‡8   AIR-ERRing ERatosthenes

by
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Two very different ancient Greek estimates of the Earth's circumference were widely adopted in antiquity. Eratosthenes and Hipparchos held for roughly 250000 stades, a stade being a tenth of a nautical mile. This value dominated for about two centuries. Later, Poseidonios and Ptolemy championed 180000 stades, which became standard among geographers for 1000^y. The ever-wheelspinning debate over what went wrong with these estimates turns out to have a very simple and neatly-precise long-available resolution.

Previous investigators have narrowly pursued strictly a metrological key to the problem: manipulating the stade's definition — without a satisfactory resolution. For 259^y now. Shock: for those 259^y ***metrologists have been barking&chasing up the wrong rabbit-hole***. The true tri-fit solution presented below is generically distinct: **just air's bending of light**.

A   Two Hugely Disparate Ancient Earth-Measures

A1   Most of us have encountered the oldest of legendary astronomical measurements — which we are about to see (§B4 below) isn't astronomical at all — the 1st precise estimate of the Earth's circumference C, by the 3rd century BC Alexandrian Greek, Eratosthenes of Kyrene. He contended it was about 250000 stades — actually nearer 256000 stades (§B3 below) — supposedly from solar observations at Alexandria and Aswan. Though dead for over 2000^y (suiciding c.195 BC) Eratosthenes today has inspired enough fans to envyize a rock-star, his apologist-army regularly launching article after article after article — Eratosthenes-Reconsidered-Reevaluated-Revigorated-Reconstructed-Redux-Reverberated — invariably trying to alibi why his circumference C was about 6/5 too high, just as invariably arguing *by pure attestationless speculation* (ever-disguised as solid ancient reality) that Eratosthenes' C just *seemed* inaccurate, only because he had adopted a stade-length much smaller than the anciently standard 185 meters, so his C was actually correct within a few percent. Unfortunately for ever-loyal Eratosthenians, most scholarly opinion and unambiguous evidence (§B3) puts the Greek stade at 185 meters, which makes Eratosthenes' 256000 stades about 19% too high.

A2   But that overestimate is rigidiously excused by an eternal Eratosthenian cult, invincibly certain that its hero musta used a runty stade c.157 meters long, though (D.Shcheglov *Isis 107.4* p.698; 2016) there's zero ancient evidence of any kind for that value. Its advocates also ignore the inconvenient fact that there were not one but two widely adopted ancient Earth-circumferences. The other was 180000 stades, 17% too low, which ultimately displaced Eratosthenes' earlier value *and was considered standard far longer*: over 1000^y. It even convinced Columbus the Earth was so small one could reach Ptolemy's Kattigara (Saigon: www.dioi.org/vols/w50.pdf, fnn 64&68, Table 25) quicker going west than east. The 180000-stade estimate is thought to be originally from Poseidonios, 1st century BC, and was adopted by Ptolemy for his millennium-dominant 2nd century AD *Geography*.

A3   Though for over 2 1/2 centuries (D.Engels *Am.J.Philology 106*:298-311 p.299), hundreds of articles have undeterrably tried to explain the ancient Earth-size mystery by fiddling with the stade, three problems prevent the issue from ever being thusly resolved:

[1] Eratosthenes' 256000 stades is over ***forty percent higher*** than Poseidonios' 180000.

[2] Assuming any stade-length that makes Eratosthenes closer to the truth simultaneously puts Poseidonios farther therefrom. And vice-versa.

[3] All such solutions fit only one of the 3 quantities in play (& that not necessarily to 1%): defying the near-universally accepted 185 meter stade, while fitting at best only one of the two standard ancient Earth-size estimates.

A4   These difficulties suggest that we look outside of metrology for a solution that fits all three. Fortunately, such has been available in the professional — and even popular — scientific literature for nearly 40^y, and it is not metrological but physical.

B   Triple-Fit Physical Solution

B1   It is commonly thought that Eratosthenes' C was obtained by desert travel between Alexandria&Aswan, combined with Sun sights at each city. But that would have produced a correct value. So, some believe that his C's origin lies elsewhere, in cleverer and less laborious stay-at-home methods. Could the "Pharos", Alexandria's legendary Lighthouse (2nd-most enduring of the Seven Wonders of the World), have been used for the purpose? That obvious possiblity is suggested by the double-coincidence that it was built **at the very time AND place** of Eratosthenes' Earth-size estimate. (Similar potentially productive space&time confluences: ‡7 §B #5, and Rawlins *Peary . . . Fiction?* 1973 pp.262-263.)

B2   As shown in 2008's *DIO 14* (www.dioi.org/vols/we0.pdf, p.2 fn 1 & ‡1 p.12), the Pharos' height h = c.300 feet. If its designer Sostratos wanted a world-record lighthouse-tallness of *exactly* 300 feet, that would equal half a stade. The equation for determining Earth-radius r, from the Pharos-flame's visibility distance v, is $r = v^2/2h$ (*ibid* eq.2), so $h = 1/2$ stade renders the equation's denominator $= 1$, reducing the equation to simply $r = v^2$ (*ibid* eq.21): the square of the visibility distance v **in stades *equals the radius of the Earth* in stades**. The Eratosthenes r implied by Eusebios is 40800 stades (*ibid* eqs.11&18) which happens to be 202 stades squared (*ibid* eq.24), conventionally rounded (www.dioi.org/jm03.pdf, Table 1) to the nearest 100 stades. The coast southwestish from Alexandria being nearly linear, with the Pharos sitting a km offshore into the sea, one could wheel-odometer-measure that the flame was visible over water out to v = 202 stades.

B3   As 1st realized in 2008, multiplying 40800 stades by 2π yields C = 256000 stades, exactly agreeing with the circumference extracted back in 1982 from Strabo's Eratosthenes Nile Map. (Hugh Thurston *Early Astronomy* Springer 1994 p.120.) The royal stade was years ago shown beyond doubt to be 185 meters by D.Engels (*op cit* p.309). Thus, 256000 stades is 19% or almost 6/5 too high; and by glaring inverse-contrast, the Poseidonios-Ptolemy value, 180000 stades, is exactly 5/6 low.

B4   Now to the shockingly elementary key to the long-intractable Earth-size mystery: if an ancient scientist had indeed accurately measured how far over the sea one could spy the Pharos' flame (§B2), and done the easy computation of the Earth's radius from this, the result would have been *wrong on the high side by factor 6/5, due to the bending of horizontal light by air*, "atmospheric refraction". (Because the curvature of a horizontal light ray is 1/6 of the Earth's curvature.) The flame-idea was not unknown in antiquity: Pliny (*Nat.Hist.* 2.65.164) noted that if a lantern were hung on the mast of a receding ship, it would disappear when sufficiently distant, due to the Earth's curvature. Realize in-passing that this measurement would be totally non-astronomical.

B5   Another obvious stay-at-home method: if 2 ancient scientists coordinated to compare times of a clear-atmosphere sunset seen from the Pharos' base&top, the difference would be unmissably large, exceeding a minute of time. Computing Earth's circumference from such data would result in *a figure too low by 5/6, again from airbending of horizontal light*. Such a value rose to domination c.200^y after Eratosthenes'. The delay's likely cause: computing it required spherical trig (D.Rawlins "Doubling Your Sunsets" *Am.J.Physics 47*:126-128, 1979, Tables I & II), not available until the 2st century BC (*DIO 22* ‡3 Table 1).

B6   So airbending of light can explain each of the anciently adopted Earth-sizes cited above (§§B4-B5) to *within one percent in both cases* (www.dioi.org/vols/we0.pdf, ‡1 eq.28). And this is accomplished without the slightest *ad hoc* manipulation of the standard length of the stade. Thus, the solution simultaneously satisfies all three desiderata: *both* Earth-sizes *and* the royal 185 meter stade. Again: all three to 1%.

C   Enlightenment by Satellightenment

C1   Readers who have difficulty accepting the foregoing analysis would do well to consult our ultra-simplified illustration-by-extremes (provided at www.dioi.org/vols/w23.pdf, *DIO 2.3* ‡8 §§A5&A7), which considers what would happen if the Earth's atmosphere bent sealevel horizontal light 6 times more strongly than the reality. On that hypothetical Earth, the light-ray's curvature would equal the Earth's, so a level light-ray would (if not for atmospheric extinction) circle the globe forever! — a new class of non-artificial satellite.

C2   The resultant misleading of refraction-innocent scientists, aiming to calculate the size of a satellightened Earth, would be spectacular.[1] Sostratos&Eratosthenes would see the Pharos' flame above the horizon — no matter how far they receded — so that §B2's v, in www.dioi.org/vols/we0.pdf, ‡1 eq.2, would be infinite, thus (by that equation) rendering Earth-radius r infinite: *a flat Earth*. On the other hand, since the Sun would never set, the (presumably-Poseidonian) double-sunset method's time t would be infinite. So, since t^2 resides in the denominator of Rawlins 1979's eq.13 (for computing r), calculated r must be infinity inverted, or **zero**, indicating the Earth to be an infinitely small point-mass.

C3   These two extreme illustrations (§§C1&C2) should satisfy anyone who at first finds it hard to accept that the two proposed stay-at-home Earth-measure methods must produce seriously disparate Earth-sizes, due to atmospheric refraction.

D   Historians-of-Science Versus Science — and Scientists

D1   Pieces of the airbend solution have appeared for decades in *American Journal of Physics* (1979), *Scientific American* (1979), *Archive for History of Exact Sciences* (1982), H.Thurston, *Early Astronomy* (1994), *DIO: The International Journal of Scientific History* (2008), and in the 1990s as opening-page applied-physics example (with credit to Rawlins), in the ubiquitous physics textbook, Halliday, Resnick, & Walker. Despite such broad availability, professional historians-of-science continue pursuing exclusively their fruitless, endless, chimeral stade-scrunching metrological search — e.g., *Isis*' 2016 December enormous **lead** article. (Which also attacks *DIO* for the "delusion" that Greek scientists were accurate [an issue taken up in ‡9 below], but has its own delusions [§E2; www.dioi.org/vols/wm0.pdf, ‡3, §D; or www.dioi.org/islg.doc, §D], e.g., on telling addition from subtraction [like *JHA*'s Editor: ‡9 §I5] and between solar and lunar eclipses.)

D2   Meanwhile, historians-of-science unanimously — **without a single exception** — ignore [a] the airbend solution and [b] the independently-verified (§B3) 256000 stades circumference of Sostratos-Eratosthenes, not mentioning either even when citing and discussing articles explicitly recommending both. And no historian has ever indicated awareness that the 6/5 air-bend factor[2] has been standard in navigation manuals — e.g., the Bowditch — for over 100^y. Astronomers can make up their own minds as to whether historians or scientists ought to write astronomical history, when it involves mathematical science.

[Observation added 2018/4/19. If there is historical reality in ‡9 §F's hypothesis of Timocharis' precise Earth-measurement, resulting in 600 stades/1° (c.300 BC), then: why did 700 stades/1° wipe it out just decades later? Potential answer: the very explosion of science — which proposedly measured Timocharis' C to within 1% — soon after also produced the Pharos and nourished math ingenuity that invented the lighthouse method of Earth-radius determination. See analogous fast-cascading enlightenment (c.1600 AD) at *DIO 3* fn 13.]

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[1] "Curvature" is defined as the inverse of the radius of the light-ray's path. Venus and the Solar System's outer giant planets all have atmospheres that would refract horizontal light to a curvature far greater than their bodies'; so (since a ray from an observer's nadir can curve into the observer!), sunrise & sunset could never occur there, even if atmospheric extinction did not inhibit such events anyway.

[2] Noticing the 6/5 connexion, between longstanding navigation-practice & both ancient Earth-size errors, is yet another entirely original and multi-perfectly-fitting *DIO* discovery (like ‡9 §J2), thus aSymmetrically-uncitable by the aHoly Trinity aTop the *JHA*: www.dioi.org/vols/wm0.pdf, *DIO 22* ‡3 p.46 & fn 1; & such academically criminal shunning is not restricted to DR: see, e.g., *ibid* fnn 5&17.