Table of Contents Page:

Back Issues

Over the last few years, numerous library subscribers have inquired about the availability of back issues, in order to render their *DIO* collections as complete as possible. We have now established procedures to get this promptly accomplished. So, libraries desiring back issues may simply email to dioi@mail.com, specifying the *DIO* issues desired and the address of the serials dep't that will receive them. If a not-yet-subscribing library wishes a complete run to date, that is OK, too. In either case, there is no charge for issues or postage.

The present *DIO 14* may be read online at http://www.dioi.org/vols/we0.pdf, whence it may be downloaded gratis; likewise, the booklet version of it and all other published *DIO* issues may be found via http://www.dioi.org/bk.htm; readers preferring instant hard copy can have any of these printed-stapled-trimmed for trivial cost at a local photocopy shop.

News Notes

From the *International Herald Tribune* 2008/1/12-13 p.1 obit for Edmund Hillary, 1953/5/29 co-conquerer of Mt.Everest: "In the annals of great heroic exploits, the conquest of Mt.Everest by Hillary[2] and [Tenzing] Norgay ranks with the first trek to the South Pole by Roald Amundsen in 1911 and the first solo nonstop trans-Atlantic flight by Charles Lindbergh in 1927." In the era B.D. (Before *DIO*) this would instead surely have read: Peary-N.Pole & Lindbergh-Atlantic. Popular history takes far too long to reach accurate equilibrium. And all-too-often never does. But we may savour justice at it blossoms.

[Note added 2008 Dec.] *DIO*'s newest winners of its $1000 R.R.Newton Award for Scientific History are S.Albers & G.Graßhoff, honoring their originality and fruitfulness.

Steve Albers was first to propose (*Sky & Telescope* 1979 March) the ingenious notion of investigating the ms records of earlier-era astronomers who had searched for satellites of any known planet at times when it had been near conjunction with then-unknown planets — in order to find out whether the latter had been accidentally recorded as possible satellites. Albers' nomination for this *DIO* prize was due to the R.R.Newton Award Committee's Charles Kowal, who (with Stillman Drake) had in 1980 taken up Albers' suggestion and made the sensational discovery of Galileo's 1612-1613 observations of planet Neptune. The remarkable 1980 history will be told by Kowal for the first time in *DIO* volume 15. (Before the committee existed, the DR-selected winner of the first RRN Award was Kowal himself, for this very discovery.)

Gerd Graßhoff's 1986 University of Hamburg thesis (Springer Verlag 1990) was based upon his novel, ultimately successful experiment (which no one [including R.Newton & DR] had thought of) to detect mass-statistical correlations between the hundreds of star-positions in Hipparchos' *Commentary* & Ptolemy's *Almajest* star catalog. This was a crucial contribution to eventual conversion of what had seemed a needlessly ever-undead controversy into a genuinely dead one — a valued rarity in cemental-cult-infested academe.

---

[1] [Note added 2009.] An 1165 AD report (*Proc. Brit. Acad. 19*:277-292 [1933] pp.280&282-283) has the Pharos-flame-replacement mosque's base 31+15+4 = 50 fathoms high, or (contra *PBA* conversion) 300 ft. (Six ft = 1 fathom ≡ outstretched hands' tip-to-tip span, one of the least infirm ancient measuring units.) The most detailed eyewitness Pharos image we have (late 1st century AD Alexandria) shows like proportions: see inset in ‡1 Fig.1. Of oldest few extant Pharos-height reports (Thiersch 1909 p.66 & *PBA*), most are in the range 300-to-306 units. For oldest of all, see InductionQuake at p.12 within.

[2] When in 1999 the body of 1924 Everest-challenger George Mallory was found 2000 ft below Everest's summit, the question arose: was he going up? — or coming down, after attaining the top? Hillary responded by opining that no conquest should count unless the conquerer returned to base. Hmmmm. And just where would that leave Brit ultra-polarhero Rob't Scott?

‡1   Eratosthenes' Too-Big Earth & Too-Tiny Universe

Sostratos-Pharos Science Behind Alexandria-Aswan Myth
Lighthouse Flame Height Exactly 300 Feet = HalfStade
Ultimate Geocentrist's Sun Smaller Than the Earth
High-Precision Ancient Science Doubly Verified

> **Dedicated to the Memory of**
> **Our Irreplaceable Friend**
> **HUGH THURSTON**
> **1922/3/28-2006/10/29**

A   Big-Science Dawn: Sostratos' Pharos, 1st Precise Earth-Measurer

A1   Over 22 centuries after Eratosthenes' legendary Earth-measure, newly-mined ancient sources finally permit arrival at the non-astronomical truth behind the most famous of ancient geographical tales, the long-suspect myth of his 600-mile-travel to compare (§§A4[a]&D3) the Sun's noon altitude at Alexandria vs Aswan. The actual method instead used hometown measures of the height & night-visibility-distance of the Alexandria Lighthouse designed by Ptolemy II's architect Sostratos, which explains the result being too high by a factor of 6/5 (eq.28), just the error (§B3) expected from air's bending of horizontal sealevel light. [This paper — and the issue's cover — were revised in 2013 to clarify Sostratos' primacy.]

A2   Rawlins 1982N (p.217 & n.26) discussed two easy stay-at-home methods which would account for the overlargeness of Eratosthenes' Earth-size, one being: measure how far over the sea a known-height lighthouse is visible at night. (Near-attestation at §A4[c].) But neither DR nor anyone else noted the coincidence that the tallest lighthouse in the world *debuted right at Eratosthenes' time&place, 3rd century BC Alexandria* (§D5) — the "Pharos" (Greek for "lighthouse"), 2nd most durable of the ancient 7 Wonders of the World, surviving for 1 1/2 millennia. until ultimately falling to earthquakes and their aftershocks.

A3   With this glimmer of where we're headed, we now plunge into solving the entire Eratosthenes Earth-measure mystery: method, place, all his data (terrestrial and celestial), and we even develop (§I) the 1st credible (if quite speculative [at least until p.12's finale]) figure ever modernly proposed for the precise height of the Pharos itself. Further, we find (§F) that royals-catering Eratosthenes was a geocentrist who rejected obvious visual counter-indicia, to promulgate the anthrocentric delusion that the Earth is appreciably bigger than the Sun. Finally, it will be shown (§K2) that air-bending ("atmospheric refraction") of horizontal light explains **both** of the equally erroneous but extremely disparate (fn 8) ancient standard Earth-sizes (Eratosthenes & Poseidonios) within c.1% in each case (§K4).

A4   Before beginning, it's best to recall the four options available for ancient Earth-measurement, and each's respective atmospherically-induced error:

[a] Kleo Method: compare Summer-solstitial noon Sun's measured altitude at different latitudes. This is the still-famous and oft-repeated "Eratosthenes" method (Rawlins 1982G), given at Kleomedes 1.10 for Alexandria vs Aswan, which will variously show (§D3, §G2, §H2, fn 7) is perfectly mythical. Negligible inherent error (§K2) for the cited cities.

[b] Mountain Method: measure the sea-horizon's angular "dip" (below 90° from zenith), as seen from a mountain peak of known height. (Rawlins 1982N App.A.) Error factor 6/5.

[c] Pharos Method: measure how far out to sea a lighthouse of known height is visible at sealevel. (*Ibid.*) Error factor 6/5. Report of similar ancient experimentation: Pliny 2.164.

[d] Sunset Method: measure difference in sea-horizon sunset-times for two known heights above sealevel. (Rawlins *loc cit* & 1979; www.dioi.org/cot.htm#bsqq.) Error factor 5/6.

Summarizing the respective methods' errors: c.0%, +20%, +20%, −17%.

(All these errors would be appreciably weaker for great heights' thinner air: fn 1.)
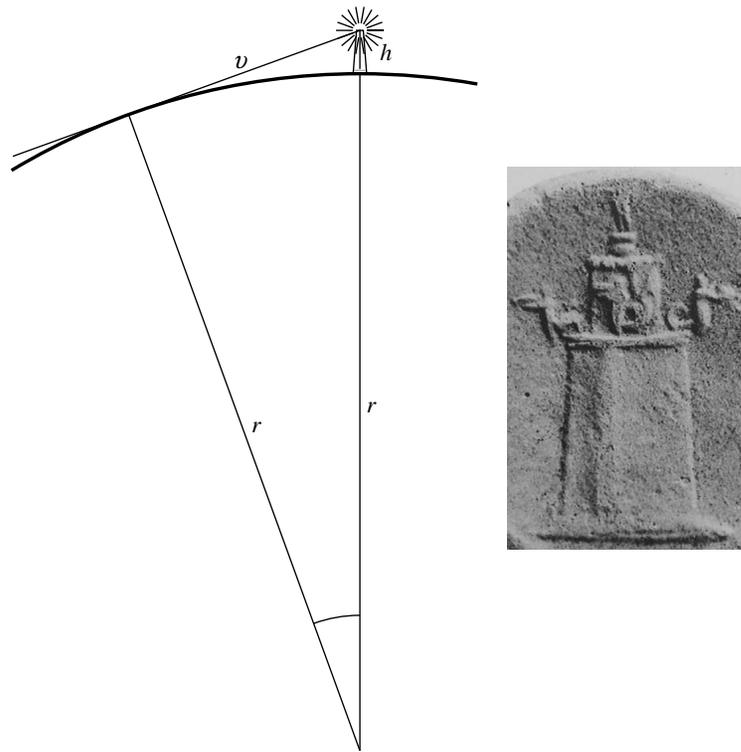
Figure 1: Pharos of height h, sealevel-visible at distance v, over Earth of radius r. (Height h [and thus v] greatly exaggerated for illustrative clarity.) The actual Pharos was constructed in 3 diminishing (as one ascended) sections, of square, octagonal, & circular cross-section, crowned by light & Poseidon statue. Inset image from Alexandria coin minted under Domitian 90-95 AD. (H.Thiersch *Pharos* 1909 pp.v&7, Tabl.I #10, Tabl.III #130.)

B   Lighthouse Math

B1   The math of the Pharos Method is so easy that it doesn't even require a diagram, though we supply Fig.1 anyway. At whatever distant point the Pharos' flame starts (due to Earth-curvature) becoming invisible to a receding observer on the sea, is where the Pharos' light-rays skim (are tangent to) the sea. Let v be this observer's distance from the Pharos, and r his distance from the Earth's center, while the Pharos' flame is $r + h$ from that center — h being the Pharos' height and r the ideally-spherical Earth's radius. At the observer's position, it is obvious that the angle between the skimming-light-ray vector and the Earth-radius vector is a right angle.

B2   Assuming an airless Earth (which permits straight-line light-rays), we can use Pythagoras' Theorem:

$$v^2 + r^2 = (r + h)^2 = r^2 + 2rh + h^2 \qquad (1)$$

Cancelling r^2 from each side and dropping relatively trivial h^2, we have the naïve Airless Lighthouse Equation which ancients would have used to determine Earth-radius r:

$$r = v^2/2h \qquad (2)$$

B3   But to find the *Real* Lighthouse Equation (based on Earth-with-atmosphere) at sealevel, one must account for horizontal atmospheric refraction, which stretches v artificially by the square root of 6/5 since horizontal light is bent with curvature equal to 1/6 of the Earth's curvature (S.Newcomb 1906 pp.198-203) so v^2 in eq.2 is augmented by factor 6/5, producing an Earth-radius high by 20%. (Curvature is defined as inverse of radius.) To return the problem to the straight-ray Pythagorean math behind eq.2 requires undoing the effect of the ray's curvature. Ancients may have suspected atmospheric refraction (‡2 fn 56), but no evidence for quantitative corrections exist until Tycho (c.1600 AD). Since the radius-estimate an ancient scientist would compute (via good Pharos-Method data) would be high by factor 1.2, the Real-Earth Lighthouse Equation is (using eq.2):

$$R = r/1.2 = v^2/2.4h \qquad (3)$$

— from which one can get an accurate estimate of the Earth's real radius R, instead of the 20%-exaggerated r one would get from the ancients' refraction-innocent eq.2.

B4   Rawlins 1979 applied very similar elementary straight-ray math & diagram to the §A4[d] Sunset Method of Earth-measure. (Though that method's resulting Earth-radius is low by factor 5/6, from air-refraction.) The pre-refraction-correction math of the §A4[b] Mountain Method (result high by 6/5, like the §A4[c] Pharos Method) is much the same.[1]

B5   Application of the Pharos Method would have been particularly simple because the shore along the Alexandria region is straight enough that one would not need to bother with ships: v could've been found by simply wheel-odometering the distance along the shore (checking by triangulation) until the Pharos light was no longer visible. The Pharos' height h was knowable via trig or by measuring ropes hung from flame, to successive sections, to sea; though, as suggested below (§I1), the exact height was probably already known.

B6   K.Pickering notes that on the nearly-linear coast just west of Alexandria, at distance c.20 nmi, the Pharos (slightly off said coast) is seen over the sea at azimuth c.40°. In this direction, the R corresponding to the sea's real curvature can be shown to be 6371 km = 3440 nautical (geographical) miles = 3959 statute mi $\doteq$ 34400 stades, so we take this as the effective value of R in the discussions below, where we use the standard 185 m Greek stade (embedded in all our fits, which thereby confirm conventional opinion [§J1] on the stade).

C   Pharos' Approximate Height

C1   Josephus *J.War* 4.613 says the flame of the Pharos was visible to ships for 300 stades (obviously a round figure for v), which would by eq.3 make it the world's then-tallest building (exceeding the Great Pyramid); yet it was never so described. Solution to Josephus' datum: the crow's-nests of tall ancient ships were roughly 1/4 of the Pharos' height, meaning (eq.3) that approximately 1/3 of Josephus' 300 stades was due to ship-height; so $v \approx$ 200 stades is an adequate rough estimate for the Pharos' visibility-distance v at sealevel.

---

[1] While seeking an explanation of Eratosthenes' result, DR has in recent years been inexplicably distracted by the §A4[b] Mountain Method. (Thurston 2002 p.66 evidenced better memory and sense.) Yet it is obviously inferior (to the §A4[c] Pharos Method): it involves measuring a small angle — and the 1% precision of agreement with Eratosthenes' actual Earth-radius would require $1'$ measuring accuracy under difficult seeing conditions. (Also, the great height required to get an angle large enough to render observer-error negligible would lead to weakening of refraction due to decreased atmospheric density-gradient, yet the error in C_N is closely [§I3] consistent with virtually full-strength sealevel refraction.) Advantageously, the Pharos Method does not even get involved with angles at all, and the requisite relative precision is attained with ease. Note: the Mountain Method would lead to two-significant-digit results; the Pharos Method, three. So the very fact that Eratosthenes expressed his Earth-radius to three (eq.13) provides yet another indication that it was based on the Pharos Method.

C2   Thus eq.3 gives us a pretty good idea of the Lighthouse's height h_L:

$$h_L = v^2/2.4R \doteq 200^2/(2.4 \cdot 34400) \doteq 0.48 \text{ stade} \approx 1/2 \text{ stade} \approx 90m \qquad (4)$$

D   Eusebios Bequeaths Us Eratosthenes' Exact Earth-Radius

D1   Eusebios, Bishop of Caesarea-Palestine, is most remembered for leaving us his invaluable *Ecclesiastical History* of the Christian church at its time of triumph.
D2   We will henceforth also owe him for the long cast-aside, here vindicated clue relayed in his *Præparatio Evangelica*, which unlocks the full truth behind the most enduring of ancient geographical legends, Eratosthenes' measurement of the Earth. The key data (Eusebios PE 15.53):[2] Eratosthenes had the Moon 780000 stades distant; and the Sun, 4080000 stades. We formally list these two Eratosthenes distances:

$$M_E = 780000 \text{ stades} \qquad (5)$$

$$S_E = 4080000 \text{ stades} \qquad (6)$$

D3   The traditional Eratosthenes Earth-circumference C_K is based upon the famous §A4[a] Kleo "experiment" (Kleomedes 1.10): Summer Solstice Apparent Noon Sun's zenith distance ($90°$ minus altitude h) was 1/50 of a circle at Alexandria but null at Aswan-Elephantine (very near Tropic of Cancer) where legend had vertical sunshine reaching well-bottom (though see Rawlins 1985G p.258) — 2 cities 5000 stades apart in latitude. (*NB*: Kleomedes 1.10 doesn't say that the 5000 stade distance was measured, merely calling it a "premis".) So:

$$C_K = 50 \cdot 5000 \text{ stades} = 250000 \text{ stades} \qquad (7)$$

If one checks this vs the Bishop Eusebios-reported solar distance S_E, we find ratio p_{BK}:

$$p_{BK} = 2\pi S_E/C_K \doteq 103 \qquad (8)$$

much too unround a number, given ancient convention (‡2 fn 37) of using powers of 10 for loosely-determined distances. (This habit is the earliest historical evidence for use of order-of-magnitude [ordmag] estimation of that which is too uncertain for more exact gauging. In this tradition, Poseidonios made the solar distance 10000 Earth-radii: ‡2 §F2 eq.15.) If we instead adopt the Eratosthenes circumference $C_G = 252000$ stades (which he'd presumably [vs fn 6] adjusted slightly for geographical convenience to a round ratio of 700 stades per great circle degree: Strabo 2.5.7), a fresh check instead produces ratio p_{BG}:

$$p_{BG} = 2\pi S_E/C_G \doteq 102 \qquad (9)$$

but this is also unacceptably non-round.
D4   However, years ago, DR analysed the Nile Map which Strabo 17.1.2 attributes to Eratosthenes, and showed (Rawlins 1982N p.212) that the underlying measure was

$$C_N = 256000 \text{ stades} \qquad (10)$$

When we check this vs Eusebios' $S_E = 4080000$ stades (eq.6), the Sun/Earth-radius ratio p_{BN} provides a pleasant shock, as we begin our realization that C_N unleashes the long-dormant Eusebios data-treasure of eqs.5&6:

$$p_{BN} = 2\pi S_E/C_N \doteq 100.1 \qquad (11)$$

---

[2]See www.tertullian.org/fathers/eusebius_pe_15_book15.htm or H.Diels *Doxographi Graeci* Berlin 1879 pp.362-363. Eq.6's S_E is so startlingly small (entailing a Sun smaller than Earth: eq.16) that Heath 1913 p.340 just can't believe it. Such inertia has prevented entertainment of the hypothesis (§F3) that pol's-pol Eratosthenes found it advantageous (& healthy: ‡2 fn 69) to be a geocentrist's-geocentrist.

D5   *This is a hit that carries us right into the heart of the Earth-measure mystery*. The obvious conclusion from eqs.6&11 is that Eratosthenes had the Sun's distance equal to 100 Earth-radii, so

$$S_E = 100r_E \qquad (12)$$

$$r_E = 40800 \text{ stades} \qquad (13)$$

the only 3-significant-digit Eratosthenes figure for the Earth's size directly based on empirical data. (Compare eq.13 to eq.7.) *All pre-Pharos C were 1-significant-digit-rough*: 400000 stades (Aristotle c.350 BC), 300000 stades (Dikaearchos c.300 BC). Yet (§I1) after the Pharos' debut, we find ordmag 100 times greater precision in 3-significant-digit eq.13.

E   Eratosthenes' Moon

E1   While placing the Sun 100 Earth-radii distant, far short of Aristarchos' solar distance, Eratosthenes nonetheless adopted the farcical lunar distance of pseudo-Aristarchos,[3] 19 Earth-radii (Heath 1913 pp.339 & 350; but see ‡2 §C5), as eq.13 verifies:

$$M_E = 19r_E = 775200 \text{ stades} \doteq 780000 \text{ stades} \qquad (14)$$

which matches[4] eq.5, Eusebios' report. (The match is far better than that figured at Heath 1913 p.340, where $2\pi/19$ is divided into the hitherto-conventional Eratosthenes $C = 252000$ stades, yielding about 760000 stades.)
E2   But if we try recovering the lunar distance from the Nile Map C_N (eq.10):

$$19C_N/2\pi = 774130 \text{ stades} \doteq 770000 \text{ stades} \qquad (15)$$

we find that it does not check with eq.5.
E3   Comparison of eq.15 to eq.14 begins a linchpin realization: *Eratosthenes' root measurement was Earth-radius, not Earth-circumference*. The historical import of this revelation will become evident below (§G2).

F   Eratosthenes' Sun

F1   Remarkably, Eratosthenes had the Moon's distance almost 1/5 of the Sun's — which goes counter to easy visual checks, since if his 19:100 ratio were true, half-Moons would occur with the Moon more than $10°$ from quadrature. (Arcsin $0.19 \doteq 11°$.)

---

[3] DR has long contended (‡2 §C1 etc) that Aristarchos' supposed ms "On the Sizes and Distances of the Sun & Moon" is not truly his but is by an uncomprehending pedant (follower, detractor, distractor?), since the work is vitiated by an error of *a factor of four* (mis-step's amateurish origin explained at ‡2 §C1), leading to a $2°$-wide Moon and thus (‡2 §C5) a $4°$ wide Earth-shadow at the Moon, which would imply central lunar eclipses' Entirety (partiality-start to partiality-end) lasting *half a day*, with c.4[h] Totality (durations too high by factors of about 3 and 2, respectively). Pseudo-A's 19[e] lunar distance required the Moon to visibly retrograde daily, and this joke-astronomy became the royally approved lunar theory in the Alexandria that elevated Eratosthenes to top academic. (Full incredible details below at ‡2 §C.) Eratosthenes' adoption of this way-too-low lunar distance (vs DR's reconstruction of c.60 Earth-radii for Aristarchos: ‡2 §C11) suggests that the acceptance of pseudo-Aristarchos' work as genuinely Aristarchos' goes way back. (It also suggests little comprehension by Eratosthenes of his lunar distance's two most ludicrous implications, as just remarked. Perhaps lunar parallax was not recognized by some scholars of the 3rd century BC, though it is obvious that Hipparchos had parallax tables only a century later: Rawlins 1991W fn 288.) Note that, by contrast with Eratosthenes (and modern scholars), Archimedes didn't fall for any of pseudo-Aristarchos' bizarre astronomy: ‡2 fn 33.

[4] A lunar distance of 19[e] implies $3°$ Earth semi-diameter as seen from the Moon, which itself was anciently gauged as having semi-diameter $1°/4$ as seen from the Earth; that is, seen at the same distance, the Moon has merely 1/12 the Earth's angular sd. Thus (by the same symmetry argument we'll use at §F3), the Moon's radius is 1/12 the Earth's so (in adopting pseudo-Aristarchos' lunar distance of 19 Earth-radii: ‡2 §C5) Eratosthenes had the Earth's volume about $12^3 \approx 1700$ of the Moon's!

F2   This bizarrity seems less likely to be the result of observation than of patch-work synthesis: melding two distances from two distinct sources, regardless of compatibility. A possible trigger: the Sun's size shrank for ascientific reasons (royally-oily Eratosthenes was a fave of the Ptolemies' theocratic Serapic regime: Rawlins 1982G p.265), the Sun's greater size having been a likely spark to the proscribed heliocentrist heresy.

F3   From Eratosthenes' 100 Earth-radii solar distance (eq.12), we see that the Earth's angular semi-diameter as seen from the Sun would be $180°/100\pi = 0°.573$, while the semi-diameter of the Sun (seen from the same 100 Earth-radii distance) was pretty accurately estimated (‡2 §C1) to be $0°.25$. Therefore, the implicit solar size s in Earth-volumes is:

$$s = (0°.25/0°.573)^3 \doteq 1/12 \qquad (16)$$

So Eratosthenes was pretending that the Sun was 12 times smaller than the Earth![5] Such cosmology doubtless delighted (and offered justifying comfort to) gov't-catering geocentrist priests, whose anti-progressive view of the universe dominated the world by force for millennia, until modern times. This discovery widens our basis for appreciating how Eratosthenes climbed to academic eminence in Ptolemaic Alexandria, promoting a cozy universe trillions of times smaller than that already proposed by Aristarchos of Samos. (See ‡2 fn 33 & §H1.)

G   Eratosthenes' Earth

G1   The Nile Map's Earth-size is now confirmed by congruence (eqs.5-14) with Eusebios' numbers, so we ask how well the map's underlying C_N (eq.10) generates the radius:

$$C_N/2\pi = 256000/2\pi \doteq 40700 \text{ stades} \neq r_E \qquad (17)$$

— no match. But the reverse process does create a match to eq.10. Starting from eq.13:

$$2\pi r_E = 2\pi \cdot 40800 \text{ stades} \doteq 256000 \text{ stades} = C_N \qquad (18)$$

This contrast (eq.17 vs eq.18) confirms the §E3 finding, so that we now have double-evidence that Eratosthenes' *radius generated his circumference* C_N, not the reverse.

G2   What is the significance of this priority? Simple: it kills the legend that Eratosthenes got the size of the Earth by the famous Kleo Method (based on measuring the distance from Alexandria to Aswan: §A4[a]), because that method's math (eq.7) *produces circumference*. By contrast, the Pharos Method (§A4[c]) directly *yields the Earth's radius*: eq.2. Thus, the clear implication of the radius' computational priority is that the Pharos Method (not the Kleo Method) was that actually used by Eratosthenes or his source to find the Earth's size. (The Kleo Method's untenability will be independently confirmed below: §K2 & fn 7.)

H   Inventing the "Experiment"

H1   As noted at Rawlins 1982N n.10, Eratosthenes was possibly unsure of whether the Mediterranean Sea's curvature matched the world's. If so then (*ibid* p.216) he may have unwittingly based his 5000 stade supposed-meridian (Alexandria-to-Aswan) & his

---

[5] Note Sun-shrinker Eratosthenes' Scylla-Charybdis narrows: bringing the Sun near enough to make it smaller than Earth, while putting the Moon not too close to the Sun (thereby inflating ‡2 eq.4's γ) but not too close to the Earth, since that would entail huge daily lunar parallactic retrogrades. (A contended Macrobius passage has Eratosthenes' Sun 27 times Earth's size: I.Kidd 1988 p.454. Did Macrobius invert the ratio? If the math of §F3 used smaller solar sd (Heath 1913 p.312-314), perhaps also rounding π to 3, then the computed Earth/Sun radii-ratio could be ≈ 3, the cube of which is 27.)

C_K ultimately upon use by another scholar (see, e.g, §I1) of the very method he questioned. It is also possible that he knew where the basic measurement came from and himself concocted the famous "experiment" as a useful illustration even though it was actually founded upon a rounding of C_N (eq.10), as titularly noted by Rawlins 1982N — and while doing so found that a round distance of 5000 stades would nearly dovetail r_E with his (defective: Rawlins 1982G n.19) gnomon observation of the solstitial Sun's culmination zenith distance, $7°12'1/2$ (*ibid* n.20 & Table 3), the rounding of which to $7°1/5 = 360°/50$ became the purported basis of his ultimately canonical $C_K = 250000$ stades.[6]

H2   Instead of walking 5000 stades or 500 nautical miles (nmi), the actual Earth-measurer walked merely (eq.4) c.200 stades or 20 nmi. Eratosthenes' "experiment" was just an *indoor* theoretical exercise whose C was swiped from Sostratos' prior *outdoor* Pharos scheme, a grab self-exposed by its preservation of the lighthouse-method's 20% systematic error from unremoved atm refraction (vs 0% for Eratosthenes' alleged method) which is thus indicated as unquantified in Sostratos' era. Had he known of (corrective) eq.3, he would have found

$$R = r_E/1.2 = 40800 \text{ stades}/1.2 = 34000 \text{ stades} \qquad (19)$$

close to the truth (§B6), 34400 stades. For naïve eq.2, perfect data would've given (§B3)

$$r = 1.2 \cdot 34400 \text{ stades} \doteq 41300 \text{ stades} \qquad (20)$$

The discrepancy with eq.13 is merely 1%, on the order of naturally occurring variations in eq.20's 1.2 factor. So the ancient mystery of Eratosthenes' C has a solution.

I   Pharos' Height: Chosen for Sostratos' Public Science Experiment?

I1   We next launch a speculative ('til eq.24) attempt at finding the Pharos' exact h. (The following reconstruction of precise v originated subsequent to §C's rough estimate of it.) The Pharos was a pioneering, literally-superlative civic-science project. So: was its height h a proud world-lighthouse-record round number of Greek feet? (Greek foot $\doteq 12''1/7$ English.) We already have evidence (§C2) that $h_L \approx 1/2$ stade, so was the Lighthouse *deliberately* constructed to be 300 Greek feet high, the flame **exactly** (vs eq.4's roughly) 1/2 stade above sealevel? — ***thereby DISAPPEARING eq.2's denominator*** (a streamlining possible only because Sostratos has-it-in-stades), as eq.2's $r = v^2/2h_L$ becomes simply:

$$\boxed{r = v^2} \qquad (21)$$

So anyone could find the Earth's radius r in stades, just by pacing v in stades and squaring it. The massive metal ring in Alexandria's Square Stoa was a public-science equinox-detector (*Alm* 3.1), so could the sailor-beacon Pharos have doubled as a huge round-Earth-measure public-demo science experiment (as the Empire State Building originally doubled as a dirigible-dock)? Was such a neat idea planned (c.270 BC, the Museum's apogee: ‡2 fn 33) by Pharos-builder Sostratos & fellow scientists, who thus should (§A2) have found $r = 40800$ stades (eq.24) before Eratosthenes? Our speculation isn't disconfirmed if 40800 is consistent with the square of a 3-digit integral v: there is only a 25% *a priori* probability that the 1/2-stade-Pharos-height theory will meet this condition. If Sostratos' r_E were, say, 40600 or 40700 or 40900 stades, our eq.21 speculation would collapse. But, rooting r_E:

$$\sqrt{40800} = 201.99 \qquad (22)$$

---

[6] Once the 5000 stades baseline led (eq.7) to $C_K = 250000$ stades, it is possible that the question of parallax was raised. Parallax correction for an Alexandria S.Solstice culminating Sun at $100r$ would shave 1% off the zenith distance and thus add 1% to the circumference, yielding c.252500 stades or (rounding low) 252000 stades (700 stades/degree) which offers an alternate explanation (vs §D3) for the origin of that famous value. If $7°12'1/2$ was not rounded to $7°1/5$, then $C = (5000 \text{ stades}) \cdot 360°/7°12'1/2 = 249711$ stades. Adding 1% yields 252208 stades $\doteq 252000$ stades.

I.e., the 1/2-stade-high-Pharos theory survives. So, using it, we'll compute out a determination of r on the assumption that Eratosthenes' measured (§B5) sealevel Pharos-visibility distance v was

$$v = 202 \text{ stades} \qquad (23)$$

(Not far from the crude §C1 estimate used in eq.4.)

I2   When these values are substituted into eq.2 (or eq.21), the result is:

$$r_{\text{E}} = v^2/2h_{\text{L}} = (202 \text{ stades})^2/(2 \cdot 1/2 \text{ stade}) = 40804 \text{ stades} \doteq 40800 \text{ stades} \quad (24)$$

which neatly matches the Sostratos-Eratosthenes radius (eq.13).

I3   To illustrate the accuracy of the work behind Sostratos-Eratosthenes' value, we check via eq.3, using the real Earth-radius $R = 34400$ stades of §B6, and (somewhat over-ideally taking the equation's 1.2 factor as exact) find that a perfect Pharos Experiment for a 1/2-stade Lighthouse would have measured $v = 203$ stades. Not only does this (compared to eq.23) evidence the care of the Greek scientists who performed the necessary measurements, but it also reminds us that (because v is squared in eqs.2&3) the relative error in the ancient experimenters' resultant r is about double that of v, so that their finding an Earth-radius 19% high (vs 20% high expected) shows experimental error of not 1% but roughly half that. NB: This point is independent of the 1/2-stade Pharos theory, and applies also to the Sunset Method (§A4[d]), whose resulting C_{P} (eqs.26&28) likewise depends upon the square of the crucial measurement. (Inverse-square of time-interval between sunsets in that instance. See Rawlins 1979.) In any case, since the 1.2 factor is not rigidly precise, the proper conclusion is that the two widely adopted ancient Earth-measures, Eratosthenes' ($r_{\text{E}} = 40800$ stades: eq.13) and Poseidonios' ($C_{\text{P}} = 180000$ stades: eq.26), are so close (eq.28) to the values expected from the Pharos and Sunset experiments, respectively, that we can regard both tiny discrepancies as *within experimental noise* (§H2).

I4   So the matches for both famous ancient Earth-size values provide as precise a validation as one could reasonably require, for the sea-horizon-refraction theory of the values' origins. They are thus a spectacular refutation of & rebuke to the ubiquitous modern cult that has misled generations of young scholars into accepting the fantasy that ancient science was unempirical: see, e.g., ‡2 §§A1, A6, B3, & *especially* the priceless gem at ‡2 fn 20.

J   Playing-Accordion with the Stade

J1   There has been a long tradition of attempting to force agreement of the Eratosthenes and Poseidonios values with each other and with reality by arguing for whatever stade-size would make-E&P-right. But it is encouraging to report that this sort of manipulation is no longer taken seriously by most specialists. Dicks, Neugebauer, Berggren, & Jones never fell for it. [Engels 1985 mashes it.]) Amusing details of testimony-twisting (used to carry out such programmes) are exposed at Rawlins 1982N App.B and Rawlins 1996C fn 47.

J2   Eqs.24-28's matches gut not only the credibility of stade-juggling-for-Eratosthenes but even (‡3 fn 13) the very *need* for it. [Note added 2013. Despite the good sense of top scholars, eminent forums&books [& Wikipedia] are the prime promoters of such folly, while popular sources (Webster's & Baedeker) correctly adopt the 185m stade.]

J3   Lack of serious instability in the Hellenistic stade is also detectable from Ptolemy's geographical evolution. In the 18th century, Pascal Gossellin 1790 noted that the macro-geographical longitude errors of Ptolemy's *Geography* (*GD*) showed exaggerations of 30%-40%. Rawlins 1985G p.264 used least-squares analyses to find the mean exaggeration (factor 1.36 ± 0.04) and explained this as the result of switching Earth-sizes.

J4   In the *Almajest* Ptolemy was under Hipparchos' influence, so he presumably adopted his C which was (Strabo 2.5.34) Eratosthenes' C_{G} (§D3). When Ptolemy switched (‡3 fn 13 & §L3) to C_{P} (eq.26) for his later *GD*, he obviously used travellers' east-west distance-estimates more than astronomically based longitudes and thus (in order to switch

his great-circle scale from 700 stades/degree to 500 stades/degree) had to stretch degree-longitude-differences between cities. So the *Almajest* longitude-degree distance from Rome to Babylon was increased by over 30% (‡3 fn 13), nearly the ratio of the prime Earth-sizes, plain evidence that the stade was a constant in the midst of geographical transformation.

K   How Atmospheric Refraction Fruitfully Explains *BOTH* Standard Ancient Earth-Size Estimates' Precise Errors

K1   As noted at §A4 & §B4, atmospheric refraction makes the §A4[d] Sunset Method of Earth-measure (Rawlins 1979) give a result low by factor 5/6. Since the actual circumference of the Earth is virtually by definition 21600 nautical miles (a nmi is now defined as exactly 1852m, nearly identical to $1'$ of great-circle measure on the Earth's globe), then given that a stade (185m) is almost exactly 1/10 of a nmi, we know the Earth's real circumference is:

$$C_{\text{o}} = 216000 \text{ stades} \qquad (25)$$

(600 stades/degree). The Poseidonios value (Strabo 2.2.2) of the Earth's circumference (which could appear only after the 2nd century BC advent of sph trig: Rawlins 1979) was

$$C_{\text{P}} = 180000 \text{ stades} \qquad (26)$$

(500 stades/degree), which agrees exactly with the §A4[d]-predicted Sunset Method's -17% error; and we have doubly found (eqs.10&18) Eratosthenes' empirical circumference

$$C_{\text{N}} = 256000 \text{ stades} \qquad (27)$$

(711 stades/degree), the $+19\%$ error of which is almost perfectly consistent with the §A4[c]-predicted Pharos Method's $+20\%$ error.

K2   While the Kleo Method (eq.7) should lead to a nearly correct circumference-estimate (for the method's near-zenith solar altitudes, refraction would be trivial), the two actual standard ancient values for the Earth's circumference are 6/5 high and 5/6 low, thus eliminating the Kleo Method right off the top — which backs up[7] our earlier elimination of it through a different approach (§G2). When we check ratios of theory and testimony, we find virtually exact hits on the horizonal-light-ray atmospheric-refraction hypothesis' 6/5 factor, for the sources of *both* attested standard C:

$$C_{\text{N}}/C_{\text{o}} = 256000/216000 = 5.93/5 \qquad C_{\text{o}}/C_{\text{P}} = 216000/180000 = 6.00/5 \quad (28)$$

which shows how dramatically successful the refraction theory has proven[8] — an ideal example of a fruitful theory, it uses the *same* mechanism (horizontal atmospheric refraction) and the *same* stade (standard 185m) to near-perfectly explain *both* of the only two widely adopted ancient Earth-size estimates. (NB: Rawlins 1996C fn 47.) Oddly, the spat attending ancients' huge shift from C_{G} to C_{P} is only scantily attested: Strabo 1.3.11 & 1.4.1.

---

[7] Other problems for accepting the Aswan-Alex tale's reality: Since the Nile is far from straight, how would one reliably measure the length of a path (really c.10% less than 500 nmi) which could not have been direct without highly arduous and dangerous travel over desert? Also, Eratosthenes placed (Kleomedes 1.10) Aswan due south of Alexandria (see also Rawlins 1982N), though travel straight from Alexandria to Aswan would have to be knowingly steered 20° east of south to hit Aswan. Finally: if the Kleo Method were actually carried out (across awful Egyptian terrain) over a N-S straight line, it would get an accurate result. (More than 1000^y later the experiment was actually done [elsewhere], successfully.) [Did an ordmag 1000-stade Nile-parallel version occur c.300 BC? See *DIO 20* ‡1 n.2.]

[8] For those who cannot immediately see why the two methods yield such different results (one over 40% higher than the other!): see *DIO 2.3* §8 §A, where extreme examples easily illustrate why one method leads to a too-high result and the other to a too-low result. (The Mountain Method is examined there instead of the Pharos Method, but the atmosphere's effect on each is similar for low mt-height.) That is, if Earth's sealevel atmosphere-density gradient were high enough, horizontal Pharos-light-rays' curvature could be the same as Earth's, so (for null extinction) the Pharos would be visible no matter how far away one receded, and this infinite v would (by eq.2) make computed $r = \infty$: a flat Earth. For the same dense atmosphere, the Sunset Method would yield $r = 0$ (*DIO loc cit*; Rawlins 1979 eq.13).

K3   But given the cascade of startling new matches above [& at this page's end], little doubt can remain that the ***unattested*** Pharos & Sunset Methods underlay the only 2 standard ancient Earth-sizes, C_G & C_P, resp. Which tells us what has often been shown in these pages (see, e.g., fn 9, ‡2 fn 38, ‡3 §A3): much of high ancient science has been lost & so is *only recoverable by reconstruction*, a finding unsurprising to most of us, yet which nonetheless eludes induction-challenged chauvinists who whenever convenient will (*DIO 11.1* p.3 & ‡2 fn 7) pretend that they cannot accept anything without extant textual explanation.

K4   But even more important than such details is the implicit general message contained in the foregoing precise vindication of the atmospheric theory that coherently explains the 2 ancient Earth-measures: the fact that both agree with the theory *to one percent* (§I3 & eq.28) overturns[9] the long-persistent delusion (§I4; ‡2 §A1, fnn 20&31) that the Greeks were mere theorists with little interest in or capacity for empirical science. *DIO* has been contending otherwise since its 1991 inception, arguing that this "blanket libel of ancient scientists" (*DIO 1.1* ‡1 fn 24) is false — and obviously so, to those possessing a genuine acquaintance with the way scientists think and work. We hope that the present paper will help diffuse a more appreciative view of the priorities, ingenuity, and perfectionism of those ancient Greek pioneers who laid the baserock-beginnings of high-precision science.

References

Almajest. Compiled Ptolemy c.160 AD. Eds: Manitius 1912-3; Toomer 1984.
Donald Engels 1985. American Journal of Philology *106.5*:298.
Eusebios PE. *Præparatio Evangelica* c.310 AD. Ed: E.Gifford 1903.
Pascal Gossellin 1790. *Géographie des Grecs Analysée* . . . , Paris.
Thos.Heath 1913. *Aristarchus of Samos*, Oxford U.
Josephus. *Jewish War* c.79 AD. Ed: H.Thackeray, LCL 1927-8.
Ian G. Kidd 1988. *Posidonius: the Commentary*, Cambridge U.
Kleomedes. *Motu circulari* c.370 AD. Ed: H.Ziegler, Leipzig 1891.
Karl Manitius 1912-3, Ed. *Handbuch der Astronomie* [*Almajest*], Leipzig.
S.Newcomb 1906. *Compendium of Spherical Astronomy*, NYC.
Pliny the Elder. *Natural History* 77 AD. Ed: H.Rackham, LCL 1938-62.
D.Rawlins 1979. American Journal of Physics *47*:126.
D.Rawlins 1982G. Isis *73*:259.
D.Rawlins 1982N. ArchiveHistExactSci *26*:211.
D.Rawlins 1985G. Vistas in Astronomy *28*:255.
D.Rawlins 1991W. DIO-J.HA *1.2-3* ‡9.
D.Rawlins 1996C. DIO-J.HA *6* ‡1.
Strabo. *Geography* c.20 AD. Ed: Horace Jones, LCL 1917-1932.
Hugh Thurston 2002. Isis *93.1*:58.
Gerald Toomer 1984, Ed. *Ptolemy's Almagest*, NYC.

InductionQuake AfterShock

This paper was 1st posted and referees alerted on 2008/3/12. But on 2008/3/15, DR happened upon the obscure sole extant ancient estimate of the Pharos' height h: 306 fathoms (Steph.Byz 735a [1825 ed. 3:1251]; Strabo [H.Jones] 8:24n), taller than any building ever. Unless Greek feet were meant. If so, h is within 2% of our eq.21, and $v = 204$ stades. But it's suggestive that 306 & 40800 are both unround by factor 1.02. Did a later scholar try estimating h by putting $r_G = (252000 \text{ stades})/2\pi \doteq 40000$ stades (Neugebauer 1975 p.654) and $v = 202$ stades (eq.23) into eq.2 to find $h \doteq 0.51$ stades $= 306$ ft? Regardless, after years of exaggerations, we now have double evidence for a conservative estimate:

$$\text{Pharos flame's height } h_L = 93\text{m} \pm 1\text{m}$$

_____

[9] Such achievements as eclipse-cycle determination (‡2 §F9) of all three of the Greek lunar periods (to a precision of one part in ordmag at least a million) might've triggered parallel enlightenment.