765433^d/25920 = 765433^d/1080$$

(1)

Current orthodoxy has been assuming that eq.1 is due to Babylon. But, using Greek relations (empirical & conventional), the following1 will trace a very few steps of plain arithmetic leading from Greek relations (eqs.6&7) precisely to eq.1. This will be accomplished on the theory that eq.1 was due to the daring Greek astronomer and innovator, Aristarchos of Samos, the earliest scientist to teach heliocentrism widely (long-persisting ancient influences of which are noted at Rawlins 1987 p.238 & nn.34-38, Rawlins 1991W eqs.23&24, & Rawlins 1991P). The Aristarchan connexion can be made more swiftly, fully, & exactly than is possible for any Babylonian-data-based explanation (even though extant Babylonian data outnumber Aristarchan data by a factor of thousands). And we’ll enjoy a series of reality-checks along the way to discovering and independently confirming the origin of this, the most central of all ancient astronomical parameters.

A2 The sole empirical foundation upon which one could firmly base a monthlength as remarkably accurate as eq.1, is the very same one which Ptolemy (from Hipparchos) cites...
1999 (£2 Refs.) (The anomalistic monthlength $V$ determined by eq.2.1 is also quite accurate: good to about 1 timesec.) The monthlength eq.2 provides is (see Copernicus 1543 [4.4]):

$$M_c = 126007.0^{\mu}31 / 102408 = 293.1^{\mu}50.0^{\mu}08^{\mu}09'09'' \ldots$$ (3)

In Aristarchos’ era, the interval between an eclipse and its 345-ago partner was always within about an hour of 126007.01 with an rms scatter (around a value a few tenths of an hour higher) of less than half a hour (0.4)`. So the error in eqs.2&3 was merely 1 part in ordmag 10 million.

A4 The Kallippic yearlength (see fn 5), basis of the Aristarchos circle’s Dionysios calendar (van der Waerden 1984-5, DIO 1.1 [111 fn 23]), was Julian before Caesar:

$$Y_K \equiv 365^{\mu}1/4$$ (4)

(“Question-in-passing: why launch the Dionysios calendar at all — since Kallippos’ calendar had the same yearlength. Speculative answer: the Dionysios calendar incorporated additionally Aristarchos’ typically large-scale-visionary Great-Year [§A7, fn 14]. He was famous in antiquity for proposing the largest universe, too: see Archimedes’ Sandreckoner.”)

A5 Eqs.3&4 determine the number of Kallippic years in each M-based 223-month saros (where we will henceforth use superscript K for Kallippic years):

$$223M_c \approx 18^{\mu}10^{\mu}40^{\mu}00^{\mu}3$$ (5)

By contrast to the Aristarchos cycle’s smooth dovetailing: had we used Meton’s monthlength in eq.5, the remainder would have come out as 10^{\mu}57’2—computing analogously for Kallippos, it would’ve been 10^{\mu}43’2’. the Geminios (18:3-6) 19756 exeligmos produces 10^{\mu}41 ’. The relative ordmag fits of these $M$ to either eq.3 or eq.6 (or eq.5): Meton 10^{\mu}3’2, Kallippos 10^{\mu}5’, Geminios 10^{\mu}6’; whereas the agreement of eq.5 (empirical) with eq.6 (Aristarchos) is 10^{\mu}5’. (For Meton’s & Kallippos’ monthlengths, see Rawlins 1991H fn 1.)

NB: Aristarchos eq.6 agrees with later “Babylonian” eq.1 to 1-part-in-76 million! in a deceptively round-looking & conveniently compact fashion:

$$S = 223M = 18^{\mu}10^{\mu}2/3 = 18^{\mu}10^{\mu}4 + 1/35 = 4868^{\mu}270$$ (6)

Thus, Aristarchos’ happy realization that deceptively crude-looking eq.6 was extremely accurate (a super-trivial rounding of empirical eq.5 [from eqs.3&4]) made possible his 4868Great Year cycle (eq.7): the prime factor 1217 (1/4 of 4868) is embedded right in eq.6.

A8 Once his Great Year was established, he needed to fit the monthlength $M$ into his vast Great Year cycle, and so computed the $M$ implied by eq.7’s $GY$. Suppose he made the clever choice to perform the division of $GY$ by 223 first, rounding the result Greek-wise (nearest timestimel): 793.0^{\mu}15’. (Apt precision: relative accuracy-degradation under 10^{-7}.) In then performing that division of the rounded result by the flagrantly sexagesimalmesacres cycle of about 270, he ensured a neatly-terminating expression for $MA$ (analog: DIO 1.2 §R12): $M = (1778037^{\mu}223)/270 \approx 793.0^{\mu}15’/270 = 293.1^{\mu}50.0^{\mu}08^{\mu}09'' = MA$ (8)

[But was eq.1 brother not son to eq.6? Despite fn 2 & p.3 fn 1, we admirably note Britton’s simple alternate (contra-Ptolemy&DR) route via degree-day division (Babylon convention [‘A’s is not known]) from eq.3 or eq.6 to $M = 293.191'00'49''$ (or 49’’), 00’00” = MA 1.]

A9 This (eq.8) is just the eq.1 we wished at the outset to explain. As Aristarchos could easily post-verify: this value of cycle-convenient $MA$ agreed with empirical eq.3 to 1 part in 36 million (an ordmag better than eq.3’s accuracy).

A10 Though a sometime critic of Ptolemy, I am here in the happy position of essentially vindicating his Almajest 4.2 assertion (repeatedly attacked since Copernicus), unaware that eq.6, not eq.2, is the more immediate source of eq.1.

A11 Reality-check: eq.5’s remainder is astonishingly close to that in Aristarchos’ (§A7; fn 14). He was highly astute in ‘proposing the largest universe, too: see Archimedes’ Sandreckoner.’

A12 [Note added 2003. Rawlins 1985K (and the 1st edition of this note) proposed the possibility that $MA$ is not known] from eq.3 or eq.6 to $M = 293.191'00'49''$ (or 48’’), 00’00” = eq.1.]
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1978022\textsuperscript{d}/4868 = 365\textsuperscript{1/4} - 15/4868 \tag{11}

In continued-fraction format, eq.11 could be written with its distinguishing number rendered sexagesimally: \[365\textsuperscript{d} + \frac{1}{4 + \frac{2}{20 + \frac{60}{20}}} \tag{12}\]

Now, it happens that the Vatican holds two rare and precious Greek ancient year-length-lists (Neugebauer 1975 p.601, cited from Vat. gr. 191 fol. 170\textsuperscript{v} & Vat. gr. 381 fol. 163\textsuperscript{v}) which express various ancient astronomers’ year-lengths as continued-fractions. Two listings are given for Aristarchos. One is obviously sidereal and so would not apply here. The other is:

Aristarchos of Samos: [365] 1/4 \textk 3 \textp 4 = [365] 1/4 20\textsuperscript{v} 60 2' \tag{13}

The remarkable match of eq.13 to eq.12 provides a gratifying extra reality-check on the foregoing proceedings.

It might be thought that eqs.11&12 do not flow specially from eq.1 because proximate eq.2 would produce the same result. Note: eq.2 would (if put into eq.10) yield 1778021\textsuperscript{d} 1\texti 33\textp, thus a remainder (in eq.11) of -16/4868, disagreeing with the Vatican ms’s Aristarchos data (eq.13): a final, discrimination-test reality-check, again revealing Aristarchos’ pre-System B possession of the precise “Babylonian” month.

[13] According to a common modern style, this would be written 20/02. Britton has suggested interpreting the “tropical” numbers in eq.13 as 365\textsuperscript{1/4}[1/20 + 1/20] = 365\textsuperscript{1/4} - 31/10052 instead of eq.12. (Though note: the idea that 60 in eq.13 indicated sixtieths did not originate with DR but with Neugebauer 1975 p.602.) This would destroy the appearance of the Aristarchan number 4868 in eq.11 and thus imply that eq.11’s display of 4868 is just a spookily spectacular coincidence. But Britton’s interpretation would (via eq.9) actually move \textm \textz closer than ever to eq.1.

[14] These lists were long regarded as mysterious gibberish (Neugebauer 1975 p.602) — until 1980, when the HHA-suppressed Rawlins 1999 treated them (sample analysis at fn 15) as consisting of continued-fraction expressions. (Notably, both ms list Greek yearlength values chronologically-prior to Babylonian values.) [Was the origin of the Greeks’ highly useful fascination with cont’d fractions related to ancients’ wide but only moderately-utilitarian use of unit fractions?] This analysis revealed both sidereal and tropical years listed under Aristarchos. (Thus, as emphasized in Rawlins 1999, Aristarchos had pre-Hipparchan knowledge of precession: indicating that, reasonably enough, the first public geomobilist was also first to perceive the Earth’s precessional wobble.) These induced yearlengths exhibited two characteristic numbers: sidereal, 152 (fn 15); “tropical”, 4868 (eqs.11-13), respectively. Both numbers are well-known to be Aristarchan: his solstice was 152\texti after Meton’s (\textj B1) and his \textg was 4868\texti long. (Note: 4868/32 = 152 1/8 [Rawlins 1999 \textb7 bracket].) There are other suggestions at Rawlins 1996C \textd of his personal contribution [directly attested anywhere at Almajest 4.2] to the evolution of 345\texti-cycle-based lunisolar theory. Note: given the chronology-ambiguities implied in \textj \textd below, it is possible that some details of what we have here been attributing to Aristarchos actually had a few debts (positive or negative; examples of latter: Rawlins 1991W \textl78 & R12) to Hipparchos’ investigations, adjustments, and/or transmissions.

C Comments on the Aristarchan Evidence

Several contemporary scholars have tried to portray Aristarchos’ pioneering heliocentrism as a virtually valueless passing-blip on the ancient screen. But the opinions of Archimedes (op cit) & Apollonios (DIO 11.14 \textf7) show that Aristarchos’ bold originality was highly regarded by the brightest ancients. However, a few historians, in anticipation of the current paper (before even seeing its evidence), have suggested there’s too little Aristarchan material to make much of. Actually, the ultra-lean evidential situation for Aristarchos only increases the strength of this paper: I can’t be suspect of picking among a large Aristarchan corpus to select only the data agreeing with my thesis, since there is so much solar corpus to filter. Those who wish to reject the paper’s proposals will now have the burden (which I have repeatedly laid on them in other contexts as well — see, e.g., the next Aristarchos-based Hipparchos-transmitted heliocentrist eqs.23&24 in Rawlins 1991W) of convincingly explaining how such mega-odds ts can meaninglessly emerge from such micro-slim materials.

References


[17] There is no question that Hipparchos used Aristarchan data: see Almajest 3.1. And the famous -15/300 remainder in Hipparchos’ canonical yearlength is actually a rounding from a cycle of not 300\texti but 304\texti (Heath 1913 p.297, Rawlins 1985H, Rawlins 1999 fn 17), perhaps of peculiarly Aristarchic origin by a route revealed in a note appended (at p.42) to DIO’s 2001 reprinting (see www.dioi.org) of Rawlins 1999: “The number of years between Meton’s famous bedrock –431 Summer Solstice [the epoch of the Metonic calendar] and the Hipparchos [solar] Ultimate-Obite epoch –127 Autumn Equinox [Rawlins 1991H eq.28: the only equinoctial ancient calendaric epoch] is 304\texti. This number is exactly one sixteenth the number of years in the ‘Great Year’ [eq.7] of Aristarchos. That is, the [the even-even interval seen in 4868/16] – on the nose.” (See in Hipparchos epoch-interval we found in \textb7 that the appearance of 15 instead of 16 in eq.11 was virtually a toss-up; it seems that Hipparchos opted for the 16, thereby adopting a 304 yr calendar that was simultaneously 1/16 of Aristarchos’ cycle [Heath loc cit] 16 times Meton’s. Note: in the 1 1/2 centuries between Meton’s 19 yr cycle and Aristarchos’ 4868 yr Great Year, the growth of Greek astronomers’ cycles reflected an outdoing of scientists’ temporal vision, up by a factor of hundreds: specifically for Meton-to-Aristarchos: about 256 [16-squared or 4-to-the-4th], paralleling a huge expansion too of man’s spatial conception of the universe, also initiated by Aristarchos: \textj A4, Rawlins 1991P fn 11, & Rawlins 1982G fn 284.