This DIO is dedicated to the ever-treasured memory of Charlie Kowal (1940-2011) — irreproducible genius, celestial & historical discoverer, brave & principled friend.

§1 Archimedes’ Hidden Measure in Degrees — Sunsize Disguise: His Solar Diameter

Hellenistic Astronomers’ High-Empiricism Confirmed by Accurate Solar Brackets
Babylonian Degree-Measure Already Greek-Adopted by 3rd Century BC

A Summary

DIO has recently discovered that Archimedes’ Sandreckoner estimate of the Sun’s diameter — which has the surface look of a crude, pedant-conventional unit-fraction range — was in truth a professional-level empirical measure, expressed instead in the sexagesimal convention of 3rd century BC astronomers: 30° ± 3'. Accurate. And couched within reasonably-cautious — and correct — uncertainty-limits. Ultimate resolution below at §E.

B Did 3rd Century BC Greek Scientists Use Degrees? Dating Strabo’s Nile Map

B1 In recent years, the superficially ambiguous evidence regarding when the Hellenistic astronomical tradition adopted Babylon’s sexagesimal measurement of angles in degrees, arcmin, etc., has led several able, prominent scholars (including sometime-Princetitutees) to doubt or cite doubt that 3rd century BC Greek astronomers used degrees (Dicks 1966 n.15; Jones 1991M n.5; B. Goldstein & Bowen 1991 pp.103-105; van Brummelen 2009 p.33 n.2).

B2 By contrast, DIO has repeatedly pointed out (e.g., Rawlins 1991W fn 53, Rawlins 1994L fn 41, & Rawlins 2008R fn 24) the probability of degrees’ use by said astronomers.

B3 But, ironically, DR’s own Rawlins 1982N paper perhaps added to the confusion since it showed that the Eratosthenes Nile Map relayed by Strabo 17.1.2 used (instead of degrees) successive halvings of circle-fractions. The exact date of the map is not known; however, the map obviously (Rawlins 2008Q eq.11) post-dates the Alexandria Lighthouse — and pre-dates the end of Eratosthenes’ career. Which sandwiches the map into the period c.270-200 BC, dating it for the 1st time. The Nile Map’s unit was a Pharos-based (Rawlins 2008Q) Earth-radius probably due to the Alexandria Lighthouse’s architect, Sostratos:

Sostratos Earth Circumference $C_S = 256000$ stades (1) which is 19% too high — but for Pharos-based Earth-measure, we expect 20% excess due to air’s bending of horizontal light-rays (to a curvature equal to 1/6 of that of the Earth’s surface), so the tight match evidences high precision (half-percent: ibid §13) empirical measurement by Greek scientists (ibid §K4), Details at op cit.

1 The Nile Map is based upon successive halvings of 7°1/2. Note parallel at §3 fn 16.
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B4 Eq.1’s $C_S$ was ultimately adopted by Eratosthenes but manipulated to (Strabo 2.5.7):

Eratosthenes Earth circumference $C_E = 252000$ stades (2) — the canonical “Eratosthenes” circumference, his slight alteration (less than 2%) perhaps effected to arrange an even multiple of 360, so that for Eratosthenes

$1^\circ = 700$ stades (3)
suggestive2 of Greek scientists’ adoption of the degree at least by the mid-3rd century BC.

C Earlier Evidence

C1 When we began looking for evidence bearing on the era when Greeks started precisely measuring celestial coordinates in degrees, the earliestdatable clue encountered was Ptolemy’s collection of 18 stellar declinations — 12 by Timocharis & 6 by Aristylls — observed in Alexandria c.300 BC & c.260 BC, resp (Rawlins 1994L, Table 3) and all expressed in degrees at Almajest 7.3. Particularly striking is the uniform rounding by Aristylls of all six of his star declinations to $1^\circ/4$ precision: a conscientiously cautious practice — perhaps intended to avoid erroneously reporting slightly discrepant empirical results, but which may (Standish 1997 DR Comm. §G12) have cost him discovery of precession, an honor which (Rawlins 1999 §D5) should instead go to his contemporary, Aristarchos. The obvious question regarding Aristylls: if his declinations were originally reported in some other measure than degrees, how likely is it that, after hypothetical subsequent transcription (into the degree-values reported by Ptolemy at Almajest 7.3), all six data would2 end up exhibiting consistency with quarter-degree rounding?

2 Early Greeks divided Earth-circumference $C$ into 60 parts (Strabo 2.5.7 & Neugebauer 1975 p.590 n.2), not 360, nor the public-treatise custom of quadrant-unit-fractions (Aristarchos&Archimedes: idem or our eq.6). [Note added 2014/4/29&2017/7/1.] So was the stade defined sexagesimally? Like our nautical mile ($C/21600$)? Or the meter ($C/4x10^3$)? Greeks expressed fractions sexagesimally. Was the formerly-unsteady, locally-varying stade imperially regularized c.300 BC as $1/60^\circ$ of $1/60^\circ$ of $1/60^\circ$ of $1^\circ$? Above-attested $1/60^\circ$: cascade’s determinant step 1. Our proposed integral unit-fraction in modern sexagesimal notation: 1 stade $\equiv$ PRECISELY $0^\circ:00:00,01$ (thus $1^\circ \equiv 600$ stades). At Egypt, latitudinal Earth-curve’s $C \approx 39,900$ km: $C/60^\circ = 184.7$ m = standard Alexandria 185 m stade, independently-validated at Rawlins 2008Q §K2. Pharos shows the early Ptolemaic empire’s enthusiasm for vast projects. Did an ordnance-1000-mile version of the unsullied but low-systematic-error Kleo-Method (Rawlins 2008Q §A4; GeoDir. 1.3.2-3), and/or a royal surveyors’ project, find the equivalent of correct $C = 39900000$ m, thus 1 stade $\equiv C/21600 = 185$ m, well before Sostratos and Eratosthenes got clever with measurement by the Pharos’ flame? See www.dioi.org/cot.htm#csqm.

3 We can test independently whether Aristylls (c.260 BC) used $60^\circ$th (fn 2) instead of $360^\circ$th of a circle for his share of the only 18 Greek declinations surviving from the 3rd century BC: those 6 in the north quarter of the sky. Try his $\zeta$ UMa declination, where Almajest 7.3 reports $\delta = 67^\circ 1/4$, which — in $60^\circ$th — exactly translates to $11^\circ5/24$ (not credible) or $11^\circ12^1/2$ (too precise, compared to the simplicity of $67^\circ 1/4$). Testing instead $11^\circ12^1/2 \& 11^\circ13^1/2$, we find that they would result in $67^\circ 1/5$ (or $1/6$) & $67^\circ 1/3$, respectively — not $67^\circ 1/4$. So this star alone eliminates the theory that Aristylls used degree-$60^\circ$th. B.Goldstein & Bowen 1991 pp.103-105 suggest he could have used two-degree “cubits” or degrees — or half-degree “points” or half-degree “Moon-breaths”. But the differences between these measures are too trivial to regard as generic. Also: [a] two-degree cubits (ibid p.104) would require overprecise $13 1/2 \& 11 1/2$ for Archimedes’ solar brackets; and even more unlikely: 33 5/8 for Aristylls’ $\zeta$ UMa declination. [b] Ancient mention of points is later than of degrees. [c] As for Moon-breaths (1/2º): these were just visual yard-sticks for eyeball-observers who lacked the rigged astrolabe — irrelevant to Aristylls’ transit-instrument observations. Further: Moon-breaths are also Sun-breaths. Wouldn’t it seem a mere superfluous for Archimedes to announce that he had empirically measured the Sun’s width to be one Sun-width? (And: how do you call a quantity equal to itself plus-or-minus 10%?)

C2 The next point coming to our attention (noted at Rawlins 2008R fn 24) was Aristarchos’ record: his $1/1720$ of a circle for solar-diameter (eq.7; Rawlins 2008R eq.3), which is $1/2$; his empirical estimate (his Hypothesis 4) that half-Moon elongation was $1/30$ of a RtAng from quadrature (ibid eq.4 or Heath 1913 pp.352-353), which is $3^\circ$; and Ptolemy’s mention (Almajest 4.2) that in the Aristarchos luni-solar scheme (Rawlins 2002A), the saros’ excess over $18^\circ$ was $10^\circ2/3$ — that is, $32^\circ$ excess for the $54^\circ$ exeligmos — an integrality which led to DIO’s reconstruction (idem) of the origin of Aristarchos’ monthlength:

$M_A = 29^d3^h50^m08^s20^"/200^" = 765433^d/25920 = 765433^d/1080$ (4)

(falsely labelled by hist.astron’s political-centrists the “Babylonian” month though unattested in Babylon c.200 BC), based upon the 4267-month eclipse-cycle (as correctly reported by Ptolemy at Almajest 4.2), a value accurate to a fraction of a timesec then and now. (For other viewpoints, e.g., Swerdlov 1980 & Engelson 2006A.) That’s one part in several million, and it’s based upon degrees not only in the expressions for saros & exeligmos but — as pointed out to DR by John Britton and John Steele — in the key rounding (Rawlins 2008Q §A8) that produces the precise degree-expression of Aristarchos’ monthlength in its original form:

$M_A = 29^d3^h19^m00^s50^"/200^"$ (5)

which later became equivalently expressed in the sexagesimal format (eq.4) we know from Almajest 4.2.

D Archimedes the Astronomer

D1 Archimedes is not usually seen as astronomer but as combo of mathematician and arms-designer. Yet the latter career could hardly have occurred without a scientist’s knowledge, drive, and thought-habits.

D2 Almajest 3.1 quotes Hipparchos’ testimony that he & Archimedes observed solstices to an accuracy no worse than $1^\circ/4$, so we know that Archimedes had 1st hand outdoor experience in solar work.

D3 His measure of the Sun’s diameter (to be analysed in what follows) gives flesh to that supposition, as well as providing a prime example of ancient scientific writers using circle-fractions for publication of empirical data actually measured in degrees, a more familiar example of which is Eratosthenes’ description of the Earth’s obliquity as 11/83 of a semi-circle, when (Aristarchos 1.12) 23 51/4° ± 1°/4 was the actual (precise but inaccurate: Rawlins 1982G eq.9) mean measurement by asymmetric (unfortunately) gnomon.

D4 Archimedes’ report (Archimedes p.224) is that the Sun’s angular diameter $d_0$ is between $1/200^\circ$ and $1/164^\circ$ of a quadrant — a right angle or $90^\circ$. Which can be expressed thusly:

Archimedes: $\text{RtAng}/200 < d_0 < \text{RtAng}/164$ (6)

What can a description of such oddity (§E1) be telling us?4

4However, one must be taught BY evidence rather than teaching TO it. E.g., Shapiro 1975 p.77 long ago realized that exact conversion of Archimedes’ brackets equaled 27° & 32°/56°. But he then spurned the discovery-opportunity here by neglecting to ponder: [a] the latter angle’s glaring nearness to 33°, [b] the pair-average’s nearness to Aristarchos’ $1/2$ (reported by Archimedes in the same opus under examination), or [c] the wisdom of computing to 4 places using a 3 significant-digit number! (The same naiveté affected even Delambre 1817 [1:104], but that was back in an era when scientists were insensitive to significant-digits.) So Shapiro didn’t calculate in reverse by simply (§E1) hypothesizing & checking to see whether $5400/33^\circ$ rounded to 164 or to a different integer. He instead swiftly concluded by just echoing establishment dogma:

“the degree was, of course, not a unit used by Archimedes”. (If anyone among our readers knows of an earlier analyst who realized the sexagesimal truth behind Archimedes’ solar brackets, please inform us so that we may add a citation here in this issue’s next printing.)
E Archimedes’ RtAngle-Unit-Fractions: His Solar Diameter Solved

E1 Since Archimedes’ predecessors, Aristarchos (c.280 BC) & Aristyllos (c.260 BC), & likely Timocharis (c.290 BC) measured astronomical angles in degrees (see fn 3 & Rawlins 2008R fn 24), let us investigate eq.6 by the hypothesis that it expresses an empirical range, originally in degrees. Archimedes is our sole reliable witness to Aristarchos’ solar diameter \( d_\odot \), making it \( 1/720 \) of a circle or a half-degree (§C2; Archimedes p.223):

\[
\text{Aristarchos: } d_\odot = 30' \tag{7}
\]

In eq.6, the number 164 is peculiar (prime factors: 2 & 2 & 41, which lead nowhere), so we are inspired to dig beneath the surface. Noting additionally that 200 and 164 differ by c.20%, we try the following hypothesis for explaining eq.6:

\[
\text{Archimedes: } d_\odot = 30' \pm 10% = 30' \pm 3' \quad \text{or} \quad 27' < d_\odot < 33' \tag{8}
\]

To test for confirmation we convert eq.8 into not only circle-fraction format (§D3) but specifically into satisfaction of another ancient schoolbook convention, *unit-fractions* (inverse-integers — as displayed in eq.6); we find the unit-fractions’ denominators by dividing \( 27' \) & \( 33' \) successively into \( \text{RtAng} = 5400' \), yielding 200 & 163 resp, which (after integralization of the latter) produces:

\[
\text{Archimedes: } \text{RtAng}/200 < d_\odot < \text{RtAng}/164 \tag{9}
\]

— the very Archimedes bracket-expression (eq.6) we’d set out to trace the origin of. Being the least ambiguous of all entries in the list of evidences for 3\(^{rd}\) century BC use of degrees, our finding now takes its place at the head of the list. And, with respect to Archimedes-as-astronomer (§D), eq.8 is absolutely accurate: the Sun never strayed outside limits 31’-33’, so not only was his solar diameter correct but his brackets were judiciously applied.

E2 From fn 2, eq.8, & Rawlins 2008R fn 24, we see that, starting from dividing Earth \( C \) into 60 parts c.300 BC, Greek science transitioned to degrees early in the 3\(^{rd}\) century BC.

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