‡3 Great Pyramid's Orientation by Star 10i Draconis. And Its Latitude by Star Thuban?

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A Greater Pyramid Misses Orientation-Polestar

A1 The Great Pyramid of Giza is oriented to the cardinal points (N, S, E, W), with a mean error of less than 3 arcminutes, counter-clockwise as seen from above. Numerous papers, positing the most impressively ingenious theories, have tried to explain how such accuracy was accomplished by the ancient pyramid-builders.

A2 The following is much less clever than most of these arguments, as we instead simply adduce a seemingly obvious use of a felicitously-placed but previously overlooked star. Indeed — incredibly — before our 2001 revelation in *Nature*, the star's Pyramid-orienting utility was cited *nowhere* in the centuries-accreting Pyramid-inspired hyper-massive pile of motley[1] litterature, whose ever-burgeoning heft has long since earned *DIO*'s awed admiration as:

The Greater Pyramid

A3 General consensus places the date of Pharaoh Khufu's Great Pyramid at about 2600 BC. It happens that a star less than 1° from the North Celestial Pole at that time is conveniently[2] placed for easy determination of true North — the 4.6-magnitude star, 10i Draconis.

A4 Why 10i Dra was never noted in this connexion — until D.Rawlins&K.Pickering (*Nature 412*:699; 2001/8/16; *N.Y.Times* Science 2001/8/28) — is a mystery. Perhaps it was simply attention-outshone by nearby Thuban (11α Draconis), nearly a magnitude brighter and oft justly noted in popular publications as one of the best polestars in history.

A5 The Pole got[3] as close as about 1/10 of a degree to Thuban — in almost exactly 2800 BC. (Throughout pharaonic times, 10i Dra was never closer than c.10 times that.) But by 2600 BC, Thuban was in a position where it was worthless for orientation, though perhaps useful in a different way, a point we'll return to below in §E.

[1] The exceptional, truly brilliant speculation was by W.Petrie, who in the 19th century measured the slope of the Great Pyramid as 14/11 (or $4/\pi$: hard to tell which), suggesting to him that the ancient designer pondered the squaring of the circle: deliberately ensuring that a circle of radius equal to the Great Pyramid's height had a circumference equal to its square perimeter.

[2] We are not being selective: remarkably, in -2612 there was no star brighter than 10i&11α Draconis within *eight degrees* of the Pole (radius of a circumpolar area equalling 200 square degrees of sky), and only 10i Dra was of use for orientation.

[3] Remember: it's the Pole that moves, not the star (other than contextually-trivial stellar proper motion). Pure but irresistible speculation: the unmissable convenience of Thuban's remarkable 2800 BC proximity to the Pole may have helped inspire Egypt's long tradition of surveying.

B Previous Attempts at Solution

The 1st modern scheme that drew serious attention to explaining the orientation problem astronomically was that of Steven Haack (*Archaeoastronomy 7* [*Journal for the History of Astronomy 15*]:S119-S125 [1984]), which proposed that an East-West line was established by the horizon position of the stars Hamal (α Ari) & Acrab (β Sco), and that precession of the line connecting them allowed estimation of the Pyramid's date. Given the several obvious problems attendant to horizon phenomena, Kate Spence's idea (cover story *Nature 408*:320-324, 2000/11/16, plus O.Gingerich pp.297-298 preface-promo) was more intelligent — e.g., not degraded by atmospheric refraction or local topography. Spence noted that the precessing North Celestial Pole came upon the line connecting stars Mizar (ζ UMa) & Kochab (β UMi) in the 25th century BC — and, when vertical, would indicate the true North point on the horizon. The theory could be used to date the Pyramid by finding when the Pole pierced the line, 2467 BC. Though quite speculative&shaky (its inversion-anomalies are explored at www.dioi.org/jd10.pdf [*DIO 13.1*] p.2), the theory was backed by *Nature*&Gingerich, and so appeared in: *Discover* 2001 Jan; *Scientific American 284.2*:28 (2001 Feb); *Mercury* July-Aug; *National Geographic 200.3*:98 (2001 Sept) — aided by the paper's computational flaw (§C1) being unknown to the public for 9 months. J.Belmonte then proposed (*Archaeoastronomy 26* [*Journal for the History of Astronomy 32*]:S1-S20 [2001] Figs.3ff) a different star-pair, Phecda&Megrez (γ&δ UMa), whose connecting line, extended, was crossed by the Pole in the mid-26th century BC; but this approach was subject to more uncertainty than Spence's theory. The difficulties with all of these imaginatively-contrived, complex, but independently-unconfirmed hypotheses are summarized at *DIO 13.1* pp.2-3 (2003).

C A Surer & Far Simpler Method

C1 Immediately after the *Nature*-cover appearance of Spence's paper, *DIO* discovered a non-trivial error in its spherical trig (creditably admitted by Gingerich, & finally by Spence). A letter (eventually Rawlins&Pickering *op cit*) was swiftly submitted to *Nature*, proposing use of stars much nearer the North Celestial Pole. *DIO*'s simple approach: find a star which, during a Winter Solstice night's near-12^h of darkness or Astronomical Twilight,[4] 1/2-circuits the Pole symmetrically, so ***its arc's mid-point is North***. Just averaging the 2 extreme azimuths yields azimuth $0^\circ \equiv$ North.

C2 As noted at §A, a perfect-candidate star exists in Draco the Dragon: 10i Draconis, magnitude 4.61. In -2612, the solstitial colure passed right through 10i Dra, rendering its Right Ascension $\alpha = 6^h$ exactly. So its semi-circular path around the Pole in a semi-day of 12 sidereal[5] hours looks like an inverted cup, set perfectly symmetrically over the Pole, as depicted in Fig.1. Averaging the two extremal East&West azimuths finds North. The method's efficacy is enhanced by several factors: [a] Azimuthal motion at those extrema is virtually nil, ensuring 2 easily **and repeatedly** measurable quantities to be averaged. [b] The method is recommended by its very simplicity. [c] That 10i Dra is under 1° from the Pole makes the measure ***less dependent on instrument-reliability*** than like use of farther stars. [d] There's no serious systematic error from atmospheric refraction. The method could have been applied anytime for roughly a century ere&aft the optimal date -2612, allowing an independent if not surprising rough date for the Great Pyramid.

[4] Darkness is when the Sun is 18° or more below the horizon; Astronomical Twilight, between 18° & 12°. At Giza for the semi-circular-path times cited, the two stars were at altitude c.29°-31°, and the Sun was always at least c.12° below the horizon, and at least 111° of azimuth distant at the extrema (LAT 18^h & 6^h), so it was dark enough for each star to be visible during its entire 12^h semi-circular path: see Tousey & Koomen *op cit*.

[5] A sidereal half-day is 2^m shorter than the solar half-day we refer to elsewhere here, but the difference is negligible with respect to our analysis.

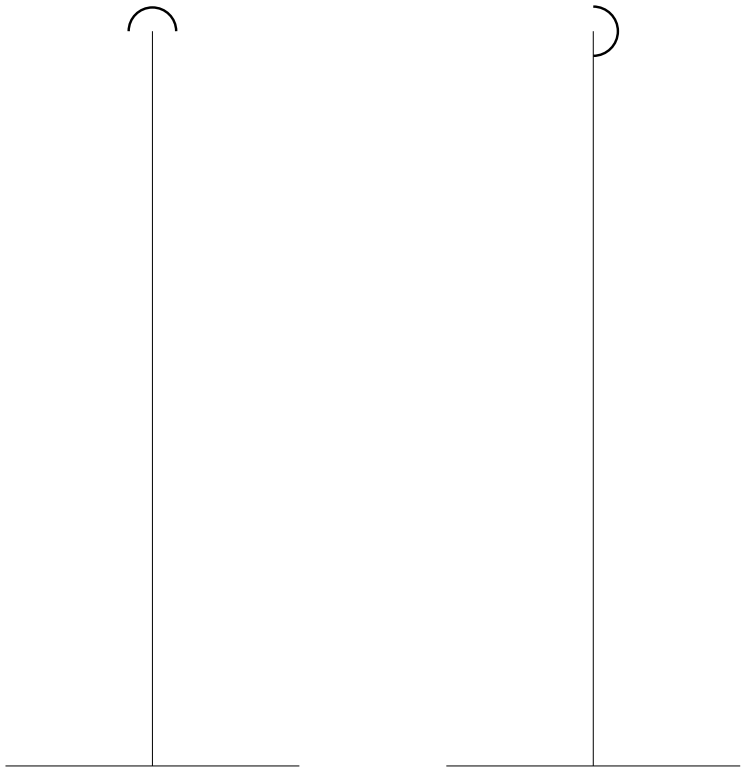

Figure 1: The Great Pyramid's geographical latitude L is $29^\circ 58'.7$. So if we include the effect of $1'.7$ of atmospheric refraction, the Celestial North Pole as seen from the Great Pyramid (and both of the other two Giza pyramids) is above the horizon's North point (azimuth 0°) by $30^\circ 00'$ — within c.2 parts in 10000. (In §E, we will consider whether the coincidence is meaningful.) In both left&right diagrams of the northern sky as seen from Giza at night, the horizon is the horizontal line at bottom. Perpendicular to the horizon in each diagram is 30° of the Giza meridian — from the horizon to the Celestial Pole. And each diagram bears a dark semi-circle centered on the Pole, depicting the counter-clockwise 12^h path of one of the two circumpolar stars here considered, whose respective start&endpoints obviously differ by 90°. The left diagram's half-circle marks the night path of star 10i Dra (magnitude 4.61) around the Pole from c.18^h to c.6^h Local Apparent Time on -2612/1/10-11, or 0^h to 12^h sidereal time, on the Winter Solstice, when night's length is the year's greatest, and the interval cited is almost-entirely in full darkness or Astronomical Twilight. (Extremal solar altitude [18^h&6^h] $h_S = -11^\circ.7$, near enough to Astron.Twilight's -12° bound.) The radius of 10i Dra's semi-circular arc is $0^\circ 58'$, so (dividing by $\cos 30^\circ$) the star oscillates $1^\circ 07'$ in azimuth. The diagram on the right shows the semi-circular night path of star 11α Dra or Thuban (magnitude 3.65) around the Pole 6^d later, -2612/1/16-17, from c.18^h to c.6^h LAT or $0^h.4$ to $12^h.4$ sidereal time. (extremal $h_S = -11^\circ.6$, again about -12°). The radius of 11α Dra's semi-circle is $1^\circ 03'$, which swings it as far east as $1^\circ 13'$ in azimuth, en route from $1^\circ 03'$ below 30° altitude to same above. Both stars visible from Giza throughout their semi-circular paths: R.Tousey & M.Koomen, *Journal of the Optical Society of America 43.3*:177-183 (1953/3).

D How Explain 3′ Systematic Error? Simple: It's Not Systematic!

D1 For decades, *DIO* has wondered: what systematic effect caused Great Pyramid orientation to be in error[6] almost 3′ counter-clockwise? Various systematic factors were considered. None were satisfactory. Then, late on 2018/9/7, *DIO*, which has been analysing Greek star observations' errors for 1/3 of a century, finally thought to apply the fruits of that work to Egyptian observership. (Considering *DIO*'s warnings to modern History-of-science's "Muffia" cult against preconception-blindness: it's only just — & beneficially humbling — to recognize the fault in oneself.)

D2 Since stars are virtually punctal, it is a surprise to many (incl. *DIO*) to learn what R.Newton was perhaps 1st to realize: ancient observations of stars were several times less accurate than those of the nonpunctal Sun. (Due to dealing with instruments in the dark?) For the best Greek astronomical observations: Hipparchos' single-datum scatter was 2′ for the Sun (*DIO 22* ‡3 §B8), but 5′ for stars (*ibid* Table 2). Aristyllos' star-scatter was perhaps even as low as 4′ [*idem*] but his reportage's 1°/4 rounding practice makes it hard to tell.

D3 When it was realized that the Giza 3′ orientation-error might be not systematic but random, it was easy to check: orientation via the technique shown at left in Fig.1 requires averaging the 2 extremal deviations, which means each's expected 5′ error (§D2) must be divided by the square root of 2 and (H.Thurston "On the Orientation of Early Egyptian Pyramids" *DIO 13.1*:4-11 eq.1) by $\cos L$, to find the expected azimuthal error of the mean: 4′, closely matching the Pyramid's actual 3′ orientation error, perhaps a piece of independent confirmation that 10i Draconis was the star used to orient the Great Pyramid — carefully&intelligently — **forty-six centuries ago**.

E Thuban & Determination of Latitude

E1 As noted in §A5, though Thuban couldn't help with orientation, it has another potential rôle: finding latitude. Fig.1's right half shows that only 6d after the left half's Solstice, Thuban's night-journey is another symmetric half-circle, but 90° out of phase with the left-half's 10i Dra path. Thuban's 12h arc is from exactly below the Pole to exactly above it. Averaging the upper&lower Thuban altitudes would be expected to produce latitude L, though the experiment would mistakenly lead to a result for L too high by 1′.7, due to (anciently-untabulated) atmospheric refraction.

E2 It's perhaps suspiciously remarkable that all 3 Giza pyramids are within 0′.4 (naked-eye limit) of refraction's 1′.7 — south of a geographical latitude L equal to ***exactly*** 30° or (since Egypt used not degrees but unit fractions) 1/12th of the Earth's circumference C. The offsets from 29°58′.3 are, respectively, 0′.4 (Khufu), 0′.2 (Khafre), & 0′.0 (Menkure). Is this triple-error evidence for deliberate ancient intent to place monuments at C/12? — a speculation originating in Rawlins, *Vistas in Astronomy 28*:255-268 (1985) pp.255-256, which tries to create a context by noting that the L of all the other prime sacred temples of Egypt are also very close to unit-fractions of C: Amarna C/13, Karnak (Thebes) C/14, Biga Island C/15. Within naked-eye precision, all three are smack-on the unit fraction indicated, depending on whether L was determined by observations of polestars (Giza, Amarna) or solstices (Karnak, Biga). (See Rawlins *loc cit* for details of the selective arithmetic.) Weighing whether these coincidences are significant is left to the reader.

[6] The west wall was probably the anchor that was originally oriented N-S by astronomical means, the north&south walls then oriented with respect to it, by normal non-astronomical surveying techniques, thus showing the same error (from their intended E-W direction) as the west wall; the three's mean error: 2′.6 counter-clockwise. (The Khufu east wall's orientation is discrepant from that by ordmag 1′.) For all 4 errors: see J.Dorner, *Die Absteckung und astronomische Orientierung ägyptischer Pyramiden*, dissertation University of Innsbruck (1981) p.77. Or *DIO 13.1* p.8 fn 11. (The east wall of the 2nd Giza pyramid, Pharaoh Khafre's, was likely set parallel to the Khufu west wall's cornerstones: see *DIO 13.1* p.3 [2003]. Easy, convenient, & natural, since [as 1st noted at *idem*]: Khafre's E side is twice as near Khufu's W side as Khufu's W&E sides are to each other!)