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LUNAR ECLIPSES, LONGITUDE AND THE NEW WORLD

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Throughout the first hundred years after the discovery of the New World late in the fifteenth century, determining the location and extent of their overseas domains was a principal preoccupation of the Spanish empire. Not only were terrestrial coordinates vital for navigation, but following the 1494 Treaty of Tordesillas, knowing with certainty the location of newly discovered territories carried significant geo-political implications as well. Further complicating the matter were the technical difficulties of calculating longitude. The Spanish monarchy turned to experts in navigation and cosmography to address the problem, and by the mid-sixteenth century had institutionalized cosmographical practice at the House of Trade (Casa de la Contratación) in Seville, the Council of Indies (the King's council in Madrid responsible for the administration of the colonies), and at the court of Philip II (1527–98). This paper examines the instruments and techniques used in one of the projects that resulted from their efforts to produce a more accurate description of the world: the project (1577–88) to determine the longitude of all of Spain's overseas territories using lunar eclipses. The first part of this article discusses the project's conception, scope, and historical precedents. The second part develops mathematical models for the methods used and instruments employed, and also assesses how these were used to yield longitudinal coordinates. The final section studies six surviving observations made using the project's protocol and compares these observations to positions computed using modern computer algorithms.[1]

1. LUNAR ECLIPSES AND THE LONGITUDE PROBLEM

The eclipse project has been described as the "first known large-scale, systematic plan of astronomical observation".[2] Several decades ago Clinton Edwards introduced the project to English-language scholarship and identified it as a crucial component of the efforts made by the cosmographer of the Council of Indies to map Spanish overseas domains. He also published a translation of the 1582 version of the printed instructions and identified a number of surviving observations.[3] Despite this and other valuable contributions, the true scope of the eclipse project remained unknown and this led some to surmise that the observations were so imprecise that they were meaningless, or that there were so few respondents that they did little to improve Spanish cartography.[4] This study challenges these conclusions by studying the methods and instruments designed for the project and comparing surviving observations with values obtained using modern computer algorithms.

The genesis of the eclipse project dates to the early 1570s when a series of administrative reforms initiated by Juan de Ovando at the Council of Indies created the post of Cosmographer and Chronicler Major of the Council of Indies.[5] One of the

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cosmographer's principal duties was to compile and maintain a comprehensive geographical description or cosmography of the New World. The law that governed the post mandated the observation of lunar eclipses for determining terrestrial longitude.[6] In doing so it codified and wove into the bureaucratic fibre of the empire's administration what had been until then a sporadic practice. Since the discovery of the New World, pilots, missionaries and government officials scattered throughout the region had sent reports of lunar eclipse local contact times to the Council of Indies.

The first person to occupy the post of cosmographer major was Juan López de Velasco, a jurist, linguist and a man of broad interests but who lacked formal training in astronomy or cosmography. His almost twenty-year tenure (1571–90) as cosmographer of the Council of Indies was characterized more by his diligence in handling the bureaucratic aspects of his job than by cultivating the fields that required knowledge of navigation, cartography and astronomy. In fact, it remains unclear who Velasco might have consulted when devising the methodology and instruments used for the eclipse project, since it is unlikely he devised then on his own.[7]

It would not be until 1577 that the heavens put forth a series of spectacular lunar eclipses visible in both Spain and the Americas that permitted simultaneous observation of the events. The scope of the eclipse project initiated by Velasco to comply with the directives to use the differences in the local times of the lunar eclipse observations to determine longitudes was truly remarkable. Between 1577 and 1588, he prepared printed instructions advising all overseas provinces about upcoming eclipses, and giving detailed instructions on how the event was to be recorded. He intended observers throughout the New World — whom he assumed had little expertise in astronomical matters — to measure the same lunar eclipse following carefully the set of instructions. The observers were told how to build a simple instrument — the Instrument of the Indies, discussed below — and to carry out the observation, log the results, notarize them, and send them to the Council of Indies for analysis. Back in Spain, cosmographers would complete the necessary mathematical computations, determine the respondents' longitude relative to Spain, and correct the appropriate maps.

Lunar eclipses were observed in Spain and in the New World for the purpose of determining longitude long before Velasco's project. Mariners routinely used the method should they chance upon an eclipse while at sea, but some civilian authorities also contributed eclipse observations. One of these early observations dates from the eclipse of 16 November 1537 and was recorded by the viceroy of New Spain, Antonio Mendoza. The viceroy was acquainted with royal cosmographer Alonso de Santa Cruz and likely acted at the cosmographer's urging.[8] The viceroy noted that the eclipse began in Mexico City "half a quarter hour after sunset". His observation was remarkably accurate. Modern computations show that the beginning of the partial phase of the eclipse (the earliest darkening visible with the naked eye) would have occurred at 17:28, while seven minutes after sunset corresponded to between 17:27 and 17:31.[9] Examples of similar reports abound. For example, a cosmographical conference held in 1566 to discuss whether the Philippine Islands lay on Spain's side of the demarcation also relied on eclipse observations provided from overseas.[10] Likewise, missionaries, particularly in the Far East, contributed to the steadily growing number of reliable eclipse observations. Martín de Rada and the Jesuit missionary Matteo Ricci also recorded eclipse observations in China.[11]

The theory behind the use of lunar eclipses to determine longitude is deceptively simple. The eclipse serves as a global synchronizing event. Observers in one part of the globe would record the local time of the contact phases of the lunar eclipse and compare it to the local time elsewhere. The difference in time can be easily translated into degrees of longitude since one-hour difference in local times equals 15° of the Earth's circumference. This assumes that clocks in both places have been synchronized to their respective local meridians and that they also keep accurate time, something that in fact was impossible given sixteenth-century clock-making technology.[12]

The principle behind using lunar eclipses to determine longitudinal distances had been well understood since Antiquity and enjoyed renewed popularity after the publication of Ptolemy's *Geography* in the fifteenth century. Within the community of astronomers, interest in determining longitude accurately was largely driven by the need to adjust astronomical motions listed in the Alfonsine Tables — all based on the meridian of Toledo — to a local reference point as determined by longitudinal distance between Toledo and the point of observation.

There is little evidence, however, of systematic efforts to carry out simultaneous observations of eclipses expressly for the purpose of determining longitudes, except for the efforts of al-Batānī and al-Bīrūnī in the ninth and tenth centuries.[13] Often when an astronomer mentioned having determined a new value for a city's longitude, at least one of the two necessary reference observations were not eclipse observations at all, but calculations based on predicted times of lunar eclipse phases taken from ephemerides. After the discovery of the New World, however, global distances could no longer be accurately estimated from terrestrial distances or from sea logs.

Further complicating the matter were the inaccurate predictions made using Alfonsine Tables. For example, we know that one of the reference texts Velasco used for the eclipse project was Cyprianus Leovitius's *Eclipsium omnium ab anno Domini 1554 usque in annum Domini 1606*.[14] If we compare the start and end times of the 16 (27) September 1577 lunar eclipse as predicted by Leovitius (for Augsburg) against modern positions we find a discrepancy of 1:21 hours for the start time and 1:14 for the end time.[15] Further exacerbating the discrepancy, Leovitius based his local times on Puerbach's Toledo–Vienna distance of 78 minutes (the modern value is 65 minutes) and then adjusted these to Augsburg which he estimated was 26 minutes from Vienna (the modern value is 22 minutes). These errors in longitude contributed 17 minutes of error to the contact times that Leovitius indicated.

With the renewed interest in determining longitude both on land and at sea, came new methods that promised to solve 'the longitude problem'. In 1514, Johann Werner (1468–1522) proposed the method of lunar distances. His method used the Moon's motion relative to fixed stars (conjunction) to determine local time (and thus longitude).[16] Werner recognized that his method required precise tables listing star positions and knowledge of the Moon's true and mean motions for a given location. Yet, although theoretically correct, his method needed stellar and lunar tables with a predictive accuracy far beyond what was available at the time.[17]

There is some evidence that pilots had used other methods that relied on a 'known' position of the Moon to determine longitude. As early as 1499 while off the coast of Brazil, Amerigo Vespucci used the conjunction of the Moon with Mars to calculate longitude.[18] Andrés de San Martín, the pilot of the Magellan-Elcano circumnavigation, used a derivative of this method to attempt to determine longitude at sea. In 1519, he observed the Moon's position relative to Jupiter, only to find the result seriously corrupted by inaccuracies San Martín rightly attributed to the astronomical tables of Zacut and Regiomontanus.[19]

In his popular *Cosmographia* of 1524, Peter Apian explained Werner's method but also advocated the use of lunar eclipses for determining longitude. To facilitate calculations, he included drawings showing the maximum phase of upcoming eclipses (calculated for the position of the city of Leiden) and instructed the reader to compare these images with local observations. By correlating the observer's local time with the ones indicated in the book, the longitudinal distance between the observer and Leiden could be calculated.[20] Gemma Frisius continued advocating the method in his many editions of Apian. But it was in his 1530 *De principiis astronomiae* that he introduced the method of determining longitude by transporting mechanical timepieces — the method that would decidedly solve the longitude problem almost two hundred years later. Disappointed by the accuracy of contemporary mechanical timepieces, Frisius later suggested keeping time using hourglasses or water clocks to adjust periodically the mechanical devices.[21]

In Part 3 of his 1544 *Quadratura circuli*,[22] Oronce Finé discussed what he considered a refinement of the method of using lunar eclipses to determine longitude. It consisted in determining the angle between the eclipsed Moon and its meridian transit, whether the transit happened before or after the eclipse. Finé recommended the use of swivelling Ptolemaic rulers that could also be used to determine the altitude of the Moon during the eclipse. He recognized that a lunar eclipse offered a rare opportunity to know the true position of the Sun from the observed position of the Moon since during an eclipse the Moon's right ascension has a twelve-hour difference from that of the Sun. By measuring angles and computing from its altitude the true longitude of the Moon, Finé hoped to side-step the errors introduced by the inaccurate timepieces into the timing the eclipse and the resulting longitude computation.

For longitude calculations when a lunar eclipse was not available, the angular measurement produced a projected meridian transit time which when compared to tables of lunar meridian transits computed for a reference location yielded terrestrial longitude. However, like Werner, Finé relied on predicting the time of the Moon's meridian transit by dividing the distance between the Moon and the meridian by the Moon's true motion. Pedro Nuñez, the royal cosmographer of the King of Portugal, noted some errors inherent in Finé's method in his *De erratis Oronti Finé*.[23] Nuñez complained the Finé seemed to ignore the fact that the Moon's apparent motion is not uniform, and thus a given lunar angular distance from the meridian cannot be used to determine how long it would take the Moon to reach the meridian.

Both Werner's and Finé's methods depended on astronomical models and tables that predicted the Moon's position accurately, one of the most challenging astronomical problems of the time and one that would not be solved until new models for the Moon's motion were developed in the eighteenth century on the basis of Newton's laws of planetary motion.[24] While a number of other problems concerning eclipse observations, such as parallax, were well understood, others, such as the distortion caused by the effect of terrestrial emanations or "vapours" on the apparent position of the Moon and which today we understand to be due to atmospheric refraction, were only beginning to be better quantified. Yet despite the number of early modern cosmographers who discussed using eclipses to determine longitude in print, I have been unable to find antecedents for the particular instrument and method Velasco selected for the eclipse project.

To launch the lunar eclipse observations, Velasco prepared a printed broadsheet containing a set of instructions and distributed it to all administrative centres (*audiencias* and *gobernaciones*) throughout the Indies. The first instructions were sent to the colonies barely in time for the eclipse of 27 September 1577.[25] The printed broadsheet was titled, "Instruction and announcement to observe lunar eclipses and the quantity of the shadows that His Majesty ordered made this year of fifteen hundred and seventy-seven and seventy-eight, in the cities and towns of the Indies: to verify their longitude and latitude".[26] There were at least four other such broadsheet instructions prepared by Velasco and sent to the Indies.

Figure 1 shows a schematic reconstruction based on Velasco's instructions of the "Instrument of the Indies" used to "observe the quantity of the shadows" at the start and end of the eclipse. It consisted of a wooden board of one *vara* (approximately 84cm) on the side from which rose a perpendicular gnomon of one-third of a *vara* in length. Using the base of the gnomon as the centre, a semi-circle one-third of a *vara* in radius was drawn on the face of the board. A plumb line was attached at the base of the gnomon so that it hung freely and intersected the semicircle, indicating the instrument's vertical reference point. The instrument was inexpensive and simple to construct. If the instructions were followed carefully, it was also easy install and use to carry out the desired lunar eclipse observations. Velasco was well aware that the success of the project hinged on making what to some might appear an intimidating astronomical observation as straightforward as possible.

Velasco began the instructions by reassuring the prospective observer that the method selected to record the eclipse was the simplest available and that the instrument needed was easy to make. The document continued with step-by-step directions on how to build the instrument, make sure the gnomon was square to the face of the instrument, and how to install it correctly by checking that it was oriented true to the meridian. It then explained how to carry out the observations on the night of the eclipse and how to record the measurements. The first set of directions instructed the observer to build a level platform containing a sundial used to measure the length of the shadow cast by the Sun at noon (relative to a centre gnomon of known length). Although the instructions do not state this, the purpose of this measurement was to determine the observer's terrestrial latitude. (Observers could also have used this measurement to determine local noon and synchronize their clocks.) By following the Sun's shadow cast by the centre gnomon on the sundial from dawn to dusk, the observers were also instructed on how to draw a line representing the east/west direction. This line served to guide the placement of the Instrument of the Indies for measuring the Moon's shadow during the eclipse.

The eclipse instructions continued with guidelines for constructing the Instrument of the Indies. The instrument had to be placed carefully along the east/west line on the platform that was built for the sundial. As to the evening of the eclipse, the instructions suggested that more than one person be present to witness the phenomenon. If the Moon rose "completely round", the observers were to continue with the measurements. When at some point during the night the Moon began to be obscured by the Earth's shadow, the observers were to mark the spot on the semi-circle drawn on the instrument's face where the shadow of the gnomon cast by the Moon fell. They were instructed to do likewise when the eclipse ended and the Moon regained its perfectly round shape. After the eclipse, observers were instructed to copy these marks onto a large sheet of paper made from four sheets (*pliegos*) joined at the edges. They had to submit two sets of measurements, one showing the length of the Sun's shadow at noon relative to the centre gnomon and another replicating the measurements made of the Moon's shadow at the start and end of the eclipse. Observers then had to notarize and make duplicates of the observation, both of which were to be sent to the Council of Indies. If because of cloudiness or obstruction the eclipse could not be observed, the responsible parties still had to measure the shadow the Sun cast at noon on that or any other day and indicate the date when the observation was taken.

The various versions of the instructions printed have subtle, but telling variations. For example, in the 1577 instruction, Velasco advised that in order to ensure that the instrument's centre gnomon was vertical, the instrument-maker should hang a small weight from the top of the gnomon in the manner of a plumb line. In subsequent instructions, Velasco recommended instead that a compass be used to make sure the top of the gnomon was equidistant from the circle drawn on the face of the instrument — a far more accurate method for installing a perpendicular gnomon. The instruction of 1577, perhaps because the eclipse would take place near dusk in some parts of the Indies and the instrument might not cast a shadow, also asked observers to have on hand a "geared clock [*reloj de ruedas*]" or, in the event one was not available, a sand hourglass to time the event. If clocks were not available, Velasco asked that time be estimated "more or less, according to the opinion and judgment of the observers".[27]

The instructions for 1581 described instead an instrument with gnomons on both faces. Because of the instrument's re-design, Velasco removed from the instruction references to placing the instrument facing away from the direction where the shadow fell at noon. This 1581 instrument, although more complex to build, would be impossible to place facing the 'wrong way', that is, away from where the Moon could cast a shadow on its face. In all instructions subsequent to 1577, Velasco also advised that the observations be reported on paper rather than on parchment. The other instructions were also changed to indicate that rather than sending the observations to the "persons in the government that sent you these instructions" (the local *audiencia* or *gobernación*), as the 1577 instruction had read, the observations were now to be sent directly to the King in care of the Council of Indies.

None of the versions of the instructions gives any indication of how the observers could use their observations to calculate their latitude or longitude. Clearly, the intention was that the mathematical computations would be done by cosmographers back in Spain — a perhaps unsatisfying outcome for a diligent observer in the Indies, but also a way to keep the resulting information secret. Sadly, we have no record of how Velasco manipulated these observations to yield latitude and longitude. Clearly, the first set of observations recorded the length of the shadow cast by the Sun's shadow at noon and could be used to determine the latitude of the observer with the help of a table of solar declinations. The second set of observations taken with the Instrument of the Indies recorded the angle made by the Moon's shadow at the beginning and end of the eclipse relative to a vertical line corresponding to the local meridian. The angles defined by the marks x_1 and x_2 (Figure 1) and the base of the gnomon are proportional to the Moon's altitude and azimuth. These angles can be used to determine local time at the start and end of the lunar eclipse. By comparing observations taken in Spain with those reported from the Indies, it would then be possible, although not easy, to calculate the longitudinal distance between two locations.

Not all the eclipses for which instructions were issued turned out to be visible from America; in fact, only five of the fourteen possible start and end times of the lunar eclipses were visible over a territory spanning from Spain to Mexico. Table 1 shows a list of eclipses visible in Spain and the Americas (1577–88) and uses modern

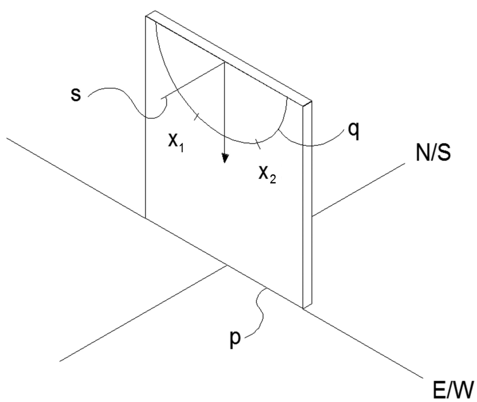


FIG. 1. The instrument of the Indies.

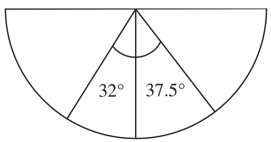
TABLE 1. Computed positions and shadow angles for observed lunar eclipses.

Date	Typ*	Lunar RA*	Lunar dec*	h in deg	Madrid 40°23′N, 3°41′W Alt° start / Az° start	Alt° end / Az° end	X° start/ X° end	San Juan 18°27′N, 66°7′W Alt° start / Az° start	Alt° end / Az° end	X° start/ X° end	Mexico City 19°24′N, 99°12′W Alt° start / Az° start	Alt° end / Az° end	X° start/ X° end
1577 Sep 27	T–	0.82	5.50	0.04	47.64	45.25	**31.26**	2.48	56.96	87.51	(28.02)	26.09	na
					138.26	227.21	**(36.03)**	85.03	109.12	31.57	na	93.40	63.87
1578 Sep 16	P	0.12	1.70	0.07	50.36	48.36	12.13	17.63	42.45	72.32	(13.59)	11.41	na
					164.96	206.12	(21.38)	94.20	105.24	46.53	na	92.23	78.58
1581 Jul 16	T	20.33	(19.20)	1.00	23.24	(6.39)	(52.03)	42.16	47.79	**35.18**	17.04	49.14	70.82
					213.38	250.17	83.21	140.34	207.88	**(22.99)**	118.20	160.08	16.42
1584 May 24	T+	16.08	(20.90)	3.01	(63.61)	(63.07)	(20.44)	(12.43)	(51.82)	76.94	15.90	(21.50)	83.25
					131.33	230.19	21.32	251.85	259.39	37.70	240.42	254.47	67.76
1584 Nov 18	T+	3.57	19.10	0.00	58.90	58.57	26.20	6.92	56.52	82.73	(21.32)	25.81	na
					125.33	235.39	(26.71)	72.12	83.06	33.29	na	77.58	**63.66**
1588 Mar 13	T–	11.56	3.20	0.51	52.74	27.72	(3.34)	29.64	73.37	60.17	(1.62)	47.86	**na**
					184.40	248.65	(60.57)	97.02	155.72	7.00	na	107.52	40.79
1588 Sep 5	T+	22.93	(7.00)	1.06	33.54	(2.91)	(44.66)	51.66	54.26	**31.58**	23.38	62.55	(89.66)
					220.91	263.29	87.07	128.98	226.87	**(27.71)**	107.04	163.35	8.47

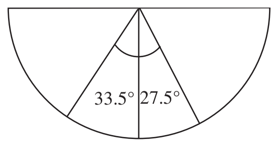
*Eclipse data from Fred Espenak NASA/Goddard Center. ( ) altitude indicates the Moon was below the horizon. An altitude of 10° or better is needed to observe an eclipse.

Observations made using the Instrument of the Indies showing observed values of X°.

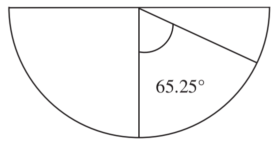
Madrid 1577 Sept.
See Fig. 8.



San Juan 1588 Sept.



Mexico 1584 Nov. (J. Juan)
See Fig. 9.



Column headings:

Date: date of the lunar eclipse.

Typ: type of lunar eclipse. T+ denotes a total eclipse where the Moon passed north of the shadow axis, T– a total eclipse where the Moon passed south of the shadow axis, P a partial eclipse.

Lunar RA: the Moon's right ascension.

Lunar dec: the Moon's declination.

Alt° start (*or* end): altitude of the Moon at the start (*or* end) of the eclipse.

Az° start (*or* end): azimuth of the Moon at the start (*or* end) of the eclipse.

X° start (*or* end): angle with the vertical of the shadow cast by the Instrument of the Indies at the start (*or* end) of the eclipse.

positions to compute the Moon's altitude and azimuth for some cities we know reported results. For example, only the ends of the total eclipses of 1577, 1584 and 13 March 1588 were visible in both Madrid and Mexico City. The end of the eclipse was not visible in Spain in the case of the 16 July 1581 eclipse. On two occasions, the printed instructions embarrassingly predicted lunar eclipses when instead solar eclipses took place (19 June 1582 and 10 May 1584).

Velasco's personal astronomical observations do not survive nor did he ever, to our knowledge, compile or draw a map based on longitudes derived from the eclipse project. Given the secrecy policy under which Velasco operated, one would not expect to find evidence of the project in material published during the reign of Philip II. Most of the information we have about the project's outcome comes from Velasco's successor as cosmographer major of the Council of Indies, Andrés García de Céspedes (*c.* 1545–1611). Céspedes, an accomplished mathematician and cosmographer, practised cosmography during a time that began to see an erosion of the secrecy policies that had regulated the dissemination of geographical information during Velasco's tenure.

Céspedes discussed the lunar eclipse observations in his *Regimiento de navegación e hydrografía* of 1606.[28] He went as far as to suggest that either his predecessor had kept the method for converting shadow observation into longitude values secret, or he simply had not known how to do the lengthy computations![29] Céspedes, nonetheless, had a favourable opinion of the project, explaining that the observations were carried out diligently as per the instructions and were made by the most skilled men in each location. In fact, Céspedes carried out a complete reform of the official navigation charts ("*Padrones reales*") issued by the House of Trade in Seville using the longitude coordinates the eclipse project yielded.[30]

2. ANALYSIS OF THE INSTRUMENTS AND METHODS

The design of the Instrument of the Indies is based on the premise that during totality in a lunar eclipse the Sun and Moon are in opposition. Therefore the Moon's right ascension has a twelve-hour difference from that of the Sun. Furthermore it can also be assumed that the Moon's declination is the same as that of the ecliptic. This sidereal coincidence solved a vexing problem for sixteenth-century astronomers. Both the Ptolemaic and Copernican models of lunar motion were recognized as being notoriously unreliable, making right ascension and declination values for the Moon equally unreliable. The Sun's movement, however, had been well understood since Antiquity and, therefore, a lunar eclipse offered the rare instance when the celestial position of the Moon was known since it was 180° away from the Sun during the event. The fact that a lunar eclipse could be observed simultaneously across almost half a terrestrial hemisphere provided cosmographers with the type of global synchronizing event that allowed for a time-sensitive computation, such as longitude, to be carried out.

The Instrument of the Indies is not unlike a north- or south-facing vertical sundial, but with the added complexity of not having its gnomon tilted by the number of degrees equal to the latitude of where the sundial is installed. The relationship between the shadows cast by the Moon at the start and end of the eclipse and that were recorded on the face of the instrument are shown in Figure 1. The instrument *p* had a gnomon *s* perpendicular to the face of the instrument. A semicircle *q* centred at the base of the gnomon was inscribed on the face of *p* and its radius equalled the length of the gnomon *s*. The instrument had to be placed due east/west with the gnomon facing either north or south, so that the face of the instrument was towards the Moon. Point *v* indicated where the plumb line intersected *q*. Observers were to place a mark x_1 where the shadow cast by the gnomon fell on line *q* at the beginning of the eclipse and another mark at the end of the eclipse. These points determined angles $X_1°$ and $X_2°$ formed between the vertical line defined by the plumb line and a line drawn from the base of the gnomon to points x_1 and x_2.

To determine the relationship between the observations recorded on the face of the instrument and the position of the Moon, we must determine the relationship between angles formed by the shadow of the Moon at both the beginning and end of the eclipse ($X_1°$ and $X_2°$) and the Moon's altitude (*alt*) and azimuth (*Az*). In Figure 2 the angles formed by the shadow may be defined by the following equations:

$$z = s \tan(Az) \quad \text{and} \quad y = s \tan(alt) / \cos(Az). \qquad (1)\ (2)$$

Therefore, the angles $X_1°$ and $X_2°$ satisfy the following relationship:

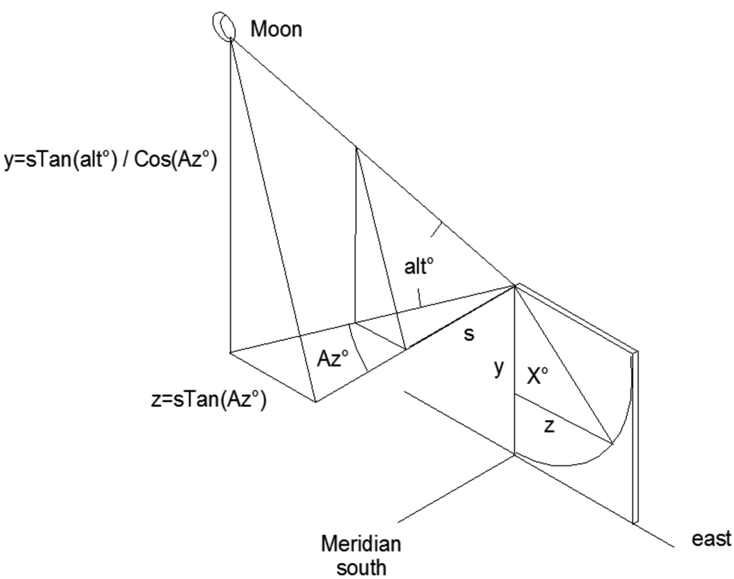


FIG. 2. Relationship between the angles formed by the shadow cast by the Moon using horizontal coordinates.

TABLE 2. Time, duration and position of lunar eclipse of 18 November 1584.

U.T. at max. eclipse	Semi-duration of partial phase	Moon right ascention	Moon declination
00:02	107	3.57	19.1

$$\tan (X°) = \sin (Az) / \tan (alt). \tag{3}$$

Equation (3) shows that the shadow angles $X°$ recorded on the face of the Instrument of the Indies are a function of both altitude and azimuth and therefore fail to yield a definitive position for the Moon.[31] Whereas Finé's use of Ptolemaic rulers permitted measuring the Moon's altitude and thus the computation of its longitude, the simple design of the Instrument of the Indies did not. Andrés García de Céspedes recognized the instrument's shortcoming. His method for solving this problem relied on taking values of the Moon's longitude from an unspecified ephemerides and then manipulating these following the procedures discussed in Regiomontanus's *Tabula directiorum* to determine the Moon's declination.[32] He realized that in order to find the Moon's declination the eclipse contact times had to be known — precisely the problem at hand. Céspedes suggested that the contact times listed in ephemerides could be used as an estimate to find a value for the Moon's longitude and thus compute its declination, adding that "although the place of the Moon might be wrong by half or up to one degree (which is not possible), in the declination there would be little difference".[33]

The design of the Instrument of the Indies was a compromise between precision and simplicity. On the night of the eclipse, observers had only to place two small marks on the face of the instrument, and did not need to worry about altering the instrument's placement by also measuring the altitude of the Moon, as would have been required by Finé's approach. There was also no need for a rare and expensive mechanical timepiece (although Velasco recommended using one). The Instrument of the Indies was designed so that someone with only a basic level of literacy could build the instrument and carry out the observation. This was a key factor in the project's success. Yet in doing so it compromised the precision of the results by relying on estimated values of lunar longitude, which, as Céspedes correctly observed, introduced a small error into the contact times computed from the observations.

Unfortunately, no two sets of observations made with the Instrument of the Indies survive that record the same lunar eclipse from two different locations. Given this, I will use computations using modern algorithms and the equation for the Instrument of the Indies (Equ. (3)) to model how the eclipse would have appeared to a local observer and how the observations were used to determine longitude. This exercise helps us understand how the instrument worked and allows us to make an approximation to ascertain the accuracy of the results.

The observation made by Jaime Juan for the eclipse of 18 November 1584 in Mexico City reported that at the end of the eclipse, the shadow formed an angle of 24.75° with the horizontal, so for this case: $X_2° = 65.25°$. Using the modern values

TABLE 3. Position of the lunar eclipse as viewed from Mexico City on 18 November 1584.

Eclipse time at max.	Eclipse time at end	Hour angle at max.	Hour angle at end	Moon's altitude at max.	Moon's altitude at end	Moon's azimuth at max.	Moon's azimuth at end
0.03	1.82	–95.25°	–68.5°	1.55°	25.81°	250.28°	257.58°

(shown in Table 2)[34] and taking the coordinates of Mexico City as 19.40°N, 99.20°W, it is possible to determine the positions of the Moon at totality (max) and at the end of the eclipse (Table 3).[35]

Table 3 shows that in Mexico City at the end of the eclipse the Moon appeared 25.81° above the horizon and had an azimuth 257.58° (from north). Using these values in Equ. (3) we can then determine that the Moon should have cast a shadow on the instrument as measured from the vertical of 63.65° at the end of the eclipse. Referring back to Jaime Juan's 1584 observation, at the end of the eclipse the shadow formed an angle with the vertical of $X_2° = 65.25°$. Juan erred by only 1.6°, a diligent observation indeed. Table 1 shows values of $X°$ computed using modern values of altitude and azimuth. The graphics below the table shows shadow angles as recorded on two other eclipse observations. As in the case of Jaime Juan's observation, these other observations correlated closely with the value of $X°$ predicted by Equ. (3).

How were these observations used to determine the longitudinal distance separating two observers recording the event using the Instrument of the Indies? Recall that Céspedes complained that Velasco had left no indication as to how the Instrument of the Indies could be used to determine longitude. The design of the Instrument of the Indies suggests that the longitude difference between two locations observing the event was calculated from the value of $X_1°$ and $X_2°$, since neither the Moon's altitude, nor the length of the shadow, nor the length of the gnomon was recorded. (We should not assume that the semi-circle was 1/3 of a *vara* in radius or that the shadow reached the circle.)

A simple difference between the angles created by the shadows cast on the face of the instrument for two different locations does not yield a time difference, and hence a valid longitudinal value. To illustrate this consider Table 4. It shows shadow angles computed using modern values calculated for Madrid and San Juan. The distance

TABLE 4.

	Madrid computed values		San Juan computed values			
	$X_1°$	$X_2°$	$X_1°$	$X_2°$	Difference between $X_1°$ in two locations	Difference between $X_2°$ in two locations
1577	31.26	(36.03)	87.51	31.57	66.25	67.60
1578	12.13	(21.38)	72.32	46.53	60.19	67.91
1584	26.20	(26.71)	82.73	33.29	55.53	60.00
1588	(44.65)	(87.07)	31.58	(27.71)	76.23	59.36

between the two cities is 62°26′ or 4.14 hrs.

In order to be able to use the wealth of lunar eclipse data at his disposal Céspedes developed two ways of using the observations from the Instrument of the Indies to find the local times corresponding to the start and end of the eclipse. The first method he described as using "the doctrine of spherical triangles". The second relied on an instrument intended to facilitate the computations. These methods are independent of the length of the gnomon used for fabricating the instrument, as well as of the radius of semi-circle *q*. They rely solely on the value of X_1° or X_2° and on an estimated value of the Moon's declination. The two sections that follow discuss the methods and illustrate how they were used.

Céspedes's Solution using Spherical Trigonometry

Céspedes explained that the angle formed on the face of the instrument by the vertical line and the Moon's shadow (X_1° or X_2° in our previous examples) defines the arc between the zenith and the Moon's position circle at the start and at the end of the eclipse. This arc, along with the declination of the Moon taken from ephemerides (recall that during an eclipse it is the same as the Sun's) and the observer's latitude, were enough to calculate the eclipse local start or end times. [36]

He used Figure 3 for his proof. (See Figure 4 for a different interpretation of Céspedes's figure.) In the figure, *abcd* is the horizon, *aec* is the meridian at the point of observation, while *bed* is a vertical line that goes from the zenith to the east and west. The celestial equator is shown as *bnd*. Point *g* indicates the Moon's position at either the start or end of the eclipse, while *agc* is the Moon's position circle either at the start or the end of the eclipse. If *nhk* is taken as the ecliptic, then *fgh* indicates the Moon's path. Given these points, he drew a meridian *gm* from the pole *m* of the

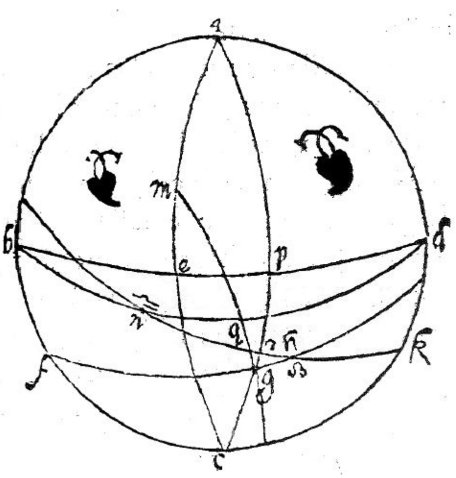


FIG. 3. Drawing accompanying Céspedes's proof of the Instrument of the Indies.

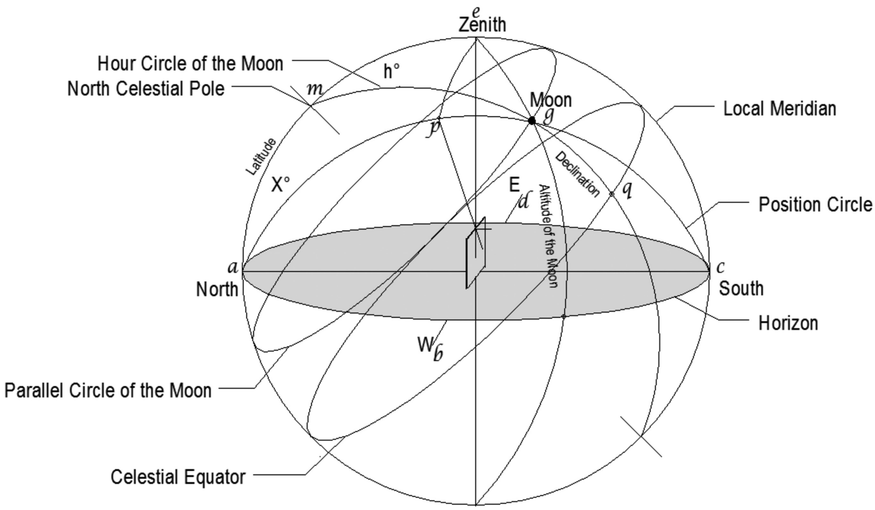


FIG. 4. Relationship between celestial and alt-azimuth coordinates associated with the Instrument of the Indies.

celestial sphere, passing through the centre of the Moon at *g* and meeting the celestial equator at point *q*. Therefore to know local time we need to know the value of $\angle cmg$ (the angular distance between the Moon and the local meridian).

Céspedes explained that arc *ma* is defined by the observer's latitude (known) and that arc *mg* is derived from the Moon's declination. (The declination is taken as a positive value if the Moon is declining north and negative if declining south of the ecliptic.) The angle *mag* is also known since it is determined by arc *pe* which is the same as $X°$ (the angle of the Moon's shadow with the vertical). Céspedes then explained that $\angle cmg$ can be solved using spherical triangles since two sides are known, *gm* and *am*, as is one angle, $\angle mag$. Therefore, using the theorem set out in Regiomontanus's *De triangulis* (Book 4, Proposition 30), $\angle amg$ can be computed.[37] Once $\angle amg$ is computed, subtracting $\angle amg$ from 180° gives $\angle cmg$, which is the distance of the Moon to the meridian or its hour angle. Given that during the eclipse the Moon is in opposition to the Sun, the Sun can be assumed to have this same hour angle ($\angle cmg$) from the opposite meridian, and thus the local time is known.

In the *Regimiento de navegación*, however, Céspedes does not work out an example showing the mathematics of his proof. What follows is my interpretation of the spherical trigonometry involved in solving the problem. Figure 5 is a subset of Figure 3 where arc *mt* has been defined as perpendicular to arc *ag*. Thus, the following parameters are known:

$\angle mat = \angle X_1°$ or $\angle X_2°$ (from the observation recorded on the Instrument of the Indies);

arc *am* = the observer's latitude (*f*);

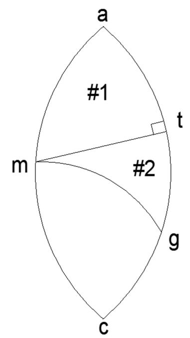


FIG. 5. Spherical triangles used to solve Céspedes's proof.

arc *mg* = 90° – Moon's declination (*d*).

The problem requires finding ∠*amg*, which when subtracted from 180° yields *cmg*, the hour angle of the Moon. Dividing the angle ∠*amg* by constructing line *mt* perpendicular to arc *ag* forms two right spherical triangles, #1 and #2. In right spherical triangle #1, two parameters are known: ∠*mat* and side *am*. So in order to find ∠*amt* and arc *mt* we use equations for right spherical triangles and find that[38]

$$\tan(\angle amt) = \cot(\angle mat) / \cos(ma) \quad \text{or} \quad \tan(\angle amt) = \cot(X°) / \cos(f). \qquad (4)$$

Having found ∠*amt*, we then find side *mt*:

$$\tan(mt) = \cos(\angle amt) \times \tan(ma) \quad \text{or} \quad \tan(mt) = \cos(\angle \text{amt}) \times \tan(f). \qquad (5)$$

In right spherical triangle #2, we now know two parameters: arc *mt* from Equ. (5) and arc *mg* from the declination. To find ∠*tmg*:

$$\cos(\angle tmg) = \tan(mt) / \tan(90° - d). \qquad (6)$$

Therefore, $\angle amg = \angle amt + \angle tmg$ and $\angle cmg = 180° - \angle amg$.

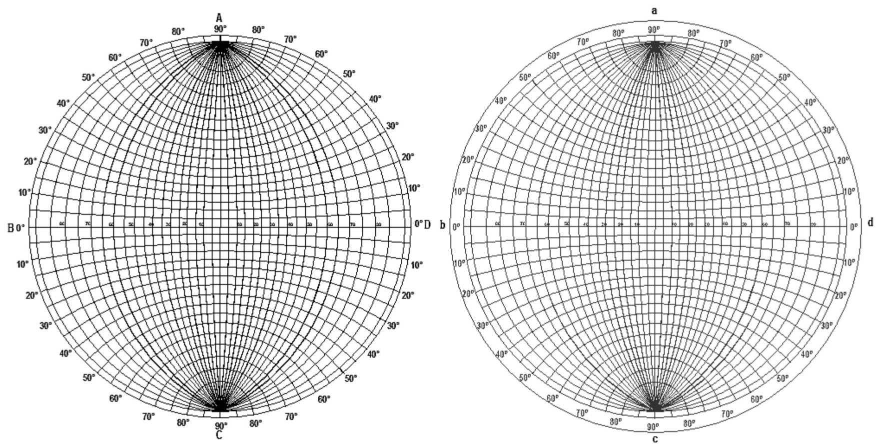
The angle *cmg* divided by 15 gives the local time. This time, however, must be adjusted by the difference between Greenwich sidereal time (GST0) and right ascension because the modern longitude values used are calculated taking GST0 into consideration. Once this adjustment is made, Céspedes's equations yield local times that conform to modern values.[39]

Céspedes's Solution Using a Computational Instrument

To simplify the lengthy computations involved in turning shadow angles into longitudes, Céspedes proposed that cosmographers use instead a computational instrument that reduced the calculation of local times determined by $X_1°$ or $X_2°$ to the correct setting of the instrument, together with simple arithmetic operations.[40] Céspedes's instrument is most useful when it is necessary to translate observations taken using one celestial coordinate system, but which need to be manipulated with values determined from another coordinate system. In this case, the shadow observations recorded on the face of the Instrument of the Indies translate easily into alt-azimuth coordinates, but the Sun's position (and therefore the Moon's during the eclipse) was available from ephemerides as ecliptical coordinates which with the aid of the instrument could be translated into equatorial coordinates (right ascension and declination).

The instrument described by Céspedes consisted of a revolving transparent disk with a universal stereographic projection; the disk was made from oiled paper and mounted on top of another disk likewise engraved with a universal stereographic projection. It used the projection typical of the Islamic *saphea* astrolabe and popularized in sixteenth-century Europe by Gemma Frisius as the "catholic astrolabe" or universal astrolabe.[41] Céspedes does not include in either section an explanation of the theory behind the instrument, referring the reader instead to his now-lost treatise on the astrolabe.

Figure 6 shows the disassembled instrument replicated using a computer-aided design (CAD) modelling tool. To use the instrument for calculating the local start or end time of lunar eclipses using observations taken from the Instrument of the Indies, one must assume the gridlines on the top disk *ABCD* represent equatorial coordinates. In *ABCD*, *A* is the north celestial point, lines going from *A* to *C* are hour lines and lines from *B* to *D* are declinations, while line *BD* is the celestial equator. The bottom disk, *abcd*, shows alt-azimuth coordinates, where line *ac* is the local meridian and lines going from *a* to *c* are position circles, while line *bd* is a line going from east to west. Recall that the angle formed by the eclipse shadows $X_1°$ or $X_2°$ determines the Moon's position circle. In this instrument the value of $90° - X°$ is reckoned from



Top (ABCD) Bottom (abcd)

FIG. 6. Top and bottom disks of the computational instrument used by Céspedes to compute local start and end times of lunar eclipses.

line *ac* or the local meridian.

The instrument is used as follows to calculate the hour angle of the Moon (the Sun's + 180°) and therefore local time.[42] On the top transparent plate *ABCD* point *A* is rotated towards *d* as many degrees as determined by the latitude of the place where the observation was taken. Once the instrument is set thus, the declination circle on *ABCD* that corresponds to the known declination of the Moon is located. Recall that positive declinations are above line *CD* and negative, below. Find the point where the declination line on *ABCD* intersects the position circle line on *abcd* as determined by the shadow angle. Only the right hand side of the *abcd* circle may be used but recall that line *a–c* is the local meridian and therefore we must find 90°– *X*°. Once the intersection of the declination and the shadow angle is found, determine the corresponding hour line on *ABCD*. Having found the hour line *h*°, the distance from the Moon to the meridian is known and therefore the location of the Sun (since it is 180° away).

The example that follows shows how this computational aid would have been used to find the local time of the eclipse observations recorded by Jaime Juan of the November 1584 eclipse as observed in Mexico City. Juan was able to record only the end of the eclipse, and he noted it formed an angle of 65.25° with the vertical. He stated that the Moon's declination according to his calculations was 19.71°. These values used to set Céspedes's instrument yield an end time for the eclipse of 19.33 hours, that is, 19:20. When compared with a computed local end time of 19:12, we find an error of 8 minutes or 2° longitude. Modern calculations, however, indicate the Moon's declination was 19.10° and thus the shadow line on the Instrument of the

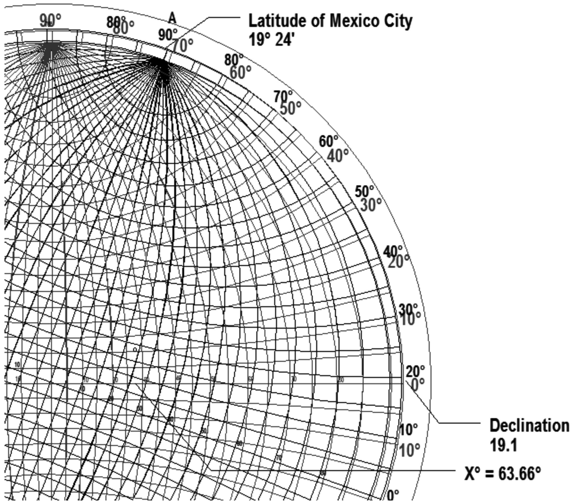


FIG. 7. Céspedes's instrument set for solving the Mexico Nov. 1584 lunar eclipse.

TABLE 5. Lunar eclipse times computed using modern algorithms versus times calculated using CAD model of Céspedes's instrument.

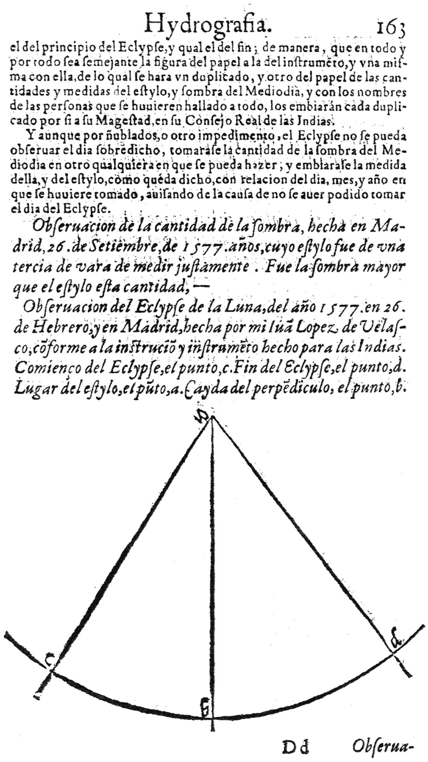
Date	Madrid Computed local start / end times	Madrid Calc. local start / end times	Madrid Error (hrs)	San Juan Computed local start / end times	San Juan Calc. local start / end times	San Juan Error (hrs)	Mexico Computed local end time	Mexico Calc. local end time	Mexico Error (hrs)
1577 Sep 27	22.03	22.33	0.30	17.87	18.10	0.23			
	1.90	2.26	0.36	21.74	21.93	0.19	19.54	19.77	0.23
1578 Sep 16	23.18	23.46	0.28	19.02	19.33	0.31			
	0.95	1.06	0.11	20.79	21.00	0.21	18.59	18.80	0.21
1584 Nov 18	22.00	22.26	0.26	17.84	18.20	0.36			
	1.57	1.73	0.16	21.41	21.73	0.32	19.20	19.40	0.20
1588 Sep 5	2.15	2.20	0.05	21.99	22.20	0.21			
	5.79	5.33	(0.46)	1.63	1.73	0.10	23.42	23.50	0.08
	Avg. error =		0.13			0.24			0.18

Indies at the end of the eclipse computed using Equ. (3) would have formed an angle of $X° = 63.66°$ from the vertical. Setting Céspedes's instrument with these values yields a local end time of 19.4 hours or 19:24, for a 12-minute error (Figure 7).

Columns in Table 5 under the heading "Calc." show some examples of lunar eclipse start and end time values obtained using the CAD model of Céspedes's instrument, and compared to local times computed using modern algorithms. The CAD software allowed the author to 'virtually' superimpose and rotate the top disk (*ABCD*) to obtain the values shown in Table 5. The table uses values of $X°$ predicted by Equ. (3) and calculated using modern values. It shows an average departure from modern positions in the range of 0.13 to 0.24 hours (8–14½ min). This error is probably due to the 1° resolution of the CAD model of Céspedes's instrument.[43]

3. OBSERVATIONS MADE WITH THE INSTRUMENT OF THE INDIES

Only two sets of original observations prepared following Velasco's instructions survive. One set was made in Mexico for the event of November 1584 and has original observations from four witnesses.[44] The other set is a single, previously unpublished observation made in Puerto Rico.[45] A third observation, purportedly of Velasco taken in 1577, appeared in print.[46] Archival research has uncovered evidence of other observation reports made following these instructions, including a number of observations from the Philippines. Unfortunately, Céspedes discussed in print only a few selected eclipse observations after hinting tantalizingly that Velasco had handed down to him a great number of lunar eclipse observations. For the purpose of this study we will discuss only the three surviving observations recorded using the Instrument of the Indies.



Hydrografia. 163

el del principio del Eclypse,y qual el del fin; de manera, que en todo y por todo sea semejante la figura del papel a la del instrumẽto,y vna misma con ella,de lo qual se hara vn duplicado, y otro del papel de las cantidades y medidas del estylo,y sombra del Mediodia, y con los nombres de las personas que se huuieren hallado a todo, los embiarán cada duplicado por si a su Magestad,en su Consejo Real de las Indias.

Y aunque por ñublados,o otro impedimento,el Eclypse no se pueda obseruar el dia sobredicho, tomarase la cantidad de la sombra del Mediodia en otro qualquiera en que se pueda hazer; y embiarase la medida della,y del estylo,como queda dicho,con relacion del dia, mes,y año en que se huuiere tomado, auisando de la causa de no se auer podido tomar el dia del Eclypse.

Obseruacion de la cantidad de la sombra, hecha en Madrid,26.de Setiembre,de 1577.años,cuyo estylo fue de vna tercia de vara de medir justamente. Fue la sombra mayor que el estylo esta cantidad,—

Obseruacion del Eclypse de la Luna,del año 1577.en 26. de Hebrero,y en Madrid,hecha por mi Iuã Lopez de Velasco,cõforme a la instruciõ y instrumẽto hecho para las Indias. Comienço del Eclypse,el punto,c.Fin del Eclypse,el punto,d. Lugar del estylo,el pũto,a.Cayda del perpẽdiculo, el punto,b.

Dd *Obserua-*

FIG. 8. López de Velasco's 1577 lunar eclipse observation as it appears in Céspedes's *Regimiento de navigación e hydrografía* (ref. 28), f. 163.

Madrid – Mexico City 1577

According to Céspedes's account, it appears Velasco personally observed the eclipses in Spain, using both the Instrument of the Indies and an astrolabe. In his *Regimiento de navegación* Céspedes published Velasco's personal observations for the eclipse of 1577 and included a drawing reproducing the tracings indicating the beginning and end of the eclipse (Figure 8). I have measured the angles directly from the 1606 edition of the *Regimiento*. The angles from the vertical indicating the start, 32°, and end of the eclipse, –37.5°, correspond reasonably well with the values calculated using modern theory of 31.3° and –36° respectively (Table 1). Céspedes found these values acceptable. He noted, however, that Velasco also used an astrolabe to measure the altitude of the Moon at the start (41°) and end of the eclipse (39°) and declared that Velasco had "fooled himself" by trusting these values. (This was indeed so, as Table 1 shows, since the altitude for the Moon at the start of the eclipse was $47\frac{1}{2}°$ and at the end, $45\frac{1}{4}°$.)

TABLE 6. Lunar eclipse times from observations versus modern values.

Date	U.T. start / end times	Local start / end times modern values	$X°$ start / $X°$ end from observation	local start / end times calculated with Cespedes's equ.*	Error (hrs)
1577 Sep 27	22.28	22.03	32.00	21.99	(0.05)
Madrid by Velasco	2.15	1.90	37.50	2.00	0.10
1584 Nov 18	22.25				
Mexico by J. Juan	1.82	19.20	65.25	19.09	(0.11)
1584 Nov 18	22.25				
Mexico by Dominguez	1.82	19.20	64.50	19.14	(0.06)
1584 Nov 18	22.25				
Mexico by Farfan	1.82	19.20	64.50	19.14	(0.06)
1584 Nov 18	22.25				
Mexico by Gudiel	1.82	19.20	64.75	19.12	(0.08)
1588 Sep 5	2.40	21.99	33.50	21.87	(0.12)
Puerto Rico by unknown	6.03	1.63	27.50	1.61	(0.02)
				avg. error =	(0.05)

* Modern values assumed for lunar declination

Céspedes reported that Velasco noted that the end of the eclipse took place at 2:16. (The document does not specify how Velasco obtained this time; he either computed them using his astrolabe observations or he used a clock.) Table 6 shows that, assuming a value for the lunar declination derived from modern calculation, Velasco's observation recorded using the Instrument of the Indies would have yielded a start time of 22:00 and an end time of 2:00. These compare favourably with modern values of 22:02 and 1:55, while the end time Velasco noted of 2:16 was 21 minutes too late.

Mexico City 1584

The sole complete set of eclipse observations made following the method outlined by Velasco's printed instructions corresponds to the eclipse of 18 November 1584 in Mexico City.[47] One of the observers was Jaime Juan who had been sent on an ambitious voyage of scientific exploration in 1583, a mission largely planned by the royal architect Juan de Herrera to test a number of longitude-finding instruments. Velasco, who had only a marginal role in planning the expedition, asked Juan to also measure lunar eclipses during his voyage. On the evening of the eclipse a group of observers, in the presence of the viceroy of New Spain Archbishop Pedro Moya de Contreras, gathered on the roof of the *casa reales* in the city of Mexico to observe the eclipse. Three other witnesses, Francisco Domínguez, Pedro Farfán and Cristóbal Gudiel, also had access to a "geared clock, very accurate" provided by the archbishop. They reported observing the end of the eclipse of between 19:27 to 19:31 and like Jaime Juan also made drawings as required by the instructions.

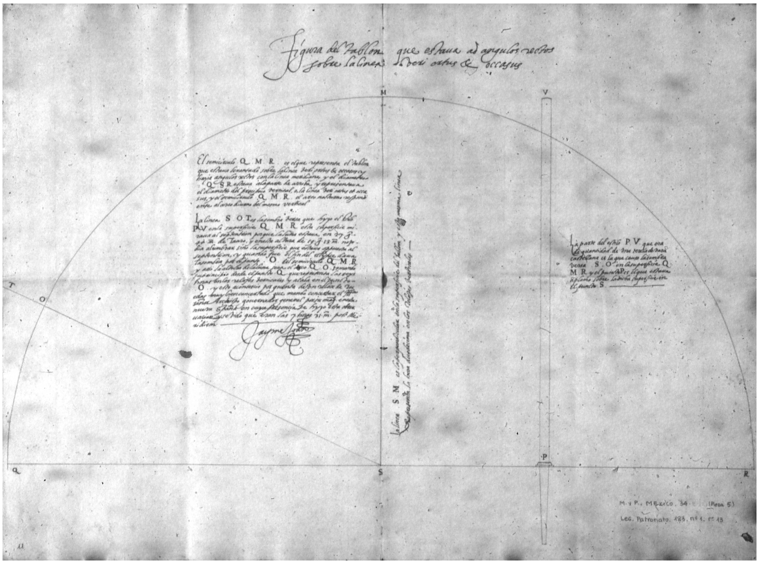


FIG. 9. Jaime Juan's 1584 eclipse observation. General Archive of Indies, Mapas y Planos, Mexico 34. Spain, Ministry of Culture.

Only the end of the eclipse was visible from Mexico — the Moon was only 25°48′ above the horizon when the eclipse ended — and the observations clearly reflect this fact (Figure 9). The set of observations made by Juan is somewhat different from the others. Juan's drawings explain the instrument setup in far more detail, include latitude calculations, and show that he carefully measured the angle cast by the Moon's shadow at the end of the eclipse using an astrolabe. He understood that the angle of the shadow cast on the instrument at the start and end of the eclipse was intended to function as a time-recording device using the same principles as those governing a vertical north-facing sundial.

In addition to dutifully recording the event according to Velasco's instructions, Juan also discussed — this time in Latin — two additional methods for determining the time of the end of the eclipse, neither of which used the observations taken by the Instrument of the Indies. (This clearly suggests he also ignored the question of how to convert the shadow observations into longitudinal coordinates.) Juan estimated the local time of the event by determining the angle between the Moon's position at the end of the eclipse and its position when it crossed the meridian and by calculating the Moon's position using a fixed star as the sidereal point of reference (Werner's method). He was well aware of the problems inherent in Finé's method and with charts of stellar positions.[48] His calculations yielded end times respectively of 19:20 and 19:22. These results are within ten minutes of the computed end time of 19:12.

As Table 6 shows, using Juan's value of $X°$ = 65.25°, Céspedes would have calculated an end time of 19:05 or a 7 minute error. The observations from the three other witnesses yield values within 3 to 5 minutes of the modern value.

San Juan, Puerto Rico 1588

The third set of lunar eclipse observations recorded using the Instrument of the Indies is a previously unpublished one made in Puerto Rico.[49] These are undated drawings made according to Velasco's instructions but without any documentation explaining the circumstances surrounding the observations. Archival records suggest the observation is from Puerto Rico, and for either 1581 or 1588. We know both eclipses were observed and their reports sent to Spain. One clue is in the response to the 1577 questionnaire from Puerto Rico dated 1 January 1582.[50] However, another letter, this time from 1588, suggests that the lunar eclipse of that year was also observed, recorded and the "best possible" results were sent to Spain.[51] The questions remain, Do the drawings at the Archive of Indies correspond to the 1581 eclipse or to the 1588 eclipse and do they correspond to the observations made in Puerto Rico?

I was able to inspect and take measurements from the four sheets of paper carefully pasted together showing the tracing of the sundial-like instrument. From these we can surmise that the observation was taken at a place near the equator (the Sun's shadow is short), but because the length of the gnomon was not noted, we have to assume the observers used, as instructed by the direction, a gnomon $\frac{1}{3}$ of a *vara* (27.3cm) in length. The drawing suggests they did so and luckily it also includes a small notation that the Sun's shadow was recorded on 2 September. (This indicates the 1588 lunar eclipse rather than the earlier ones.) Given a solar declination of 7°40′ on that date, the drawing implies that the observers were near latitude 19°N. The latitude of San Juan, Puerto Rico is 18°27′N.

From the drawing showing the shadow cast by the Moon at the beginning and the end of the eclipse we can surmise that the eclipse straddled the meridian in the location where the observation was recorded and that both the start and the end of the eclipse were visible. Only the eclipses of 16 July 1581 and 5 September 1588 meet these conditions for Puerto Rico. A physical measurement using a protractor of the shadow angles recorded in the observations indicates that at the beginning of the eclipse the shadow was 33.5° from the vertical and –27.5° at the end. On the basis of how these value correlate with those predicted using modern algorithms, I believe the drawings are indeed from Puerto Rico and for the lunar eclipse of 5 September 1588.

Table 6 shows the contact times determined by the shadow angles recorded on the observation and compares these to modern values. Once the observation is reduced to start and end times using Céspedes's equations, they yield a start time of 21:52 and an end time of 1:36, which when compared to modern computed times of 22:00 and 1:38 yield errors of 2 and 7 minutes respectively.

Sources of Errors in Observational Technique

Céspedes conceded that the results obtained from either of his two methods needed several adjustments. First, the time found using these calculations had to be adjusted by the time it took the Sun to pass from first contact to mid-eclipse. This time varied according to the type of eclipse, with total eclipses taking less time to reach mid-eclipse. This adjustment, he assured his reader, was only for those interested in "refining things too much [*adelgazar mucho las cosas*]", since the largest angle possible is about one degree (semi-diameter of the Moon plus the semi-diameter of the shadow of the Earth). He correctly estimated the error this contributed to the start or end times of the eclipse to be in the order of 2 to 3 minutes.

Secondly, and as noted above, he also warned that consulting ephemerides in order to determine the declination of the Moon used for the computation supplied only an estimated value. (Recall that this value had to be found in astronomical tables because of the Instrument of the Indies's inability to determine a value for the altitude of the Moon.) He considered that the error introduced by the declination value on the value of $\angle cmg$ and in the resulting time "will not be something sensible, especially for such remote distances".[52] An error in the order of 1° in declination value yields an error in local contact times of only ±1 minute. Moreover, if the same lunar declination value is used to compute local times of the same eclipse in two locations, the error cancels out and does not affect the difference in geographical longitude separating the two locations.

There are, however, other inaccuracies that result from the use of the shadow angles that Céspedes did not take into account. Determining precisely the start and the end of the eclipse is very difficult with the naked eye since the penumbral shadow that surrounds the partial phase can distort the observation. The instructions tried to overcome this by recommending that several people witness the eclipse and concur on when it began and ended.

The value of $X°$ is further compromised by distortions caused by atmospheric refraction. This is particularly the case for eclipses that happen near the horizon where the apparent altitude of the Moon can appear up to $\frac{1}{2}°$ higher. Refraction is negligible when the Moon has an altitude of more than 30°.[53] Even if we assume a maximum $\frac{1}{2}°$ error in both altitude and azimuth, this translates into less than 0.15° in the value of $X°$ and therefore represents about a minute from the start or end time. Interestingly, since the computation of local time does not depend on the comparison of the values of $X°$, distortions caused by parallax due to terrestrial distance do not affect the results. (In this case parallax is understood as the effect whereby the apparent position of the Moon changes with the observation point.)

Table 6 shows that the eight surviving observations of eclipse contacts recorded shadow angles that yielded times within 2 to 8 minutes of modern calculations. These errors, however, have been computed using modern declination values since we do not know the declination values Céspides used. Thus the average error of –3 mins indicates only a general approximation to the accuracy of the observations. Granted this, they still compare favourably with the level of accuracy Regiomontanus and Copernicus achieved in their eclipse observations, but do not approach the accuracy of Tycho Brahe's observations of the same lunar eclipses.[54] Given that the error factors discussed above can distort the observations by at most 5 minutes, we must attribute the remaining error to the subjective nature of naked-eye observations in determining the start and end of the lunar eclipse.

The eclipse project set in motion by the Cosmographer Major of the Council of Indies during the years 1577–88 was ambitious and was indeed the first coordinated worldwide attempt at systematic astronomical observations. Although not all the lunar eclipse observations have survived, this study shows that we must concur with Andrés García de Céspedes when he comments that the observers were diligent and that the Instrument of the Indies served its purpose adequately. The success of the project hinged on devising an instrument that could be cheaply and easily built and on employing observational techniques that were equally as simple. Furthermore, by integrating the project into the bureaucratic fibre of the Spanish empire, Juan López de Velasco and the Council of Indies were able to demand compliance and put in place a project that assembled standardized lunar eclipse observations from throughout the empire.

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3. Clinton R. Edwards, "Mapping by questionnaire and early Spanish attempts to determine New World geographical positions", *Imago mundi,* xxiii (1969), 17–28. Rodríquez-Sala published what were believed to be the sole surviving observations in María Luisa Rodríguez-Sala, *El eclipse de Luna: Misión científica de Felipe II en Nueva España* (Huelva, 1998).
4. Edwards's opinion is echoed in Barbara M. Mundy, *The mapping of New Spain: Indigenous cartography and the maps of the relaciones geográficas* (Chicago, 1996), 55–56. Randles, however, conceded that towards the end of the sixteenth century the Spanish had achieved remarkable accuracy in their longitude calculations, but still asked, "w[ere] such close figure[s] a fluke or the result of a real mastery of the process of measurements?". W. G. L. Randles, "Portuguese and Spanish attempts to measure longitude in the 16th century", *Vistas in astronomy*, xxviii (1985), 235–41, p. 238.
5. Edwards, "Mapping by questionnaire" (ref. 3), 17–18. For more on the reforms, see Juan Manzano Manzano, "La visita de Ovando al Real Consejo de las Indias y el Código Ovandino", in *El Consejo de las Indias en el siglo XVI* (Jornadas Americanistas, Valladolid, 1970), 111–23, pp. 119–21,

and Ernesto Schäfer, *El Consejo Real y Supremo de las Indias: Su historia, organización y labor administrativa hasta la terminación de la Casa de Austria* (2 vols, Madrid, 2003), i, 136–9.

6. Ordinance 118, in Consejo de Indias, *Ordenanzas reales del Consejo de Indias: Gobernación y estado temporal* (Madrid, 1585), 21v.
7. For more on Velasco see chap. 4 of Portuondo, *Secret science* (ref. 1). For the sake of brevity, I refer to persons with compounded last names by the final name, thus, López de Velasco becomes "Velasco".
8. General Archive of the Indies, Seville (AGI), P-184, R. 27 (27).
9. The sidereal circumstances surrounding the viceroy's report can be reconstructed using positions computed using modern computer algorithms. We know that the beginning of the partial phase of the eclipse took place at 01:05 UT on 17 November 1537 (calculated as the time of maximum eclipse minus the semi-duration of partial umbral phase). Since the time distance from Greenwich (UT) to Mexico City is 6:36 hours, the event would have been visible in Mexico City at 17:28 on the evening of 16 Nov. Sunset on 16 Nov. in Mexico City took place between 17:20 and 17:24. The viceroy noted the start of the event at "half a quarter hour after sunset" ($7\frac{1}{2}$ minutes after sunset), thus only minutes from the actual start time. The fact that the eclipse occurred so soon after sunset clearly influenced the accuracy of the reported time. The time-keeping device used, perhaps a sand-clock, would have been started at the moment of sunset. All predictions in this article use historical eclipse data courtesy of Fred Espenak, NASA/Goddard Space Flight Center. See http://Sunearth.gsfc.nasa.gov/eclipse/LEcat/LEcatalog.html.
10. AGI, P-49, R.12, Bloque 2.
11. Biblioteca de la Academia de Historia (Madrid) (BAH), Colección Muñoz, xxxiii, f. 249–265. "Relación de Juan Bautista Román, factor de las Islas Filipinas en Macao. Copia de la carta que me envio el P. Matheo Reci Italiano, Religioso de la compañia de Jesús que reside con el P. Miguel Ruggeiro en la ciudad de Junquinn cabeza de esta provincia de los Cantones en los Reinos de China", 28 September 1584, Macao.
12. Few texts explain the "longitude problem" in its historical context better than the classic E. G. R. Taylor, *The haven-finding art* (London, 1956). For an explanation of different methods of finding longitude during early modern era, see Charles H. Cotter, *A history of nautical astronomy* (London, 1968), 180–208. For a comprehensive survey of the topic, see W. J. H. Andrewes (ed.), *The quest for longitude: The proceedings of the longitude symposium, Harvard University, Cambridge, Massachusetts, November 4–6, 1993* (Cambridge, MA, 1993).
13. Richard L. Kremer and Jerzy Dobrzycki, "Alfonsine meridians: Tradition versus experience in astronomical practice, *c*. 1500", *Journal for the history of astronomy*, xxix (1998), 187–99. Steele cites only seven astronomers known to have systematically observed eclipses during the period 1200–1600. John M. Steele, *Observations and predictions of eclipse times by early astronomers* (Dordrecht and Boston, 2000), 133. For eclipse observations by Muslim astronomers, see S. S. Said and F. R. Stephenson, " Solar and lunar eclipse measurements by medieval Muslim astronomers, I: Background", *Journal for the history of astronomy*, xxvii (1996), 259–73, p. 262.
14. The impressive book Velasco used is still in the library of El Escorial: Cyprianus Leovitius, *Eclipsium omnium ab anno Domini 1554 usque in annum Domini 1606…* (Augsberg, 1556). His notes are in Royal Library of El Escorial, K-III-8, f. 311v–318v: "Copias del Eclipsium omnium ab año Domini 1554 usque in annum 1606 de Cypriani Leovitii", *c*. 1581.
15. Using Peurbach's contact times and adjusting them for Augsburg, Leovitius found the following for the eclipse of 16 (27) September 1577: "relinquuntur hor: 12. minut: 47. Unde principium consequitur hor: 10. minut: 54. Finis hor: 14. minut: 40." Therefore, he expected the start of the eclipse to take place at 22:54 and the end at 2:40. Modern computations adjusted for Augsburg (48°26′N, 10°56′E) place the start time at 21:33 and the end time at 1:26, for a 1:21 and 1:14 error respectively (eclipse data taken from Table 6).
16. J. A. Bennett, *The divided circle* (Oxford, 1987), 53–56. For a translation of Werner's method, see Appendix C in Andrewes (ed.), *The quest for longitude* (ref. 12), 380, 384–5.
17. Astronomers throughout the sixteenth century routinely complained about the predictive accuracy

of ephemerides, whether these were computed using the Alphonsine or the Prutenic Tables. Tycho expressed his concern in his correspondence. Victor E. Thoren, *The Lord of Uraniborg* (Cambridge, 1990), 16, 326. Thoren cites correspondence in *Tychonis Brahe opera omnia*, ed. by J. L. E. Dreyer (Copenhagen, 1913–29), v, 106–7. Michael Mastlin and Wilhelm IV, Landgrave of Hesse-Kassel, also questioned the accuracy of the Prutenic Tables. Richard A. Jarrell, "The contemporaries of Tycho Brahe", in *The general history of astronomy*, ii: *Planetary astronomy from the Renaissance to the rise of astrophysics*, A: *Tycho Brahe to Newton*, ed. by René Taton and Curtis Wilson (Cambridge, 1989), 22–32, p. 28.

18. Frederick J. Pohl, *Amerigo Vespucci, pilot major* (New York, 1945), 65–67.
19. Martín Fernández de Navarrete, *Disertación sobre la historia de la náutica y de las ciencias matemáticas que han contribuido a sus progresos entre los españoles*, ed. by Carlos Seco Serrano, lxxv–lxxvii: *Biblioteca de autores españoles* (Madrid, 1954–55), lxxvii, 333–4.
20. Peter Apian, *Cosmographicus liber Petri Apiani mathematici studiose collectus* (Landshut, 1524). Regiomontanus preferred to calculate eclipse local contact times from altitude observations made during the eclipse and to determine times using spherical trigonometry. John M. Steele and F. Richard Stephenson, "Eclipse observations made by Regiomontanus and Walther", *Journal for the history of astronomy*, xxix (1998), 331–44, p. 337.
21. A. Pogo, "Gemma Frisius, his method of determining differences of longitude by transporting timepieces (1530), and his treatise on triangulation (1533)", *Isis*, xxii (1935), 469–85. Sixteenth-century geared clocks had an expected accuracy of within a quarter-hour a day. Bennett, *Divided circle* (ref. 16), 60.
22. Oronce Finé, *Quadratura circuli.... De invenienda longitudinis locorum differentia* ... (Paris, 1544), 73–91. In his *Protomathesis*, Oronce Finé also discussed the mathematics behind the method of using lunar eclipses for determining longitude, but does not discuss instruments. Oronce Finé, *Protomathesis* … (Paris, 1532), 145v–46.
23. Pedro Nuñez, "De erratis Orontii Finoei liber unus", in *Petri Nonii Salaciensis opera* (Basel, 1592).
24. Eric G. Forbes and Curtis Wilson, "The solar tables of Lacaille and the lunar tables of Mayer", in *The general history of astronomy*, ii: *Planetary astronomy from the Renaissance to the rise of astrophysics*, B: *The eighteenth and nineteenth centuries*, ed. by René Taton and Curtis Wilson (Cambridge, 1995), 55–68.
25. The viceroy of New Spain complained that the instructions arrived on 9 Sept. AGI, Mexico 69, R. 5, N. 83, "Carta del virrey Martín Enríquez al rey", 19 October 1577.
26. Biblioteca Nacional de España (BN), MS3035. "Instrucción y advertimiento para observación de los eclipses de la luna, y cantidad de las sombras", 28 May 1578. This is the earliest known version of the eclipse instructions.
27. "[A] poco mas o menos, segun el parecer, y arbitrio de los que lo miraren." BN, MS3035, f. 40v.
28. Andrés García de Céspedes, *Regimiento de navegación e hydrografía* (Madrid, 1606), 128–45v.
29. "Although Juan López de Velasco sent the instructions of what had to be done in the Indies for observing the eclipse, he did not give [in the instructions] order or doctrine on how the start or end time of the eclipse could be known from such observation … nor have I found in all the papers I was given such a doctrine." García de Céspedes, *Regimiento de navegación* (ref. 28), 163v–64.
30. For more on the reform see M. I. Vicente Maroto and Mariano Esteban Piñeiro, *Aspectos de la ciencia aplicada en la España del Siglo de Oro*, 2nd edn (Valladolid, 2006), 390–414.
31. I thank the anonymous referee for pointing out the following: "The plane that passes through the gnomon and any given point of the semicircle (the $X°$ value) — call it the shadow plane — intersects the plane of the lunar orbit in a line. That is, for a given observation of $X°$, the Moon could be anywhere along a lengthy arc of its path! The 'Instrument of the Indies' works ONLY if you have independent evidence, taken from an ephemerides, for the lunar longitude, a value NOT observed by the instrument."
32. "El primero Problema de las tablas de Direccion de Iuan de Monterregio." He also suggested that an

astrolabe could be used to get the declination. In chaps. 15 and 16 he explains how to translate ecliptic longitude into declination using an instrument discussed in the second part of this article. García de Céspedes, *Regimiento de navegación* (ref. 28), 164–164v, 166–167.

33. "A esto digo que por las Efemerides, o algunas tablas, se sepa quando sera el principio del Eclipse, y para esta hora se puede hazer la cuenta de la declinacion, que aunque en el lugar de la Luna se errasse medio grado, o uno (lo qual no es possible) en la declinacion aura poca diferencia." *Ibid.*, 165.
34. The values computed using modern computer algorithms used in this and other tables rely on estimated values of the parameter ΔT (the difference between Terrestrial Dynamical Time (TD) and Universal Time (UT)). ΔT reflects the change in the Earth's rate of spin due to the effect of the tides. The modern positions used in this article extrapolate values for ΔT of the period 1500–1600 from 3 to 2.33 minutes based on F. R. Stephenson, *Historical eclipses and Earth's rotation* (Cambridge, 1997). Recently Morrrison and Stephenson have revised their original values of ΔT during the period of 1500 –1600 A.D. to between 3.3 and 2 minutes ± 20 sec. L. V. Morrison and F. R. Stephenson, "Historical values of the Earth's clock error ΔT and the calculation of eclipses", *Journal of the history of astronomy,* xxxv (2004), 327–36.
35. The following equations are adapted Fred Espenak's catalogue of historical lunar eclipses at http://sunearth.gsfc.nasa.gov/eclipse/LEcat/LEcatkey.html:

    "The altitude 'alt' and azimuth 'az' of the Moon during any phase of an eclipse depends on the time and the observer's geographic coordinates. Neglecting the effects of atmospheric refraction and lunar parallax, 'alt' and 'az' are calculated as follows:

    $h = 15 \times (\mathrm{GST0} + t - ra) + l,$

    $a = \arcsin [\sin d \sin f + \cos d \cos h \cos f],$

    $A = \arctan [-(\cos d \sin h) / (\sin d \cos f - \cos d \cos h \sin f)],$

    where h = hour angle of the Moon (in degrees); a = altitude (in degrees); A = azimuth (in degrees); GST0 = Greenwich Sidereal Time at 00.00 UT; t = Universal Time; ra = right ascension of the Moon (in hours); d = declination of the Moon (in degrees); l = observer's longitude (east +, west –); f = observer's latitude (north +, south –)."
36. García de Céspedes, *Regimiento de navegación* (ref. 28), 164–5.
37. "[P]or la 30 del libro 4 de los triangulos de Iuan de Monterregio." García de Céspedes, *Regimiento de navegación* (ref. 28), 164v. This theorem states that when two sides of a non-right triangle are known together with the angle opposite one of them, the remaining side and two angles may be found. Joannes Regiomontanus, *Regiomontanus: On triangles. De triangulis omnimodis*, transl. by Barnabas Hughes (Madison, 1967).
38. The following are the spherical trigonometric identities for right-spherical triangles used to solve this problem: $\tan B = \cot A / \cos c$; $\tan a = \cos B \times \tan c$, and $\cos B = \tan a / \tan c$, where A, B, C are the angles at the vertices with C as the right angle, and a, b, c are the arcs on the opposite sides determined by the angles at the centre of the sphere.
39. To illustrate how Céspedes's equations determine local contact times consider the lunar eclipse of 18 Nov 1584 and the modern values as set out in Tables 1 and 2. The given values are the observer's latitude $am = 19°24'$N, the Moon's declination $mg = 70.90°$ and the predicted value of $X°$ (from Equ. 3) $mat = 63.66°$. Using Equ. 4 and 5 we find respectively, $\angle amt = 27.70°$ and $mt = 17.32$, while Equ. 6 yields $\angle tmg = 83.80°$. Therefore since $\angle amg = \angle amt + \angle tmg$ and $\angle cmg = 180 - \angle amg$, we find a value of $\angle cmg = 68.50°$ for a distance of the Sun to the meridian that corresponds to 4.57 hours. Once adjusted by the difference between Greenwich sidereal time (GST0) and right ascension (4.57 + (3.8 – 3.57) = 4.80), it yields a local end time of 19.20 or 19:12, the same as predicted by modern values.
40. The instrument's construction appears in García de Céspedes, *Regimiento de navegación* (ref. 28), 32–33v.
41. Gemma Frisius, *De astrolabo catholico* (Antwerp, 1550). The *saphea* was well-known in Spain. Its construction and use was discussed in a treatise by Ali ibn Khalaf and included in Alphonso X "the Wise", *Libro del saber de astronomía* (13th century). The projection was employed in the late

sixteenth century in two instruments attributed to Juan de Herrera and discussed in R. Moreno, K. Van Cleempoel and D. King, "A recently discovered sixteenth-century Spanish astrolabe", *Annals of science*, lix (2002), 331–61. (Juan de Herrera and Céspedes were well-acquainted; the cosmographer taught at the mathematics academy Herrera instituted at Philip II's court.) The *saphea* was also known in Northern Europe before Gemma's reintroduction. The fourteenth-century abbot Richard of Wallingford describes in his *Albion* a very similar instrument to the one Céspedes proposed. J. D. North (ed.), *Richard of Wallingford* (3 vols, Oxford, 1976), ii, 188–91; iii, 36. For a study of the use of the universal astrolabe for coordinate conversion, see J. D. North, "Coordinates and categories: The graphical representation of functions in medieval astronomy", in his *The universal frame* (London, 1989), 1–16.

42. García de Céspedes, *Regimiento de navegación* (ref. 28), 166–8.
43. The CAD model of Céspedes's instrument was drawn with the stereographic projection grid at 5° intervals.
44. AGI, Mapas y Planos, Mexico 34.
45. AGI, Mapas y Planos, Teóricos-1 & 2. "Observación astronómica de la luna hecha en Puerto Rico demostrada en circulos ... 1600?"
46. García de Céspedes, *Regimiento de navegación* (ref. 28), 163.
47. AGI, I-740, N. 103. Drawings formed part of AGI, P-183, N. 1, R.13, and are currently in Mapas y Planos Mexico-34.
48. "Magnam enim habet propter varium lunae motum varietatem et non levem errandi suspicionem." Rodríguez-Sala, *El eclipse de Luna* (ref. 3), 166.
49. AGI, Mapas y Planos, Teóricos-1 & 2. "Observación astronómica de eclipse de luna hecha en Puerto Rico demostrada en circulos, c. 1600." The drawings formed part of AGI, P-175, R. 40.
50. AGI, Santo Domingo, 155, R. 9, N. 65. Juan de Céspedes, "Carta del gobernador de Puerto Rico al rey", 20 September 1580. Also, AGI, Santo Domingo, 155, R.10, N. 66. Juan Malgarejo, "Carta del gobernador de Puerto Rico al rey", 3 February 1582.
51. AGI, Santo Domingo, 155, R. 11, N. 118. Diego Menéndez, "Carta del gobernador de Puerto Rico al rey", 7 October 1588.
52. García de Céspedes, *Regimiento de navegación* (ref. 28), 165.
53. A. F. Aveni, *Skywatchers* (Austin, 2001), 104.
54. John M. Steele, *Observations and predictions of eclipse times by early astronomers* (Dordrecht and Boston, 2000), 143, 150, 153–4.