## DIO

## The Ptolemy-Venus Double-Date: Experimentum Crookis

Are Ptolemy's Apologists Right: Did His Proofs HAVE to Cheat?

## Were His Data REQUIRED to Contradict Each Other?

Are Admittedly Fabricated Data THE GREATEST Astronomy?

## Table of Contents

Page:
$\ddagger 5$ Ancient Solutions of Venus \& Mercury Orbits $\ddagger 6$ The Crucial-Test V-Bomb [Hey-Nobody's-Perfect] $\ddagger 7$ Unveiling Venus
by Dennis W. Duke 55
by Dennis Rawlins 70 by Hugh Thurston 91

In the 2002 March Isis (HistSciSoc), Harvard's O.Gingerich tries countering H.Thurston by defending fraudulent "observations" (Ptolemy's Venus data, incl. 2 different dates for the same event!), claiming the math was so hard that Ptolemy was forced to hoax. The present $D I O$ is comprised of demonstrations that a problem which stumped Gingerich's Greatest ancient astronomer is easily solved: even by a highschooler, as the below program proves. [Thurston’s geocentric method ( $\ddagger 7 \S E)$ finds identical orbital elements even more easily.]

## The "Essentially Insoluble" * - Solved by 10th Grade Math

10 Q\$="Q":IF Q\$<>"C"THEN F=2 ELSE F=1:REM"Q"equant,"C"eccentric
$20 \mathrm{~F} \$=" \mathrm{e}=\# \# \# . \# \# \# \quad \mathrm{~A}=\# \# \# \# . \# ": I F \mathrm{Q} \$<>$ "C"THEN PRINT"Eqt"ELSE PRINT"Ecc"
$30 \mathrm{~S}=3: \mathrm{E} \$=" \mathrm{n}=\# \# \quad$ \# $: \mathrm{G} \$=\mathrm{F} \$+" \quad \mathrm{r}=\# \# \# . \# \# \# ": \mathrm{H} \$=\mathrm{E} \$+\mathrm{G} \$$
$40 \mathrm{PI}=3.141593: \mathrm{C}=180 / \mathrm{PI}: \mathrm{C} \$=\mathrm{E}$ \$ +F : IF Q $\$=" \mathrm{C} " \mathrm{GOTO} 70$
$50 \mathrm{E} 0=1 / 60: \mathrm{A}=60: \mathrm{X} 0=\mathrm{E} 0 * \mathrm{COS}(\mathrm{A} / \mathrm{C}): \mathrm{Y} 0=\mathrm{E} 0 * \mathrm{SIN}(\mathrm{A} / \mathrm{C})$
60 PRINT USING C\$;N,E0*60,A
$70 \mathrm{G}(1)=+(43+24 / 60): \mathrm{V}(1)=148+34 / 60:$ REM AD138Evening
$80 \mathrm{G}(2)=+(48+17 / 60): \mathrm{V}(2)=015+08 / 60:$ REM AD140Evening
$90 \mathrm{G}(3)=-(47+09 / 60): \mathrm{V}(3)=064+06 / 60:$ REM AD140Morning
100 FOR I=1 TO 3
$110 \mathrm{G}(\mathrm{I})=\mathrm{G}(\mathrm{I}) / \mathrm{C}: \mathrm{V}(\mathrm{I})=\mathrm{V}(\mathrm{I}) / \mathrm{C}: \mathrm{L}(\mathrm{I})=\mathrm{V}(\mathrm{I})-\mathrm{G}(\mathrm{I})$
$120 \mathrm{~L}=\mathrm{L}(\mathrm{I}) * \mathrm{C}: \mathrm{L}=\mathrm{L}-360 * \mathrm{INT}(\mathrm{L} / 360): \mathrm{L}(\mathrm{I})=\mathrm{L} / \mathrm{C}$
130 NEXT I
140 FOR I=1 TO 3:G=L(I)-A/C:IF Q\$="C"GOTO 160
$150 \mathrm{H}=\mathrm{E} 0 * \mathrm{SIN}(\mathrm{G}): \mathrm{G}=\mathrm{G}-\mathrm{ATN}\left(\mathrm{H} / \mathrm{SQR}\left(1-\mathrm{H}^{\wedge} 2\right)\right)$
$160 \mathrm{QX}=\mathrm{COS}(\mathrm{G})+\mathrm{E} 0: \mathrm{QY}=\mathrm{SIN}(\mathrm{G}): \mathrm{GQ}=\mathrm{C} * \mathrm{ATN}(\mathrm{QY} / \mathrm{QX}): \mathrm{IF} \mathrm{QX}<0 \mathrm{THEN} \mathrm{GQ}=\mathrm{GQ}+180$ $170 \mathrm{R}=\mathrm{SQR}\left(\mathrm{QX} \mathrm{N}^{\wedge} 2+\mathrm{QY}^{\wedge} 2\right): \mathrm{LQ}=\mathrm{GQ}+\mathrm{A}$
$180 \mathrm{X}(\mathrm{I})=\mathrm{R} * \operatorname{COS}(\mathrm{LQ} / \mathrm{C}): \mathrm{Y}(\mathrm{I})=\mathrm{R} * \mathrm{SIN}(\mathrm{LQ} / \mathrm{C})$
$190 \mathrm{M}(\mathrm{I})=\mathrm{TAN}(\mathrm{V}(\mathrm{I})): \mathrm{B}(\mathrm{I})=\mathrm{Y}(\mathrm{I})-\mathrm{M}(\mathrm{I}) * \mathrm{X}(\mathrm{I}):$ NEXT I
200 FOR I=1 TO 3
$210 \mathrm{~J}=\mathrm{I}-3 * \mathrm{INT}(\mathrm{I} / 3)+1$
220 IF $\operatorname{SGN}(\mathrm{G}(\mathrm{I}))=\mathrm{SGN}(\mathrm{G}(\mathrm{J}))$ THEN $\mathrm{SH}=\mathrm{PI}$ ELSE $\mathrm{SH}=0$
$230 \mathrm{P}(\mathrm{I})=-(\mathrm{B}(\mathrm{I})-\mathrm{B}(\mathrm{J})) /(\mathrm{M}(\mathrm{I})-\mathrm{M}(\mathrm{J})): \mathrm{Q}(\mathrm{I})=(\mathrm{M}(\mathrm{I}) * \mathrm{~B}(\mathrm{~J})-\mathrm{M}(\mathrm{J}) * \mathrm{~B}(\mathrm{I})) /(\mathrm{M}(\mathrm{I})-\mathrm{M}(\mathrm{J}))$
$240 \mathrm{U}(\mathrm{I})=(\mathrm{V}(\mathrm{I})+\mathrm{V}(\mathrm{J})-\mathrm{SH}) / 2: \mathrm{H}(\mathrm{I})=(\mathrm{V}(\mathrm{I})-\mathrm{V}(\mathrm{J})-\mathrm{SH}) / 2$
$250 \mathrm{~T}(\mathrm{I})=\mathrm{TAN}(\mathrm{U}(\mathrm{I})): \mathrm{F}(\mathrm{I})=\mathrm{Q}(\mathrm{I})-\mathrm{T}(\mathrm{I}) * \mathrm{P}(\mathrm{I}):$ NEXT I
260 FOR I=1 TO 3: $\mathrm{J}=\mathrm{I}-3 * \mathrm{INT}(\mathrm{I} / 3)+1$
$270 \mathrm{C}(\mathrm{I})=-(\mathrm{F}(\mathrm{I})-\mathrm{F}(\mathrm{J})) /(\mathrm{T}(\mathrm{I})-\mathrm{T}(\mathrm{J}))$
$280 \mathrm{D}(\mathrm{I})=\mathrm{T}(\mathrm{I}) * \mathrm{C}(\mathrm{I})+\mathrm{F}(\mathrm{I})$
$290 \mathrm{~S}(\mathrm{I})=\mathrm{SQR}\left((\mathrm{P}(\mathrm{I})-\mathrm{C}(\mathrm{I}))^{\wedge} 2+(\mathrm{Q}(\mathrm{I})-\mathrm{D}(\mathrm{I}))^{\wedge} 2\right)$
$300 \mathrm{R}(\mathrm{I})=\operatorname{ABS}(\mathrm{S}(\mathrm{I}) * \operatorname{SIN}(\mathrm{H}(\mathrm{I}))): \mathrm{NEXT} \operatorname{I}$
$310 \mathrm{X} 0=\mathrm{X} 0-\mathrm{C}(3) / \mathrm{F}: \mathrm{Y} 0=\mathrm{Y} 0-\mathrm{D}(3) / \mathrm{F}$
$320 \mathrm{R} 0=\mathrm{R}(3) * 60: \mathrm{N}=\mathrm{N}+1$
$330 \mathrm{E} 0=\mathrm{SQR}\left(\mathrm{X} 0^{\wedge} 2+\mathrm{Y} 0^{\wedge} 2\right): \mathrm{E}=60 * \mathrm{E} 0: \mathrm{A}=\mathrm{C} * \mathrm{ATN}(\mathrm{Y} 0 / \mathrm{X} 0): \mathrm{IF}$ X $0<0$ THEN A=A+180
340 IF Q\$<>"C"'THEN PRINT USING H\$;N,E,A,R0 ELSE PRINT USING G\$;E,A,R0
350 IF N $<$ S AND Q\$<>"C" GOTO 140

* See $\ddagger 6 \S$ B4. The above BASIC program performs the method developed within from $\ddagger 6 \S \mathrm{D}$ (teen-age-level ruler\&compass crux) through to $\S($ (iterative application to equant planet model).


## $\ddagger 5$ Ancient Solutions of Venus \& Mercury Orbits by Dennis Duke

## A The Question

A1 In chapters IX-XI of the Almagest ${ }^{2}$ Ptolemy produces a rather small set of observations for each of the five planets, most of which he specifically claims to have made himself, and proceeds to systematically use those observations to derive each of the parameters of his final planetary models: a rather complicated crank mechanism for Mercury, and the equant model for Venus, Mars, Jupiter, and Saturn. Wilson, Newton, Rawlins, Swerdlow, and Thurston have analyzed Ptolemy's presentations. ${ }^{3}$ Each concludes that Ptolemy simply did not do what he wrote that he did in his Almagest presentations on Venus and Mercury; and Newton, Rawlins, and Thurston claimed the same for the outer planets Mars, Jupiter and Saturn. Instead, they show that Ptolemy very likely already knew the values of the parameters of his model and adjusted his 'observations' to make his 'derivations' of those parameters appear direct and simple.
A2 So if Ptolemy inherited the values of the parameters, or if he derived them himself from some prior analysis that he chose not to leave us, then the question is: how would one use ancient data to derive fairly accurate values for these parameters? As many previous commentators have assumed, the most plausible scenario is that the ancients had somehow managed to assemble a rather substantial set of observations, perhaps over many decades. So one way to seek an understanding of the question is to assemble for ourselves a set of observations that could plausibly have been available to an ancient astronomer - and then to try analyzing those observations in the context of Greek geometrical models, to see what parameters emerge.
A3 The principal observations used for the inner planets are greatest elongations. Ptolemy defined the elongation of an inner planet as the difference in longitude of the planet and the mean Sun, and we shall assume that his predecessors did likewise. Since the longitude of the mean Sun is obtainable only in the context of a theory of the Sun's motion, we know that insofar as elongations are used to fix the parameters of planetary models, the existence of a reasonably good model of the Sun is then a prerequisite. Now measurements of planetary longitudes were generally made relative to the longitudes of reference stars, and the tropical longitudes of reference stars must be measured with respect to the Sun (and need not coincide with the observation of the planet), so any error in the solar model will be directly transmitted as an error in the planet's tropical longitude. Thus the elongations will be somewhat immune to the simplest errors in the solar theory, such as a misplaced equinox. If, however, the errors in the solar theory grow with time (as, e.g. in Hipparchus' solar theory), then pairs of morning and evening elongations at the same longitude of the mean Sun will incur errors, and these errors will be most apparent when (eq. 1 at $\S \mathrm{E} 2$ ) we compute the difference of the absolute values $\eta$ of the elongations.

[^0]
## B The Data Required for Solution

B1 In order to generate samples of historical data I use the planetary models of Bretagnon and Simon, ${ }^{4}$ which yield geocentric longitudes and latitudes for the Sun and the planets as far back as 4000 BC , and to far greater accuracy than needed for this investigation. Figs. $1 \& 2$ show the evening (i.e. positive) and morning (i.e. negative) elongations of Venus as a function of the Sun's mean longitude using positions computed at five day intervals over $400 \mathrm{BC}-150 \mathrm{BC}$. The outer envelopes of values thus determine the morning and evening greatest elongations as a function of solar mean longitude. ${ }^{5}$ Fig. 3 shows the sum of the absolute values of greatest morning and evening elongations as a function of solar mean longitude, while Fig. 4 shows the algebraic sum of the greatest evening and morning elongations, i.e. the difference of their magnitudes. Figs. $5-8$ show the corresponding results for Mercury. For comparison and later reference, the figures also include the corresponding result from using the Almagest models of Venus, Mercury and the mean Sun to generate planetary positions. And although I am using charts throughout this paper, the ancient analyst was most likely using tables of numbers. We do know, though, that the analysts were very proficient at using tables. They could not only interpolate, but also find local maxima, minima, and rates of change of their tabulated functions. Presumably all of the analysis I do with graphs below was done just as well with tables in ancient times.
B2 The set of data so collected is clearly of far greater quality than we can reasonably expect for an ancient data collection, and the issues associated with that point will be addressed below. But for now let us assume that such a data collection is available and see how it might be analyzed in the context of Greek geometrical models. This will show us at least what is ideally possible, and hence provide an initial frame of reference for the later analysis of more realistic sets of observations.
B3 Of the possible Greek geometrical models we shall consider three. First is the simple model known at least as far back as Apollonius: a concentric deferent of radius $R$ with an epicycle of radius $r$. Second is an intermediate model which the Almagest refers to only indirectly, ${ }^{6}$ but which we can be fairly sure was at least considered at some point: an eccentric deferent with radius $R$ and eccentricity $e$, and an epicycle of radius $r$. And third is the Almagest model (excepting Mercury): an eccentric deferent with radius $R$ and eccentricity $e$, and an epicycle of radius $r$ which moves uniformly about the equant, a point which lies on the apsidal line a distance $e^{\prime}$ from the Earth. In all three models the planet revolves uniformly around the epicycle with a period in anomaly, and the center of the epicycle revolves on the deferent with (for the inner planets) the period of the mean Sun. In the first two models the center of uniform motion is the center of the deferent, while in the third model the center of uniform motion is the equant.
B4 The parameters of the models are therefore: (1) the mean motions in longitude and in anomaly, (2) the direction of the apsidal line, and its change in direction with time, (3) the radius $r$ of the epicycle, (4) the eccentricity $e$ of the deferent, (5) the distance $e^{\prime}$ between the Earth and the center of uniform motion (the equant), and (6) the values of mean longitude, anomaly, and apogee at some initial time. In addition, for the inner planets Ptolemy makes the assumption that the direction of the line from the center of uniform

[^1]motion to the epicycle center is parallel to the line from the Earth to the mean Sun. We do not know whether astronomers earlier than Ptolemy assumed this, but in the following we shall assume that they did.

## C Finding the Orbital Elements

C1 Now for all five planets, various period-relations were very well known and clearly could have been used to derive the mean motions. In addition, a single observation of longitude at a known time $t$ is sufficient to fix the initial values once the other parameters are decided. So for all five planets the principal problem is to find values for the direction $\lambda_{\mathrm{A}}$ of the apsidal line, the epicycle radius $r$, the deferent eccentricity $e$, and the distance $e^{\prime}$ of the equant from the Earth.
C2 For Venus and Mercury, the most obvious quality we notice is that the greatest elongations are not constant as the planet traverses the zodiac. Presumably this was realized very early, and so the ancient astronomers would have known that the simplest Apollonius model with a simple epicycle on a concentric deferent could not work. Now for a given distance $R$ between observer and epicycle center, and a given radius $r$ of the epicycle, the greatest elongation $\eta$ results when the line of sight from Earth to the planet is nearly tangent to the epicycle, so that $r$ is effectively determined by the simple relation $r=R \sin \eta$. Since $\eta$ is observed to be not constant, then either the epicycle radius $r$ or the distance $R$ to the epicycle center, or both, must be varying. The ancient Greek analysts apparently always chose to keep the epicycle radius $r$ fixed. It is quite plausible, however, that they realized that they could estimate the epicycle radius by simply observing the average greatest elongation, which will occur when the epicycle center is its average distance from the observer. Using the conventional norm $R=60^{\mathrm{p}}$ and the data shown in Figs.1\&2, which yield an average elongation of about $46^{\circ} .2$, the implied epicycle radius is $43^{\mathrm{p}} 1 / 3$. When rounded, this agrees exactly with the value attributed by Pliny [2.6.38] to Timaeus of $46^{\circ}$, which implies an epicycle radius of $43^{p} 1 / 6$, the value Ptolemy uses in the Almagest. The average value of the elongations for Mercury, shown in Figs.5\&6 is about $22^{\circ} .3$, which also rounds to the value of $22^{\circ}$ that Pliny [2.6.39] attributes to Cidenas and Sosigenes, and which leads to $r=22^{\mathrm{p}} ; 28,35 \approx 22^{\mathrm{p}} 1 / 2$, again the value Ptolemy uses. Thus the epicycle radii for the inner planets follow simply from knowledge of the average greatest elongations, and since they were apparently known long before Ptolemy's time, we might have some confidence that enough greatest elongations were observed to provide adequate estimates of the average.

## D The Apsidal Line

D1 The next task is to determine the direction $\lambda_{\mathrm{A}}$ of the apsidal line. One idea is that apogee is the direction in which the sum of greatest evening and morning (absolute) elongations is minimum, and hence the epicycle is farthest from the observer, while perigee is the direction in which the sum is largest. This method is also the least sensitive to any error in the computed position of the mean Sun. Ptolemy alludes to this method in Almagest X. 2 when he says, "Furthermore, it has also become plain to us that the eccentre of Venus carrying the epicycle is fixed, since nowhere on the ecliptic do we find the sum of the greatest elongations from the mean [Sun] on both sides to be less than the sum of both in Taurus, or less than the sum of both in Scorpius." Thus Fig. 3 shows that Venus' apogee is around $52^{\circ}$ and that perigee is about $180^{\circ}$ away, around $232^{\circ}$. It might have also occurred to the ancient analyst to ask for the direction in which the morning and evening elongations sum (algebraically) to zero. Fig. 4 shows that this occurs at about $60^{\circ}$ and $236^{\circ}$ for the real data. The positions for apogee and perigee that result from these two methods are not equal
due to various technical reasons: the true orbits are elliptical, and the true mean Sun does not lie on the line between the center of Earth's orbit and the center of Venus' orbit, ${ }^{7}$ etc. It is also clear from Figs. $3 \& 4$ that the symmetry method that Ptolemy claimed to be using to find the direction of the apsidal line should have produced a result much closer to $60^{\circ}$ than to $52^{\circ}$, whereas he in fact produced a result, adjusting for his $1^{\circ} /$ cy apsidal precession (from his epoch to the end-epoch of Figs.3\&4), close to $52^{\circ}$. This suggests that he was using a symmetry method with data adjusted to give a result that he had inherited, and that result was fairly accurately derived from more accurate data using the sums method.
D2 Consideration of the sums of greatest elongations for Mercury (see Fig.7) shows that the direction of the apogee for the real data is at about $220^{\circ}$ and perigee is about $180^{\circ}$ away at $40^{\circ}$. On the other hand, the difference method (see Fig.8) gives an apogee for Mercury at about $205^{\circ}$ and a perigee at about $28^{\circ}$. The Almagest model data give an apogee at just over $180^{\circ}$ and, by construction, double perigees about $120^{\circ}$ away from apogee.

## E Eccentricity

E1 Finally, we look at the determination of the eccentricity and the position of the equant. Let's begin by thinking in terms of the simplest model that might work, the intermediate model which has an eccentric-deferent and an epicycle. In this model the center of the deferent and the center of uniform motion are the same, and so $e=e^{\prime}$. The catch, however, is that there is one way to estimate $e$ and another way to estimate $e^{\prime}$, and as we shall shortly see, these two different methods give different estimates.
E2 First we estimate $e^{\prime}$. If the mean Sun is at longitude $\lambda_{\bar{\odot}}$ and the longitude of the apogee is at $\lambda_{\mathrm{A}}$, then the equant size $e^{\prime}$ is given by

$$
\begin{equation*}
\frac{e^{\prime}}{R}=\frac{\sin c}{\sin (\alpha-c)} \tag{1}
\end{equation*}
$$

where $\alpha=\lambda_{\bar{\odot}}-\lambda_{\mathrm{A}}$, and $c=\left(\eta_{\mathrm{M}}-\eta_{\mathrm{E}}\right) / 2$. [Note: $\eta_{\mathrm{M}} \& \eta_{\mathrm{E}}=$ morning \& evening $\eta$, respectively. Each $\eta$ represents absolute magnitude.] This method is only useful, however, when the mean Sun is well away from apogee or perigee, and when the two elongations are close enough in time that the apogees of the observations are not significantly different. If we analyze the real data using such a model we can estimate $e^{\prime}$ when the longitude of the mean Sun is near quadrant. The resulting estimated value of $e^{\prime}$ is about $1^{\mathrm{p}} .85$, assuming $R=60^{\mathrm{p}}$.
E3 On the other hand, the ancient analyst would estimate $e$, the eccentricity of the deferent, using elongations as close as possible to the apsidal line. In fact, if $\eta_{\mathrm{P}}$ and $\eta_{\mathrm{A}}$ are elongations at perigee and apogee, respectively (and evening or morning doesn't matter by symmetry, so one could also just average the morning and evening elongations near apogee and perigee), then the eccentricity $e$ is given by

$$
\begin{equation*}
\frac{e}{R}=\frac{\sin \eta_{\mathrm{P}}-\sin \eta_{\mathrm{A}}}{\sin \eta_{\mathrm{P}}+\sin \eta_{\mathrm{A}}} \tag{2}
\end{equation*}
$$

E4 Near the apogee of the real data the eccentricity $e$ is about $0^{\mathrm{p}} .9$ (assuming $R=60^{\mathrm{p}}$ ). What is particularly clear is that the ratio $e^{\prime} / e$ is close to 2 for the real data. Therefore these two relatively simple analyses send a clear signal that for Venus the center of the deferent is closer to the Earth than the center of uniform motion, in contradiction to the assumption of the intermediate model. It is plausible, then, that it was the need to reconcile this contradiction that led to the creation of the equant model that we find for Venus in the Almagest. It is certainly the case that for Venus, and only for Venus, Ptolemy presents an analysis that closely parallels the above to explain the problem that needs to be resolved.

[^2]E5 Repeating the equant determination using Mercury data leads to $e^{\prime}=3^{p} .25$ and $e=4^{\mathrm{p}} .6$, implying that the center of uniform motion is closer to the Earth than is the center of the deferent, and thus opposite to the situation with Venus. This is an unavoidable consequence of the fact that the center of Mercury's orbit is farther from the Sun than the center of Earth's orbit. ${ }^{8}$ We can only surmise that perhaps the difficulty of reconciling the different sizes of the eccentricity and the equant led to the creation of a special model for Mercury. Perhaps it is also possible that Ptolemy (or whoever invented the model) could not find a way to make the crank mechanism account for the variation in greatest elongation without also introducing a double perigee, but in absence of more detailed analysis we can say very little with certainty.
E6 Each of these determinations of the equant and the eccentricity involves getting a small number from the difference of two experimentally measured larger numbers, and hence all the estimates unavoidably have substantial relative errors. Thus it is not surprising that the values used in the Almagest, whoever they might originate from, differ at the $15-20 \%$ level from our more exact estimates which use accurate data.

## F The Time Factor

F1 Overall, then, we see that given an adequate (and some might say extravagant) base of historical data, it is plausible that straightforward analysis using techniques that we expect were accessible to ancient astronomers leads to just the results for Venus and Mercury that we find in the Almagest. However, all the above is based on accurate values of longitudes sampled at 5 day intervals over a period of 250 years. In reality, of course, the measurements would not have been so accurate, nor the sampling nearly so regular, nor the interval necessarily so long. In addition, especially for Mercury, one should account for the fact that the planets are not always visible.
F2 For Venus the primary issue is the length of the time interval over which observations were available. To take an extreme example, let us suppose that Ptolemy only had data over the time period explicitly mentioned in the Almagest, 127-141 AD. The available data ${ }^{9}$ are shown in Figs. $9 \& 10$ and reflect the synodic periodicity of Venus' orbit. Estimating the direction of the line of apsides from Fig. 10 alone would be hazardous at best, and even if you somehow knew the direction of apogee there are not enough observations near apogee, perigee or quadrant to make a good estimate of the eccentricity or the equant lengths. In fact, unless one includes observations prior to about 50 AD , one could not know that the maximum sum of elongations was actually in Scorpio, as Ptolemy tells us in Almagest X.2. ${ }^{10}$

F3 How many years of Venus observations are enough to get useful results? Fig. 11 shows the results of collecting data for 100 years. Since the general trend of the curve is at least partially defined now at a number of points, one can get an impression of the

[^3]longitudes of the maximum and minimum sums, and by interpolation estimate any needed values. So it appears that if the ancient analyst had access to about a century's worth of data, he would be able to use that data as we have discussed to estimate the needed parameters in the intermediate model. More data might have been available, but the fact is we have no direct evidence that such data series ever existed. What we are showing in this paper is that if the data existed, then it is plausible that the ancient analyst could use the data to estimate values of the parameters of geometrical models.
F4 For Mercury the primary issue is not so much the length of the time interval of the observations as the difficulty of observing Mercury at different times of the year. To once again take an extreme example, we use the time interval 127-141 AD. In order to make sure we record Mercury only when it is visible, we now generate longitudes every 6 minutes. However, we also compute the altitudes of the Sun and Mercury assuming we are at Alexandria, and we record the observation only if the Sun is $5^{\circ}$ or more below the horizon and if Mercury is $5^{\circ}$ or more above the horizon. The results are shown in Figs.12\&13. It is sometimes said ${ }^{11}$ that the shallow angle of the ecliptic during Spring mornings and Fall evenings would make the observations of Mercury difficult if not impossible. In the context of Ptolemy's Almagest analysis of Mercury, there are two needed observations that are conspicuously missing: the morning of 131 Apr 4 and the evening of 138 Oct 4. Precise calculation shows, however, that according to the visibility conditions being used here, ${ }^{12}$ Mercury was visible on the first date for about 16 minutes and on the second date for about 8 minutes. And although these intervals of opportunity are narrow, a few minutes of visibility obtained for about a week on both sides of the target date. However, even allowing for considerable further degradation of the data, and omitting the Spring morning and Fall evening elongations, it is clear that adequate data might well have been available to allow a determination of model parameters as discussed above. Furthermore, the idea that the difficulty in observing Mercury would lead to relatively fewer observations of Mercury is not supported by the historical records, since, as pointed out by Swerdlow, ${ }^{13}$ the Astronomical Diaries ${ }^{14}$ contain nearly three times as many observations of Mercury as of Jupiter and Saturn, and LBAT 1377, ${ }^{15}$ a text devoted to Mercury, contains more observations than all the surviving Diaries.

## G The Outer Planets

G1 For the outer planets the observation of choice is the opposition, at which the planet, the Earth and the mean Sun are aligned (with the Earth in the middle). In the Almagest Ptolemy applies an elegant geometrical analysis using three oppositions, to determine the direction of the apsidal line and the size of the equant. Evans, however, has suggested a much simpler method that uses a time history of oppositions to accomplish the same goal. ${ }^{16}$ To locate the apogee of the deferent, one considers the average distance between

[^4]oppositions as a function of the longitude of the oppositions. This function is minimum at the longitude of the apogee. Examples for Mars, Jupiter, and Saturn are shown in Fig.14, using oppositions that occurred over the (arbitrary) interval 250 BC-150 BC. Then by picking an opposition as near as possible to apogee and a second opposition at some other longitude, one can compute in the context of the intermediate model the effective size of the equant (or eccentricity - they are equal in the intermediate model) as seen from different longitudes of the (second) opposition using the formula
\[

$$
\begin{equation*}
e / R=\sin c / \sin a \tag{3}
\end{equation*}
$$

\]

where $a=\lambda-\lambda_{0}, \alpha=\omega_{t}\left(t-t_{0}\right)$, and $c=\alpha-a$. In these equations $\omega_{t}$ is the mean motion in longitude, the opposition at apogee has longitude $\lambda_{0}$ at time $t_{0}$, while the second opposition has longitude $\lambda$ at time $t$. Assuming that the distance $R=60^{\mathrm{p}}$ is constant, one finds for all three outer planets an effective equant size that decreases as you move away from apogee, as shown in Fig.15. Or inversely, if one chooses to keep the effective equant size constant, then it must be that the distance $R$ is increasing as the planet moves away from apogee. This is, of course, precisely what happens to the distance between the center of uniform motion and the center of the epicycle in the Almagest equant model.
G2 Evans has also shown another approach to motivating the Almagest equant model that uses the varying width of opposition loops and their unequal spacing in longitude. ${ }^{17}$ It is certainly possible that either method, or perhaps both, provided the motivating factors that first exposed the inadequacy of the intermediate model, and then suggested a solution. In any event, though, the fact that the anciently attested values for all five planets agree so well with the results from modern calculation shows that the ancient observations, however they were collected and analyzed, must have been adequate for the purpose.

## H Practicability

In summary, the various analyses show simple and accessible methods whereby ancient astronomers might well have used time histories of the longitudes of planets, combined with a solar model and the longitudes of a few bright stars near the ecliptic, to estimate the parameters of their models. The primary technical supplement to the geometrical models might well have been extensive sets of tables, just as Ptolemy himself eventually uses in the Almagest. Coupled with the nature of Ptolemy's own presentations in the Almagest, as discussed by Wilson, Newton, and Swerdlow, these results therefore suggest that such practice predates the analyses Ptolemy left us in the Almagest.

[^5]Evening Observations of Venus $400 \mathrm{BC}-150 \mathrm{BC}$


Figure 1: See $\S \S B 1 \& C 2$.

## Morning Observations of Venus $400 \mathrm{BC}-150 \mathrm{BC}$



Figure 2: See $\S \S B 1 \& C 2$.

Sum of Greatest Elongations of Venus $400 \mathrm{BC}-150 \mathrm{BC}$


Figure 3: See $\S \S B 1 \& D 1$.

Difference of Greatest Elongations Venus $400 \mathrm{BC}-150 \mathrm{BC}$


Figure 4: See $\S \S B 1 \& D 1$.

Evening Observations of Mercury 400 BC -150 BC


Figure 5: See $\S \S B 1 \& C 2$.

## Morning Observations of Mercury 400 BC -150 BC



Figure 6: See $\S \S$ B1\&C2.

Sum of Greatest Elongations of Mercury $400 \mathrm{BC}-150 \mathrm{BC}$


Figure 7: See $\S \S$ B1\&D2.

Difference of Greatest Elongations Mercury $400 \mathrm{BC}-150 \mathrm{BC}$


Figure 8: See $\S \S$ B1\&D2.

Obs ervations of Venus 127-141 AD


Figure 9: See §F2.

Sum of Greatest Elongations of Venus 127-141 AD


Sum of Greatest Elongations of Venus 230 BC -130 BC


Figure 11: See §F3.

Figure 10: See $\S$ F2.

Observations of Mercury 127 AD - 141 AD


Longitude of the Mean Sun (degrees)

Figure 12: See §F4.

Sum of Greatest Elongations of Mercury 127-141 AD


Figure 13: See §F4.

Apogees of Outer Planets


Figure 14: See §G1.


Figure 15: See §G1.

## $\ddagger 6$ The Crucial-Test V-Bomb [Hey-Nobody’s-Perfect] How Claudius Ptolemy Could've Solved Venus' Orbit Honestly Greatest Elongations Exceeded by Greatest's Elongations

## by Dennis Rawlins

Can RN-DR be accused of cruelty to dumb animals, given the tightness of the evidential vise they've closed on the poor [Ptolemy defense-corps]? To watch prominent scholars thrashing about in such pathetic credibility-death agonies is akin to viewing Animal-Rights films of stoats caught in spring traps trying to weasel out.
[Scholars who wish never to find themselves in the excruciating\&logicbending position of Believers who've spent decades cornering themselves into having to keep forever alibiing Ptolemy's Venus, ${ }^{2}$ stellar, ${ }^{3}$ \& etc ${ }^{4}$ pretensions, are urged to ponder DIO $10 \odot 2$ pp.83-84. Watching Muffiosi forgive sin after Ptolemy sin, B.Rawlins recalls Some Like It Hot's finale: in-love Osgood hitch-pitches in-drag "Daphne", who reluctantly protests that "she" smokes, dyes, is barren, etc, etc. But Osgood forgives all. Desperate, Daphne finally bellows the ultimate impedimentum-crucis-bomb: I'M A MAN!!! Osgood: Well, nobody's perfect.]

## A Muffiosi Laud Deliberate Fraud When You're Cornered

A1 On 1983/6/4, at a conference in Aarhus, Denmark, DR announced ${ }^{5}$ that Owen Gingerich's "Greatest Astronomer of Antiquity" - the infamous ancient plagiarist C.Ptolemy - had been faking "observations" with such profligate-sloppy haste (similar cases: fn 14) that he actually gave (fn 24) the same Venus event 2 different dates over a month apart. [But Osgood Gingerich is still in love. . . .] In the long history of the oldest science, no (other?) astronomer ever pulled off a blunder so gross. And do not miss the central point: this unique test-of-integrity arrived in an already-existing-for-centuries context of professional astronomers' multi-independently-founded suspicion that Ptolemy was an astronomical faker of equally unique massivity. (Ptolemy-defense lawyers feign obliviousness to all this, implying [§I5] that these 2 uniquenesses' connexion in the same Ptolemy is JUST A BIG ACCIDENT.) Ptolemy's 136 AD Venus fake-pair - doubly-bungled and contradicting each other - is as pure an experimentum crucis as one gets in an ancient dispute. If this isn't proof of fraud, what is? In a sane field, such a glaring, unambiguous blunder would prove Ptolemy's long-suspected fakery beyond the slightest question, and the controversy would swiftly end. But, below, we will find that Ptolemy's double-dating has instead handed us an equally unambiguous experimentum crookis, published right in history of science's top journal (HistSciSoc's Isis), showing that his defenders are now hopelessly beyond even the baldest evidential testing of their faith, and will twist \& even (fn 12) wholly-invent whatever it takes ( $D I O 4.3 \ddagger 15 \mathrm{fn} 42$ ) to escape reckoning.
A2 Adding to his double-dating farce, Ptolemy claimed that, with his very own putative ${ }^{6}$ eyes, he actually saw greatest elongations which were ( $\S 15$ ) greater than greatest elongations, another historically unique astronomical-mathematical achievement.

[^6]A3 Swerdlow \& Gingerich are religiously determined never to admit their blatantly obvious logical loss of the Ptolemy controversy. Now unambiguously cornered, the flush forces of OrthoDoxy ${ }^{7}$ have, in selfdelusional response to a long cascade of crushing evidential disasters for Ptolemy-cultcowdom, ${ }^{8}$ laboriously concocted a numbingly elaborate, mirrormaze-sinuous (note fn 61), and hilariously self-contradictory ${ }^{9}$ bluff - inventing out of pure nothing a claim that Ptolemy had no other choice but to commit a detailed and conscious fraud (an example certain worshippers have taken rather too much to heart). The Muffia's scenario: to establish Venus' orbit, Ptolemy needed Venus in certain configurations that didn't actually occur, so he "shaded" (fn 52) the observational data to make them happen. Well, even were this a well-founded proposal, it is no excuse ${ }^{10}$ for faking data. And the following paper now shows something further: besides being (fn 10) amoral \& irrelevant, the apologists' lawyeresque defense-strategy excuse isn't even true. I.e., no ancient scholar "needed" to forge specially-placed data, since easily-obtainable regular real data fully sufficed for determinating Venus' elements (see $\S \mathbf{G}$ ), if an able mathematician were doing the data-analysis. Which suggests why Ptolemy (like the dimmer end of his curiously varied spectrum of modern defenders) never figured it out, though as we show within ( $\S \S D 4 \& E 13)$ the problem can be solved graphically with ease.
A4 DIO wishes to thank supercomputer specialist Dennis Duke (Florida State University) who, during a conversation of 2002/6/28, suggested that we should look into how the ancients actually could have determined Venus' orbit. His mass-data method ( $\ddagger 5$ ) was 1st distributed just a few days later (early July). On 2002/7/18, DR's Ptolemaic-math iterative

[^7]method was sent (by fax) to Keith Pickering, Dennis Duke, \& Hugh Thurston - and DR hinted to Thurston that he might be the ideal scholar on Earth to devise a geocentrist-Greekgeometry version of this approach; soon after, Hugh sent his deft proof (here at $\ddagger 7 \S \mathrm{D}$, gutting OG's Isis apology) to OG's JHA, which took over $1 / 2$ a year to find it errorless and so of course refused it. [It was instantly added to this DIO ( $\ddagger 7$, below), which was handed out a few weeks later at the 2003/6/19-22 Univ Notre Dame hist. astron. symposium.]

## B Dumb Venus Tricks. Ancient, Etc.

B1 The History of Science Society's Isis recently published Thurston 2002S, a detailed coverage of R.Newton-DIO findings in ancient astronomy. Following this article appeared Gingerich 2002 (composed by the Louis Agassiz of the evolution of ancient astronomy), ${ }^{11}$ attempting to blunt the Thurston article's force, but conspicuously challenging not a digit of it. OG's response never even mentions DR or DIO - the subjects of most of the article he's "replying" to - thus ducking Isis' \& Thurston's generous \& prominent display of DIO's jewelbox of precise analyses \& reconstructions (the distillate of several skilled scholars' decades of devoted inductive researches in the ancient astronomy area), featuring trim hypotheses' fits to 1 part in ordmag 10000 and up (Thurston 2002S p.60: Hipparchos' lunar numbers), 1000000000 (ibid p.62: AstronCuneifText 210). [But note: the closest fit cited (ibid pp.61-62: Mars: 1-part-in-100000000000 is false: see DIO $11.2 \ddagger 4$ p.30. However, the DR-discovered Mercury fit (DIO $2.1 \ddagger 3 \S \mathrm{C} 3$ ) is even closer (1-part-in-1000000000000 [a trillion!]) and since its input is totally attested (every digit is right in Almajest 9.3), its validity is now (DIO $11.2 \ddagger 4 \mathrm{fn} 21$ item [b]) unquestioned.] OG acknowledges no intelligence at all in these findings (not even in the $1 / 1000000000$-precision DIO $1.1 \ddagger 6$ reconstruction now on public display at the British Museum), instead discerning Greatest-
${ }^{11}$ Lovably gregarious academic power-operator Louis Agassiz was the leading US fence against natural selection because: [a] He was a religious fanatic in Harvard-professorial garb; [b] committed to an increasingly untenable position, he expended his creativity not upon progressive research but upon devising convincing-to-him alibis against each successive awful enemy-evidence apparition. But he didn't spread lies about dissenters, nor attack heretic \#1's character in anonymous ref-reports, while (DIO 9.1 News Notes) ducking face-to-face debate with him. Given Gingerich's history (\& see DIO 4.3 $\ddagger 15$ ), he might be seen as the resurrection less of Agassiz than of Galileo-arrester Cardinal Bellarmino. Which ironizes the prominent rôle of Mennonite OG (no kneejerk apologist for Holy Church) in PBS' 2002/10/29 Nova on Galileo. (Though OG was among the saner commentators, watch him \& Nova go with the myth [Rawlins 1991P §F3] of stellar parallax as heliocentrism's long-awaited alleged-watershed proof.) This show's piety is so neatly apt to the (mutual) eternal-holy-war era of new-national-symbolton Shrubya (DIO $2.1 \ddagger 1$ §A10); it cravenly ameliorates the Galileo Affair's prime significances: [a] Exploratory reason vs evidence-immune [fn 13] religious faith. [b] A heaven-touting but earthly-profit\&power-seeking institution's centuries of concerted, lethally-brutal commitment to truth-suppression, isn't an unloadable-at-later-convenience little oops. It forever destroys an eternal-truth-claiming Church's intellectual (\& religious) credibility. Nova portrays rebellious \& strictly-bastard-siring Galileo as a "good Catholic". (Establishment pol-scientists [e.g., R.Millikan, D.Hughes (DIO $1.1 \ddagger 8$ §B3), R.Jastrow] ever pseudo-meld religion\&science, akin to missionaries' notorious willingness to graft local religions onto theirs.) Nova joins Pope JP2 in exploiting Galileo's statement that scripture cannot disagree with science, without asterisking such humor with the slightly relevant reflection that: had Galileo said otherwise, the intellectual zeroes of his day would have dragged him from his Church-decreed imprisonment, straight to the stake. Galileo got into enough trouble promoting astronomical dissent, without inviting independent harassment for theological heresy, too. (My Harvard SocSci prof S.Beer used to note: oldtime scholars might question either Church or State; but tolerance by at least one was required for advancement, so nobody thrived if alienated from both.) The situation was proclaimed to all by the then-recent [1600] burning-alive of Bruno, hero of rebellion against theological dictatorship, incredibly referred-to by Nova as a "new-age charlatan who denied the divinity of Christ"! This is just regurgitated 1913-edition Catholic Encyclopedia justification-apologia for religious murder: see DIO $4.3 \ddagger 15 \mathrm{fn} 33$ for the original $C E$ source.
class genius in C.Ptolemy's most hilariously inept frauds (Venus): OG spends most of his Isis space speculating ${ }^{12}$ - unencumbered by a gram of textual support ${ }^{13}$ - that "insightful" and deliberate "ingenuity and brilliance" by Ptolemy "the greatest astronomer of antiquity" secretly underlay two grossly false and mutually-contradictory Venus fakes which quite inadvertently ${ }^{14}$ produced his now-notorious super-Einsteinian ${ }^{15}$ claim (fn 24) - unique in astronomical history - that he 1sthand-observed the same celestial event at two different times and two different places (and with two different values!) $37^{\mathrm{d}}$ and $37^{\circ}$ before\&after itself (136/11/18 \& 12/25).
${ }^{12}$ It appears to be hitherto unremarked that Gingerich's explanation of Ptolemy's "brilliant" (Gingerich 2002 pp.72\&73) bungled fake is pure speculation: $100 \%$ gas - and in $100 \%$ disagreement with the 1 st-hand representations of the very ancient astrologer whose integrity OG is supposedly defending. I.e., the sole "basis" for the NS-OG theory is: evidence-contradicted scholars demand an escape-hatch, even if it is completely, utterly made-up - simply designed to the specs of necessity. (Similarly, see, e.g., fn 13 or [a different cult] DIO $9.3 \ddagger 6 \S$.) Ptolemy doesn’t (fn 18) provide any of the Swerdlow-Gingerich scenario. (In 1909, Twain [Essays (ed. C.Neider) 1963 NYC pp.420f] gibed that Shakespeare-biographies were nearly-total "plaster-of-Paris". But NS-OG's biography of Ptolemy's Venus is unqualifiedly-total plaster-of-Swerdlow.) So we must here choose between 2 theories:

Theory A: Swerdlow\&Gingerich are the greatest genii in our field's history, to have elicited so much detailed understanding of Ptolemy's mind, especially considering that he was carrying on his purported ingenuity "silently" (fn 57) according to Gingerich 2002 p.73. (OK, OK, there is a tiny glitch here: Ptolemy is not at all silent; instead, as always, honest Ptolemy consistently \& explicitly contradicts his own agile modern alibiers' reconstruction-fantasies. See him do it again\&again at Almajest 10, where Ptolemy repeatedly says he "observed" Venus at specific positions \& times, nowhere stating or even implying that these data were fudged in the manner speculated by NS\&OG.) Thus, in the absence of the slightest remark from "silent" Ptolemy supporting OG's \& NS' reconstruction, we must depend entirely upon their brilliance to accept their view.

Theory B: Swerdlow\&Gingerich have reached the end of the road and must admit the very Ptolemy fakery they both (Gingerich 1976 \& Swerdlow 1979 p.524) originally denied (see fn 53); but, instead of having the frankness to acknowledge the late, Muffia-hated R.Newton's victory in the Ptolemy controversy, both keep contending that the dispute's losers are the true experts, and the winners are mere cranks (Rawlins 1991W §I7 displays \& discusses similar Muffiose perversion) - all this while themselves convincingly imitating the key feature of cranks: clinging to long-held opinions despite avalanche after avalanche of evidence against them. (Same lesson at fn 23, fn 53, fn 55.)
${ }^{13}$ At least Swerdlow 1989 p. 59 openly states that his alibis are "speculative". But, after $1 / 3$ century of denigrating Ptolemy-skeptics, Gingerich is now intensely aware that, if NS-OG's baseless Venus speculation (vainly dodging clear Venusian proof that the skeptics were right all along about Ptolemy) isn't accepted, then future scholarship must condemn his long history of attacks upon the sanity of those who in fact turned out to more perceptive than he. Thus, in his mind, his Venus-apology gas has become solid, immutable dry-ice Reality. (Similar unflagged Muffia transformation: Rawlins 1991W fn 242.) So he quite naturally supposes that those who cannot share in the construct of his genius must be cranks! (See, e.g., the typically slanderous part [not quoted in Gingerich 2002] of his 2000 referee report to Isis: summary here at fn 23.) You know, some authorities on crank science (a subject DIO has explored a good deal more than most academic journals) just might wonder if defending - and even elevating to Greatest-Brilliance status - the most disastrously bungled fraud in astronomical history isn't an almost deliciously ideal example, perhaps even unsurpassable [in the spirit memorialized at DIO $2.3 \ddagger 6 \mathrm{fn} 18$ ], of the evidence-cornered crank mind at work. (See fn 12 \& fn 23.) Of course, Harvard-grad DR is comforted by firm surety that no prof at my eminent alma mater could possibly be a crank - a certainty bolstered by such other Harvard paragons of mental balance as: A.Counter, A.Hynek, T.Leary, J.Mack, W.Pickering, C.West. (Lest irony be deliberately misunderstood by someone: it goes without saying that [despite the occasional loon], I am hugely grateful for the wonderful education [in both matter and spirit] which Harvard provided myself and my wife.)
${ }^{14}$ For other careless Ptolemy gaffes, see papers cited at fnn $15 \& 61$ and Thurston 2002S nn.13\&14.
${ }^{15}$ Physically impossible time\&space warps regularly grace archonal tracts. See DIO $1.1 \ddagger 4$ p. 29 (preprint cited in Note [C]) \& ibid $\ddagger 8$ fn 16; Thurston 1998A $\odot 12 \& \odot 16 ;$ DIO 10 §C4, Fig. 9 \& fn 119.

B2 Hitherto unnoted: Ptolemy's joke implies that Venus' synodic motion stopped dead for $37^{\mathrm{d}}$ straight! - which tops even biblical Joshua, in the astronomical miracle dep't.
B3 To a scholar not glued forever to a tragic longago initial mistake (and not even his own mistake) - thereby irrevocably face-committed, by decades of hyper-ironic slander of the very Ptolemy-skeptics now utterly vindicated - Ptolemy's Venus disaster is simply a case where a Venus-Earth resonance (8-5) blocked kindergarten fabrication-options; as a result, the Almajest 10 Venus fakes were even more hilariously transparent than Ptolemy's usual: his ignorant preference (for imposing an inappropriate method upon Venus) so cornered him that in ultimate desperation he had to cheat Venus' actual 136/11/18 elongation [from the mean Sun] upward by over $1^{\circ} 3 / 4$ [nearly seven times the Sun's semidiameter]. (This apology's inspirer does admit the $1^{\circ} 3 / 4$ : Swerdlow 1989 p.42.) Perhaps the weirdest point here: the forged 136/11/18 geocentric Venus position disagrees by over $1^{\circ} .4$ with the very Venus orbit which Ptolemy faked it to "prove". ${ }^{16}$ This seeming absurdity is just a natural upshot of Ptolemy's clumsy attempt to force a ludicrously inapplicable simpleton-crude method upon a delicate orbit-determination problem, which had obviously already been solved years earlier by far better ancient analysts. (See other and parallel indications at $\S 19$ \& fn 55, $\ddagger 5 \S$ C2, Rawlins 2003J §G4; also Rawlins 1996C §L4 and the inspired reconstructive-extrapolation of Jones 1999E p.258.) Both revealingly huge discrepancies are hardly deniable: see OG's own Fig. 4 at QJRAS 21:253 (1980) p.261, or Swerdlow at JHA 20:29 (1989) p. 37 Table 1. The rms error of the eight Almajest Venus greatestelongation "observations" is ordmag a degree.
B4 Given that Gingerich \& Swerdlow call "silly" ${ }^{17}$ physicist R.Newton's thoroughly founded conclusion that Ptolemy was a clumsy hoaxer, it is strange to see OG now claiming that Ptolemy's Venus fakes (and Gingerich 2002 agrees that these allegedly-outdoor ${ }^{18} 1$ sthand "observations" are indeed based upon indoor concoctions) were just a matter of creating greatest elongations at mathematically convenient (if wildly false: §B3) places, an ever-so-clever ${ }^{19}$ ploy which Ptoleny was forced-into (equally-Ptolemist Noel Swerdlow confidently agrees) ${ }^{20}$ to crack an "otherwise essentially insoluble problem" (Gingerich

[^8]2002 p. 72; see also here at fn 57). Below (§D4), we will show that a simple ancient-style ${ }^{21}$ method (whose central step is highschool level) will solve this "insoluble" problem - and will do so without "requiring" any forging of reality whatever. Contrast-question: had an ancient heliocentrist (frustrated by geocentric data's inconvenient\&confusing retrogradeloop zigzag apparent motion, seen from a moving Earth) faked "observations" as if he were viewing the universe from the Sun instead of the Earth - just to simplify things enough for even a Muffiose talent to pseudo-crack ${ }^{22}$ the "essentially insoluble problem" of finding Venus' eccentricity \& apogee (which indeed would be alot simpler to find if the observer were standing on the Sun!) - then: would Ptolemists be as forgiving of such embarrassingly-crude heliocentric dumbdown-fraud as of the geocentrist brand?
B5 In any case, I heartily recommend OG's elaborate apology, to all who seek the outer limits of unfalsifiability. For decades, this same ${ }^{23}$ religiously evidence-immune author has (even in anonymous referee reports) indicated to everyone within range that his opponents are the cranks of the Ptolemy controversy. His perception (dissent from which marks one a crank in OG's eyes) is the epitome of reason and justice: a serial faker, massive plagiarizer, propagandist for a geocentrist mini-universe, and author of astrologers' bible, was: "the greatest astronomer of antiquity". For more, see www.dioi.org under Sky \& Telescope.
conveniently-flexible, all-thumbs illusionism-prank is what the MacArthur Foundation funded (sixfigure grant) to alibi an ancient hoaxer who so tanglefooted his Venus greatest-elongation fakes that the same 136 AD event ended up possessing two different dates thirty-seven days apart. (See fn 24. Again: what has $6^{\mathrm{d}}$ uncertainty to do with this? - especially for an effect which is not linear but nearly quadratic! Similarly: what has $1^{\circ} / 12$ uncertainty (fn 20) to do with the Almajest 10 Venus hoax's rampant ordmag $1^{\circ}$ errors? I.e., why even bring up the $1^{\circ} / 12$ alibi in the 1 st place? - unless one is, heedless of coherent logic, just aiming to stack up as many alibis as possible, for deliberate squid-ink-pagefill-obscurantist purposes. [Is this the brand of scholarship John MacArthur had in mind to encourage? See DIO $4.3 \ddagger 15 \mathrm{fn} 24$ for what has become of the MacArthur awards.]) One can only conclude that it must have meant ALOT - to some Very, Very Special Archons - that Ptolemy's unique double-dating embarrassment-to-then-orthodoxy (Rawlins 1987 p. 236 item 4) be extensively, prominently, \& authoritatively salved-sanitized-skwushed. We admiringly note that: upon such deeds are great careerisms reared.
${ }^{21}$ See fn 59 for the critical importance of this point to historians. The openminded ones, anyway.
${ }^{22}$ Keep in mind (fn 9) that Gingerich is not claiming that the actual solution was accompished by Ptolemy's fakes; no, OG thinks (Gingerich 2002 p. 73 Fig. 1 caption) that Venus' elements were already known (otherwise, Ptolemy could not have computed his fakes) - by an unspecified method which of course is the one which Ptolemy should have explained in the Almajest. Never-say-die loyalists cannot face this self-evident point - or the equally obvious item: Ptolemy was not the ancient who found the Almajest Venus elements. (A point understood long ago by R.Newton 1985 p.12.) But at least we all now agree with R.Newton that the whole Almajest 10 Venus discussion is fraudulent.
${ }^{23}$ As to those Ptolemists who have attacked skeptics as cranks: we note in passing here [\& fn 13] that the prime symptom of the crank mentality is rigid imperviousness to incoming contrary data. Some have also expressed disapproval of "polemics" (note irony of fn 17), as in an OG anonymous 2000 referee report - which simultaneously refers to Ptolemy-skeptics as a tiny bunch of paranoids! Amused observers of such last-ditch-desperate (DIO 10 fn 172 ) mud-hurlings are urged to check out the relative status, in the genuine scholarly community, of the board of DIO vs that of the JHA. The paranoia-charge against skeptics is just a broadbrush echo of an OG 1977 libel of DR: DIO $4.3 \ddagger 15$ $\S$ H6. The resort to authority-vs-heretic classification is the Ptolemy cult's standard creationist-levelargumentation (DIO $1.2 \S \S E 4 \& N 16$ ) for its creationist-level sudden-miracle Claudius Let-There-BeLight Ptolemy-godhead perception of high ancient astronomy's nascence. (See, e.g., Gingerich 2002 p.71.) For decades, Ptolemists have been planting loyal scholars (either strange enough or tractable enough to assent to such creationism) into prominent positions in the AAS-HAD and JHA - the inevitable resultant damage to rational discourse in the history-of-astronomy community will carry down through yet more decades, long after many of us are dead. Thus, the better part of a century of communal reasoning is being maimed by a cult's political \& scientific skills' predictably inverse correlation.

## C Other Significant Oddities of Ptolemy's Venus Presentation

The Almajest Venus chapters are peculiar in ways additional to merely supplying us with the nonpareil hilarity of double-dating ${ }^{24}$ the very same event.
C1 Ptolemy reports contemporary observations of Venus not taken by astrolabe. He does this for no other planet.
C2 Which explains another strange coincidence: of the five rare ${ }^{25}$ Almajest Catalog stars with $1^{\circ} / 4$ endings, $40 \%$ are used to measure the position of Venus. (See DIO 2.3 $\ddagger 8 \mathrm{fn} 20$. These five are the only stars whose positions we know he didn't steal from Hipparchos.) This suggests that, when grossly (§B3) forging these observations to make them agree with the requirements of his amusing Venus frauds, he in each of these cases did not change Venus' reported angular distance from the reference star - but moved Venus where he wanted it by simply fudging the star's position: the star's shift just carried Venus with it.
C3 Further, when choosing a mean motion for Venus, Ptolemy most probably confused a sidereal and tropical period-relation (Rawlins 1985K \& Rawlins 1987 n .7 ) - which so affected the original highly accurate motion (see Rawlins 1985K) that it fell from one of the two best, ${ }^{26}$ into ridiculous inaccuracy. (Note Rawlins 2003J §E3.)
C4 One of Gerald Toomer's most important discoveries is that the several tables of Venus' mean motion are discordant (Toomer 1984 p. 425 n .29 ). This is true for none of the other four planets.
C5 My conclusion is that much (if not all) of the Almajest Venus section was lifted from an ambitious but inferior source which Ptolemy did not use for the other planets.

[^9]
## D Solving the "Insoluble": Muffia\&Co. vs 10th Grade Math

D1 Gingerich 2002 p. 72 alibis Ptolemy's childishly botched (§B3) fabrications of greatest elongations of Venus by saying that Ptolemy had (fn 57) to fake impossible positions for Venus in order to pry the planet into convenient line-ups - without which the orbital elements couldn't be solved-for. Such an obviously false claim provides us one of the dizziest pinnacles of this rare treat of a paper.
D2 In fact, finding Venus' orbital elements (and in the very same geometric style which Ptolemy himself adopted) ${ }^{27}$ is obviously possible. The mean motion is easy to find from stationary point data. (See Rawlins 1987 n. $28 \& D I O 2.1 \ddagger 3$ fn 17. Any error in an adopted mean-longitude-at-epoch would so obviously affect stationary points' positions that correction would be trivially simple.) Thus, the elements that required determination from greatest elongations were: the deferent's eccentricity $e \&$ apogee $A$, and the radius $r$ of Venus' epicycle. Now, it is typical of Ptolemy's Euclidean-geometric approach that if he seeks $n$ unknowns, he uses exactly $n$ equations of condition. (So, in this case, he would have needed just three greatest-elongation observations.) One of Ptolemy's weaknesses (typical of a non-scientist) is a failure to understand the preferability of overdetermination. (See Rawlins 1991W fn 224 \& Rawlins 1996C fn 103.) We will now explain the method we have devised (for finding $e, A, \& r$ ) that follows this approach. It is chosen both for simplicity and for its nonanachronistically Ptolemaic character. Indeed, our Venus method is more ${ }^{28}$ Ptolemaic than Ptolemy's own Venus analysis (Almajest 10).
D3 The Ptolemy-alleged solar motion around the Earth is mathematically equivalent to terrestrial heliocentric motion. We will use this equivalence to simplify the problem conceptually - noting in passing that DR has long held ${ }^{29}$ that the best ancient astronomers were heliocentrists anyway.
D4 We can then take three greatest-elongation observations of Venus (preferably spaced very roughly $120^{\circ}$ apart: §G1) and graphically draw the line-of-sight for each: through the Earth's position in its own orbit (a function of the solar mean longitude, a known function of time). Once we have these three lines, we simply determine a circle, the Venus orbit (the geocentrists' "epicycle"), such that it is tangent to all three lines. Easy, since the bisectors of any two of these three lines must go through the center of the circle we seek. So there will be three two-line intersections, all at the same point: the circle's center. This key part of the problem can be accomplished graphically by a highschooler (less elementary mathematical equivalent: $\S \mathrm{E} 3 \mathrm{ff}$ ), which is why it's so inspirational to watch eminent professors deem this simple task "essentially insoluble" (§B4).

[^10]Table 1: Ptolemy-Selected Venus Greatest Morning Elongations (Real Data)

| Date\&Time $t$ | GrElong $G$ | Solar Mean Longtd $L$ | Venus’ True Longtd $V$ |
| :---: | :---: | :---: | :---: |
| $129 / 05 / 0616^{\mathrm{h}}$ | $-44^{\circ} 52^{\prime}$ | $43^{\circ} 06^{\prime}$ | $358^{\circ} 14^{\prime}$ |
| $127 / 09 / 2218^{\mathrm{h}}$ | $-48^{\circ} 20^{\prime}$ | $179^{\circ} 41^{\prime}$ | $131^{\circ} 21^{\prime}$ |
| $134 / 02 / 1515^{\mathrm{h}}$ | $-44^{\circ} 34^{\prime}$ | $324^{\circ} 00^{\prime}$ | $279^{\circ} 26^{\prime}$ |

(Here [and in similar tables below], we follow ancient convention in defining elongation as the planet's true geocentric longitude $V$ minus the Sun's mean longitude $L$.)

## E The Simple Eccentric Solution

E1 For Venus, Ptolemy uses the equant. (He pretends that he proved the equant model from Venus. Anyone who [like Neugebauer 1975 p.155] accepts this should note the provocative coincidence that Ptolemy's most hilariously and oversimplistically bungled planetary fake orbit-derivation was that for Venus. ${ }^{30}$ The equant model requires that our approach be iterative. But we will start by introducing the reader to the general problem by first considering a noniterative case: an eccentric-model solution for the three Ptolemyselected greatest elongations of Table 1 , which lists the real data (not his faked $t, L, \&$ $V$, most of which were [fn 24] in error by ordmag $10^{\circ}!$ ). That choice [reality] will hold for all three tables in this paper: each provides time $t$ (Julian calendar, Alexandria Mean Time, midnight epoch), greatest elongation $G$ (evening or eastern elongation is positive), solar mean longitude $L$, and true Venus geocentric longitude $V$ for these greatest morning elongation cases:
E2 We start with the Earth's circular orbit centered at $(0,0)$ with unit radius. and we will use the data of Table 1 to locate (in the $x-y$ plane) the center of Venus' circular orbit. The opening steps are intermediate highschool math: the line corresponding to observation i (i running from 1 through 3) in Table 1 must go through the point

$$
\begin{equation*}
x_{\mathrm{i}}=-\cos L_{\mathrm{i}} \quad y_{\mathrm{i}}=-\sin L_{\mathrm{i}} \tag{1}
\end{equation*}
$$

and the slope $m_{\mathrm{i}}$ (in the $x-y$ plane) of line i is:

$$
\begin{equation*}
m_{\mathrm{i}}=\tan V_{\mathrm{i}} \tag{2}
\end{equation*}
$$

The equation for line i is:

$$
\begin{equation*}
y_{\mathrm{i}}=m_{\mathrm{i}} x_{\mathrm{i}}+b_{\mathrm{i}} \tag{3}
\end{equation*}
$$

Substituting eqs. $1 \& 2$ into eq. 3 determines $b_{\mathrm{i}}$ (line i’s intercept):

$$
\begin{equation*}
b_{\mathrm{i}}=y_{\mathrm{i}}-m_{\mathrm{i}} x_{\mathrm{i}} \tag{4}
\end{equation*}
$$

- so we have now completely determined all of the three lines that will locate Venus' orbit (since all three lines are virtually tangent to it):

$$
\begin{array}{lll}
y_{1}=m_{1} x_{1}+b_{1} & \rightarrow & 0.0308 x_{1}+y_{1}=-0.7058 \\
y_{2}=m_{2} x_{2}+b_{2} & \rightarrow & 1.1363 x_{2}+y_{2}=+1.1307 \\
y_{3}=m_{3} x_{3}+b_{3} & \rightarrow & 6.0188 x_{3}+y_{3}=-4.2815 \tag{7}
\end{array}
$$

[^11]E3 We next determine the intersections of the three possible pairings of these lines: pt. $\mathrm{A}=$ the intersection of lines $1 \& 2$; pt.B, lines $2 \& 3$; pt.C, lines $3 \& 1$. Again, this is early highschool math (standard 2 linear-equations-in-2 unknowns problem):

$$
\begin{array}{ll}
x_{\mathrm{A}}=+1.6614 & y_{\mathrm{A}}=-0.7570 \\
x_{\mathrm{B}}=-1.1085 & y_{\mathrm{B}}=+2.3903 \\
x_{\mathrm{C}}=-0.5972 & y_{\mathrm{C}}=-0.6874 \tag{10}
\end{array}
$$

E4 Next, we find the longitudinal direction $B_{\mathrm{k}}$ (where k equals $\mathrm{A}, \mathrm{B}$, or C ) of the bisector of the two lines passing through each of these points. Using the data in Table 1, we have:

$$
\begin{align*}
& B_{\mathrm{A}}=\left(V_{2}+V_{1}+180^{\circ}\right) / 2=\left(131^{\circ} 21^{\prime}+358^{\circ} 14^{\prime}-180^{\circ}\right) / 2=154^{\circ} 47^{\prime} 1 / 2  \tag{11}\\
& B_{\mathrm{B}}=\left(V_{3}+V_{2}+180^{\circ}\right) / 2=\left(279^{\circ} 26^{\prime}+131^{\circ} 21^{\prime}-180^{\circ}\right) / 2=123^{\circ} 24^{\prime}  \tag{12}\\
& B_{\mathrm{C}}=\left(V_{1}+V_{3}+180^{\circ}\right) / 2=\left(358^{\circ} 14^{\prime}+279^{\circ} 26^{\prime}-180^{\circ}\right) / 2=228^{\circ} 50^{\prime} 1 / 2 \tag{13}
\end{align*}
$$

E5 We have already (back in §E2) performed an equivalent of the next step: a point and a direction determine a line (a bisector in these cases). Each line's slope is the tangent of its $B_{\mathrm{k}}$ (§E4, eqs.11-13)

$$
\begin{equation*}
m_{\mathrm{k}}=\tan B_{\mathrm{k}} \tag{14}
\end{equation*}
$$

Each line's intercept $b_{\mathrm{k}}$ is then found (as in eq.4) by fitting the line to the point given in $\S E 3$; thus, we find the equation for each of the bisectors:

$$
\begin{array}{lll}
y_{\mathrm{A}}=m_{\mathrm{A}} x_{\mathrm{A}}+b_{\mathrm{A}} & \rightarrow & +0.4707 x_{\mathrm{A}}+y_{\mathrm{A}}=0.0250 \\
y_{\mathrm{B}}=m_{\mathrm{B}} x_{\mathrm{B}}+b_{\mathrm{B}} & \rightarrow & +2.1068 x_{\mathrm{B}}+y_{\mathrm{B}}=0.0549 \\
y_{\mathrm{C}}=m_{\mathrm{C}} x_{\mathrm{C}}+b_{\mathrm{C}} & & \rightarrow \tag{17}
\end{array}
$$

E6 Solving the above equations as three pairs, we are gratified to find that all three intersections are identical, thus precisely ${ }^{31}$ placing the center V of Venus' circular orbit in the $x-y$ plane:

$$
\begin{equation*}
x_{V}=0.01827 \quad y_{V}=0.01644 \tag{18}
\end{equation*}
$$

Since it is the deferent not the epicycle that is off-center in the Venus model, we simply translate the center negatively by the amounts indicated in eq. 18 - in order to move Venus’ orbit onto the center $(0,0)$ of the $x-y$ plane. This makes the Earth's orbit eccentric with an aphelion in the 3rd quadrant - which for a geocentrist fixes ( $180^{\circ}$ distant) the deferent's apogee $A$, at a position given precisely by eq.18. Applying Pythagoras' Theorem and an arcsin to eq. 18, and (in ancient style) multiplying the eccentricity $e$ by 60 , we have located the deferent's center at:

$$
\begin{equation*}
e=1^{\mathrm{p}} 28^{\prime} \quad A=41^{\circ} 59^{\prime} \tag{19}
\end{equation*}
$$

[^12]Table 2: Ptolemy-Era Venus Greatest Evening Elongations (Real Data)

| Date\&Time $t$ | GrElong $G$ | Solar Mean Longtd $L$ | Venus' True Longtd $V$ |
| :---: | :---: | :---: | :---: |
| $138 / 07 / 0920^{\mathrm{h}}$ | $44^{\circ} 40^{\prime}$ | $106^{\circ} 10^{\prime}$ | $150^{\circ} 50^{\prime}$ |
| $136 / 12 / 1401^{\mathrm{h}}$ | $47^{\circ} 30^{\prime}$ | $261^{\circ} 36^{\prime}$ | $309^{\circ} 06^{\prime}$ |
| $132 / 02 / 2123^{\mathrm{h}}$ | $47^{\circ} 58^{\prime}$ | $329^{\circ} 44^{\prime}$ | $17^{\circ} 42^{\prime}$ |

E7 The radius of the Venus epicycle is then easily found by using any of the bisectors, starting with the half-angle $H$ between that bisector and either of the two lines it is half-way between. We check all three:

$$
\begin{align*}
& H_{\mathrm{A}}=\left(V_{2}-V_{1}-180^{\circ}\right) / 2=23^{\circ} 26^{\prime} 1 / 2  \tag{20}\\
& H_{\mathrm{B}}=\left(V_{3}-V_{2}-180^{\circ}\right) / 2=-164^{\circ} 02^{\prime} 1 / 2  \tag{21}\\
& H_{\mathrm{C}}=\left(V_{1}-V_{3}-180^{\circ}\right) / 2=-129^{\circ} 24^{\prime} \tag{22}
\end{align*}
$$

Then, using our pre-translation data, we find the length $v_{\mathrm{k}}$ of the vector from intersection k (eqs.8-10) to the pre-translantion center of Venus' orbit:

$$
\begin{align*}
& v_{\mathrm{A}}=\sqrt{\left(x_{\mathrm{A}}-x_{V}\right)^{2}+\left(y_{\mathrm{A}}-y_{V}\right)^{2}}=\sqrt{1.6431^{2}+0.7734^{2}}=1.8161  \tag{23}\\
& v_{\mathrm{B}}=\sqrt{\left(x_{\mathrm{B}}-x_{\mathrm{V}}\right)^{2}+\left(y_{\mathrm{B}}-y_{\mathrm{V}}\right)^{2}}=\sqrt{1.1268^{2}+2.3739^{2}}=2.6277  \tag{24}\\
& v_{\mathrm{C}}=\sqrt{\left(x_{\mathrm{C}}-x_{\mathrm{V}}\right)^{2}+\left(y_{\mathrm{C}}-y_{\mathrm{V}}\right)^{2}}=\sqrt{0.6155^{2}+0.7038^{2}}=0.9349 \tag{25}
\end{align*}
$$

E8 Highschool trig then produces the Venus orbital radius $r_{\mathrm{k}}$ :

$$
r_{\mathrm{A}}=v_{\mathrm{A}} \sin H_{\mathrm{A}}=1.8161 \sin 23^{\circ} 26^{\prime} 1 / 2=0.7225=43^{\mathrm{p}} 21^{\prime}=60^{\mathrm{p}} \sin 46^{\circ} 15^{\prime}
$$

$$
r_{\mathrm{B}}=v_{\mathrm{B}} \sin H_{\mathrm{B}}=2.6277 \sin 164^{\circ} 02^{\prime} 1^{\prime} / 2=0.7225=43^{\mathrm{p}} 21^{\prime}=60^{\mathrm{p}} \sin 46^{\circ} 15^{\prime}
$$

$$
\begin{align*}
& r_{\mathrm{B}}=v_{\mathrm{B}} \sin H_{\mathrm{B}}=  \tag{27}\\
& r_{\mathrm{C}}=v_{\mathrm{C}} \sin H_{\mathrm{C}}=0.9349 \sin 129^{\circ} 24^{\prime}=0.7225=43^{\mathrm{p}} 21^{\prime}=60^{\mathrm{p}} \sin 46^{\circ} 15^{\prime}
\end{align*}
$$

(The perfect ${ }^{32}$ agreement of all three calculations of $r_{\mathrm{k}}$ is gratifying, but it is a mathematical not an empirical triple-verification.)
E9 Merging the results of eq. 19 and $\S$ E8, we now possess all three of the Venus orbit elements we set out (§D2) to find - the epicycle radius $r$, as well as the deferent's eccentricity $e \&$ apogee $A$ :

$$
\begin{equation*}
r=43^{\mathrm{p}} 21^{\prime} \quad e=1^{\mathrm{p}} 28^{\prime} \quad A=41^{\circ} 59^{\prime} \tag{29}
\end{equation*}
$$

E10 However, the foregoing will not find the actual elements of Venus, because the eccentric model (even the equant model: $\S G$ ) is not faithful to the actual motions of Earth (deferent) and Venus (epicycle). Thus, a different choice of observations at the outset (Table 1) can give a contrasting result. E.g., consider a trio of Ptolemy's evening greatest elongations:

[^13]E11 Applying the development of $\S \S E 2 \& E 8$ to the data of Table 2 will produce:

$$
\begin{equation*}
r=43^{\mathrm{p}} 21^{\prime} \quad e=1^{\mathrm{p}} 18^{\prime} \quad A=86^{\circ} 21^{\prime} \tag{30}
\end{equation*}
$$

E12 The $r$ values of eqs.29\&30 are close to the truth. As will be noted below at fn 51, the $e$ results are inappropriately (since this is not Ptolemy's model!) close to the value adopted in the Almajest, and the mean (c. $64^{\circ}$ ) of the $A$ values for eqs. $29 \& 30$ is within $10^{\circ}$ of the Almajest value. (See eq. 49 below for Ptolemy's $e, A, \& r$.)
E13 Note: all the key foregoing math (§§E3-E6) can be performed graphically in a matter of minutes by a teenager, using standard ruler\&compass. It's a classic Euclidean problem, which teachers of the 10th-grade at Gilman School (Baltimore) tell me their students find not especially challenging. Try it for yourself.

So much for Ptolemists' "insoluble"-alibi for Ptolemy.

## F Oft-Neglected Caution for Eccentric Orbits: Greatest-Elongation Sightlines Are Not Tangents

F1 We interrupt here for a moment, in order to highlight an educational item which is closely relevant to our method's efficacy: despite an almost universal instinctive presumption, greatest elongations do $n o t^{33}$ in general [though see $\ddagger 7 \mathrm{fn} 2$ ] occur precisely when the terrestrial observer's line of sight is tangent to the sighted inferior planet's orbit. Not even for the simple uniform-motion eccentric model - unless eccentricity is null. Nonetheless, for small $e$ (or small $r$ ), tangency is virtually ${ }^{34}$ the case.
F2 If one correctly (unlike Ptolemy) indoor-computes greatest elongations \& associated Venus geocentric longitudes for a well-spaced trio selected from Ptolemy's own set of elongations - e.g., AD 127 morning, 132 evening, 134 morning - using his equant Venus model (not real observations), and substitutes these six data into the method of the present section, the deduced elements come out: $r=43^{\mathrm{p}} 10^{\prime}$, $e=1^{\mathrm{p}} 15^{\prime}, A=54^{\circ}$. Or, if we instead use the greatest elongations of AD 138 evening, 140 evening, 140 morning (again, accurately calculated from Ptolemy's Venus equant model), the data for which are provided right in lines 70-90 of p.54's program, the readout is: $r=43^{\mathrm{p}} 08^{\prime}, e=1^{\mathrm{p}} 14^{\prime}, A=57^{\circ}$. Both sets of results are virtually identical to Ptolemy's Venus elements (eq.49). (But if we instead generate these 3 gr.elongs. using nontrivial $e=5^{\mathrm{p}} 1 / 4$ (while retaining eq.49's Ptolemaic $A=55^{\circ}, r=43^{\mathrm{p}} 1 / 6$ ), then our p. 54 program will output: $e=5^{\mathrm{p}} .7, A=58^{\circ}, r=41^{\mathrm{p}} 5 / 6$, which are only approximately equal to the input elements; all these discrepancies are due to greatest elongations' non-tangency, as discussed at $\S \mathrm{F} 1$.)

[^14]Table 3: Dionysian-Era Venus Greatest Evening Elongations (Real Data)

| Date\&Time $t$ | GrElong $G$ | Solar Mean Longtd $L$ | Venus’ True Longtd $V$ |
| ---: | :---: | :---: | :---: |
| $-271 / 4 / 0506^{\mathrm{h}}$ | $46^{\circ} 53^{\prime}$ | $9^{\circ} 10^{\prime}$ | $56^{\circ} 03^{\prime}$ |
| $-265 / 8 / 3107^{\mathrm{h}}$ | $44^{\circ} 41^{\prime}$ | $154^{\circ} 38^{\prime}$ | $199^{\circ} 19^{\prime}$ |
| $-258 / 1 / 2512^{\mathrm{h}}$ | $48^{\circ} 09^{\prime}$ | $300^{\circ} 15^{\prime}$ | $348^{\circ} 24^{\prime}$ |

## G The Equant Iteration

G1 Once we adopt ${ }^{35}$ the equant model, the ability to arrive at a solution in one direct line (as in §E) vanishes - and we must iterate, as Ptolemy himself does (fn 27) for his analyses of the outer planets. To test the equant case, we will choose elongations from the Dionysian era (when the equant may have actually debuted) - and we will use ones rather symmetrically separated. Table 3 provides the data for a sample equant investigation:
G2 When we switch from the eccentric model to the equant model, we lose the luxury of starting from a known position on the deferent (as in eq.1); instead, we must initially estimate the deferent elements ( $e \& A$ ) and input the implied Earth position, in order to launch the analysis which produces an improved set of deferent elements - which we then re-input: standard iterative technique.
G3 In order to save time, we will here start from the assumption of fairly accurate elements. But no matter where one starts out, the iteration will succeed - it just might require one or two more re-inputs to get the same result, which will be just as stable as one likes (conditional only upon the patience to keep iterating). Presumably aware that Venus' orbit is nearly circular, and thus that the deferent elements had to be nearly those of the

[^15]Earth's orbit, an ancient heliocentrist ${ }^{36}$ might well start with round values close to the real Earth elements then ( $e=1^{\mathrm{p}} 03^{\prime}, A=64^{\circ}$ ):

$$
\begin{equation*}
e=1^{\mathrm{p}} \quad A=60^{\circ} \tag{31}
\end{equation*}
$$

Or, in cartesian coordinates, the Earth orbit center starts at:

$$
\begin{equation*}
x_{\mathrm{E}}=e \cos A=0.008333 \quad y_{\mathrm{E}}=e \sin A=0.014434 \tag{32}
\end{equation*}
$$

G4 Since most discussions of the equant are long on diagrams (or series expressions) but short on exact equations, $D I O$ will as a public service provide the process here, all on one line, eliciting true longitude $\lambda$ from mean longitude $L$ :
$g=L-A \rightarrow u=g-\arcsin (e \sin g) \rightarrow \lambda=\arctan [\sin u /(e+\cos u)]+A$ (33) where $\lambda$ is the Sun's true longitude (or, for the geocentric model, the true longitude of the epicycle's center) - at a distance $R$ from ( 0,0 ):

$$
\begin{equation*}
R=\sqrt{(e+\cos u)^{2}+(\sin u)^{2}} \tag{34}
\end{equation*}
$$

For the 3rd $L$ in Table 3, we apply eqs.33\&34 to yield $(R, \lambda)$ [polar coordinates]:

$$
\begin{gather*}
g_{3}=300^{\circ} 15^{\prime}-60^{\circ} \rightarrow u_{3}=240^{\circ} 15^{\prime}-\arcsin \left(\sin 240^{\circ} 15^{\prime} / 60\right)=241^{\circ} .079 \\
\lambda_{3}=\arctan \left[\sin 241^{\circ} .079 /\left(1 / 60+\cos 241^{\circ} .079\right)\right]+60^{\circ}=301^{\circ} .922 \\
R_{3}=\sqrt{\left(1 / 60+\cos 241^{\circ} .079\right)^{2}+\left(\sin 241^{\circ} .079\right)^{2}}=0.9920 \\
\text { Eqs.36\&37 can be converted to cartesian coordinates: } \\
X_{3}=R_{3} \cos \lambda_{3}=0.5246 \quad Y_{3}=R_{3} \sin \lambda_{3}=-0.8420 \tag{38}
\end{gather*}
$$

Since greatest-elongation-sighting line \#3 goes through eq. 38's point at longitudinal angle $348^{\circ} 24^{\prime}$ ( $V_{3}$ in Table 3), this line is now completely determined; so we can proceed much as we did back in $\S \mathrm{E}$ : for any line, if we know [a] its slope $(V)$ and $[\mathrm{b}]$ the coordinates of a point (eq.38) that's on the line, we have determined the line - and can find its equation (as in $\S \S E 2 \& E 5$ ). From [a]\&[b], the 3rd line's equation is determined as:

$$
\begin{equation*}
0.2053 \cdot X_{3}+Y_{3}=-0.7343 \tag{39}
\end{equation*}
$$

(analogous to eq. 7 in the foregoing eccentric-model development); and to find lines \#1\&\#2, one simply applies our equant-model eqs.35-39 to the data of the first two rows of Table 3. (The results will be analogous to our eccentric-model eqs.5\&6).
G5 And the rest of the process is parallel to $\S$ E's development. (It is not continued in our text, but its details will be evident in the BASIC program printed at p.54.) It works for the equant model just as well as it did for the eccentric model. Again, the central step is finding a circle that is tangent to three known (greatest-elongation) lines: §E3ff. The eventual solution for the center of the Earth's orbit in the Venus equant case ( $\S \mathrm{G} 4)$ is, in cartesian coordinates:

$$
\begin{equation*}
c_{\mathrm{E}}=-0.000784 \quad d_{\mathrm{E}}=+0.003286 \tag{40}
\end{equation*}
$$

or, in polar coordinates:

$$
\begin{equation*}
e=0^{\mathrm{p}} .2027 \quad A=103^{\circ} .4 \tag{41}
\end{equation*}
$$

[^16]G6 At this point, we realize that $c_{\mathrm{E}} \& d_{\mathrm{E}}$ are simply the first gauging of our positional error (on the $x-y$ plane) in the initial estimate (eq.31) of the deferent (Earth orbit) center. So we now just shift that center according to the indication of eq. 40 and then repeat the eqs.31-41 procedure - a method which (by repetition) will quickly reduce the error to virtual nullity. I.e., eq. 40 will shrink to $(0,0)$.
G7 But trial shows ${ }^{37}$ that this iterative process's convergence is at least an ordmag quicker if we alter the initial center by half of the vector $\left(c_{\mathrm{E}}, d_{\mathrm{E}}\right)$, not the full amount. Thus, the next turn in our iteration starts by merging eqs. $32 \& 40$ according to that principle:

$$
\begin{equation*}
x_{\mathrm{E}}=e \cos A-c_{\mathrm{E}} / 2=0.008333-(-0.000784) / 2=0.008726 \tag{42}
\end{equation*}
$$

$$
\begin{equation*}
y_{\mathrm{E}}=e \sin A-d_{\mathrm{E}} / 2=0.014434-(+0.003286) / 2=0.012791 \tag{43}
\end{equation*}
$$

or:

$$
\begin{equation*}
e=0^{\mathrm{p}} .9290 \quad A=55^{\circ} .70 \tag{44}
\end{equation*}
$$

This $x_{\mathrm{E}} \& y_{\mathrm{E}}$ solution is then re-inserted into eq. 33 and the process down to eq. 44 is repeated - again\&again, until satisfactory stability is attained. There is some oscillation of $e \& A$ as the iterative math swoops-in onto the correct solution. But after only three repetitions, the fit is already ordmag $1 \%$ - which would have been more than adequate precision in antiquity, given the likely uncertainty of the empirical input.
G8 By contrast, there is very little flutter in the Venus epicycle radius $r$. On the initial assumed $e \& A$ (eq.31), it comes out as $r=43^{\text {p }} .344$. Throughout the entire iteration, it hardly varies at all from the eventual exact solution, which for this case is $43^{p} .343$.
G9 Continued iterative loops produce the following successive solutions:

$$
\begin{aligned}
& r=43^{\mathrm{p}} .343 \quad e=0^{\mathrm{p}} .8916 \quad A=55^{\circ} .59 \\
& r=43^{\mathrm{p}} .343 \quad e=0^{\mathrm{p}} .8811 \quad A=56^{\circ} .18 \\
& r=43^{\mathrm{p}} .343 \quad e=0^{\mathrm{p}} .8806 \quad A=56^{\circ} .51 \\
& r=43^{\mathrm{p}} .343 \quad e=0^{\mathrm{p}} .8817 \quad A=56^{\circ} .61 \\
& r=43^{\mathrm{p}} .343 \quad e=0^{\mathrm{p}} .8824 \quad A=56^{\circ} .61 \\
& r=43^{\mathrm{P}} .343 \quad e=0^{\mathrm{p}} .8826 \quad A=56^{\circ} .60
\end{aligned}
$$

The swiftness of convergence is obvious. Further looping settles in on a precise ${ }^{38}$ solution:

$$
\begin{equation*}
r=43^{\mathrm{p}} .343 \quad e=0^{\mathrm{p}} .8825 \quad A=56^{\circ} .59 \tag{45}
\end{equation*}
$$

which may be expressed as:

$$
\begin{equation*}
r=43^{\mathrm{p}} 21^{\prime} \quad e=0^{\mathrm{p}} 53^{\prime} \quad A=57^{\circ} \tag{46}
\end{equation*}
$$

[^17]G10 That these eq. 46 elements are not quite correct is inevitable since (see also $\S F$ ) the method we are using here is attempting to simultaneously satisfy two elliptically-nonuniform effects (Earth's motion \& Venus') with two non-elliptical models: an equant (Earth) and a uniform circular motion (Venus). (I.e., real nonuniform-motion of Venusian periodicity is not accounted for by anything in the Ptolemy model.) By contrast, if one applies the foregoing method (eqs.31-46) to greatest-elongation "observations" computed from Ptolemy's Venus model, then the resulting solution (analogous to eq.46) will be very close to Ptolemy's $r, e, \& A$ (eq.49). Readers should test this for themselves. (Neat sample results of such testing are provided in §F2.) Especially Swerdlow \& Gingerich. For now we come to a simpler test - not of orbital elements but integrity elements: will these much-exalted ${ }^{39}$ Experts have the guts to explore this easy demonstration that they have mutually disgraced the history-of-science field by years of prominent (and heresy-slandering) promotion of their laughable ${ }^{40}$ claim that our highschool-math-level problem here is insoluble? Haven't we been through this drama before? - see fn 40 - with the crucial differences that the previous player (an unexcelled scholar in his area) [a] was courageously concerned to set the record straight ${ }^{41}$ and $[b]$ wasn't habitually decreeing that disagreement from his opinion was a sign of insanity, ${ }^{42}$ scholarly kookery ${ }^{43}$ (!), or dishonesty ${ }^{44}$ But, then, ever-obsessively-dedicated kook slayer Owen VanHelsing Gingerich is admittedly a very special case.

## H Reflections; and Appreciating The Greatest's True Greatest

H1 As we see by comparing eq. 30 to eq. 48 , the eccentric solutions are quite sensitive to our selection of the three input greatest-elongations. Different choices will produce quite different values for $e$ and for $A$ - a point already extremely obvious via comparison of eqs. $29 \& 30$ !
H2 But this is much less true of the equant analysis - a point which could ${ }^{45}$ well have helped clue ${ }^{46}$ ancients to the preferability of the equant. Any well-spaced Dionysian-era trio will produce $e$ just under ${ }^{47} 1^{\mathrm{p}}$ and $A$ around ${ }^{48} 57^{\circ}$, with scatter over a range of about

[^18]$\pm 6^{\circ}\left(\mathrm{rms} 4^{\circ}\right)$ - mere averaging of results from a few trios would have shown this. Thus, fairly consistent inductions of Venus' $e, A, \& r$ would have required under a decade of careful observations. Indeed, 3 of the last 4 Venus elongations ( 138 evening, 140 evening, 140 morning) available ${ }^{49}$ to Ptolemy would have sufficed in barely two years! (He actually recorded [badly] both 140 events.) For these three greatest elongations, substituting real data into lines 70-90 of our p. 54 program yields:
\[

$$
\begin{equation*}
r=43^{\mathrm{p}} 24^{\prime} \quad e=0^{\mathrm{p}} 50^{\prime} \quad A=64^{\circ} \tag{47}
\end{equation*}
$$

\]

Note that our results (eqs.46\&47) are in good agreement with the more reliable estimates ${ }^{50}$ found by R.Newton using least squares, fitting very extensively to the actual orbit of Venus (R.Newton 1977 p.311):

$$
\begin{equation*}
r=43^{\mathrm{p}} 22^{\prime} \quad e=0^{\mathrm{p}} 50^{\prime} \quad A=60^{\circ} 12^{\prime}, \tag{48}
\end{equation*}
$$

H3
Ptolemy's values ${ }^{51}$ for Venus were (Almajest 10.1-3):

$$
\begin{equation*}
r=43^{\mathrm{p}} 10^{\prime} \quad e=1^{\mathrm{p}} 15^{\prime} \quad A=55^{\circ} \tag{49}
\end{equation*}
$$

His alleged method of finding them was a childish forgery-of-reality, in order to make the problem simple enough for his limited mentality to solve it (note $\S$ B4's hypothetical parallel dumb-down heliocentrist-forgery) - a hoax so bungled that its two dates for the same event (fn 24) accidentally disagreed by over a month, thereby creating a truly unquestioned Greatest:

THE most hilariously inept fraud in the entire history of astronomy.

[^19]
## References

Almajest. Compiled Ptolemy c. 160 AD. Eds: Manitius 1912-3; Toomer 1984
DIO: The International Journal of Scientific History (www.dioi.org)
J.Evans 1984. American Journal of Physics 52:1080.
O.Gingerich 1976. Science 193:476.
O.Gingerich 1980. QJRAS 21:253.
O.Gingerich 2002. Isis 93.1:70.

Alexander Jones 1999E. ArchiveHistExactSci 54.3:255.
Karl Manitius 1912-3, Ed. Handbuch der Astronomie [Almajest], Leipzig.
R.Newton 1977. Crime of Claudius Ptolemy, Johns Hopkins U.
R.Newton 1985. Origins of Ptolemy's Astronomical Tables, U.Maryland.
O.Pedersen 1974. Survey of the Almajest, Odense U.
D.Rawlins 1985G. Vistas in Astronomy 28:255.
D.Rawlins 1985K. BullAmerAstronSoc 17:852.
D.Rawlins 1987. American Journal of Physics 55:235.
D.Rawlins 1991P. Journal for Hysterical Astronomy $1.1 \ddagger 7$.
D.Rawlins 1991W. DIO\&Journal for Hysterical Astronomy 1.2-3 $\ddagger 9$.
D.Rawlins 1996C. DIO\&Journal for Hysterical Astronomy $6 \ddagger 1$.
D.Rawlins 2003J. DIO $11.2 \ddagger 4$.
B.Schaefer 2002. Sky\&Tel 103.2:38.

Noel Swerdlow 1979. American Scholar (ФВК) 48:523. Review of R.Newton 1977.
N.Hamilton-Swerdlow 1981. JHA 12:59. Review of R.Newton 1976.

Noel Swerdlow 1989. JHA 20:29.
Hugh Thurston 1994P. DIO $4.2 \ddagger 6$.
Hugh Thurston 1998A. DIO $8 \ddagger 1$.
Hugh Thurston 2002S. Isis 93.1:58.
Gerald Toomer 1984, Ed. Ptolemy's Almagest, NYC.
B.van der Waerden 1970. heliozentrische System . . . griech, pers \& ind Astron, Zürich.

## I Appendix: A Boobonic Plague of Upside-Down-Apologia

I1 Instead of using the valid methods demonstrated here in $\S$ G or $\ddagger 5$, Ptolemy preferred to fake Venus "observations" using a crudely-rearranged (utterly impossible-fantasy) version of the very theory he claimed he was trying to prove from the faked data. (If such a lying inversion of genuine empirical investigation [and plain truth] isn't a crime against science, then there's no such thing.) See, e.g., Gingerich 2002 or Swerdlow 1989. ${ }^{52}$ Both papers

[^20]are by leading losers of the Ptolemy Controversy, who lack the simple integrity to admit their defeat. ${ }^{53}$ (But, then, their praise of Ptolemy's fakery suggests that honesty is not quite at the top of these archons' list of desirable human virtues.) Instead, they have painted themselves into the corner of having to say: well, OK, so Ptolemy did fake the data - how wonderfully clever of him! If you can't believe this (and who would blame you?), then: read both papers. (Some special enticement-sample logical-gems are collected in fn 52.)
$\mathbf{I 2}$ Bottom-lines for Ptolemy's finding $r, e, \& A$ : [a] He picked a terrible method (§B1). [b] He could only make it "work" by faking data (§I5). [c] He fumbled the fakes so badly that he even double-dated two (§B). [d] He had a poor ${ }^{54}$ eccentricity $e$
13 Gingerich 2002 p. 72 nearly swoons in admiration of such achievements, designing the boldly-invert term "approximations" for Venus places that Ptolemy super-unapproximately faked in order to get them on-the-nose-precisely where his crackpot method needed them (emph added): "Such approximations ${ }^{55}$ are characteristic of our most insightful scientists, who see them as a way to tackle otherwise intractable problems." What are the standards
than the number of digits to the left of the decimal point on a MacArthur grant-checque) which have convinced NS of the bizarre proposition that the astrologer-geometrist-plagiarist who faked clumsy pseudo-proof after clumsy pseudo-proof (oblivious to the most elementary empirical considerations, e.g., fn 24, or Thurston 1998A $\odot 16$ ), was actually a proto-Isaac Newton when no one was looking: an immortal scientist and a deft empirical investigator who carried out competent clandestine derivations (there are some wonderfully accurate parameters embedded in the Almajest, e.g., Mars' mean synodic motion, Mercury's inclination) - derivations which, despite his high integrity and the vast bulk of his encyclopaedic output, he nowhere reported, so that all are now regrettably Missing. Among the achievements implicit in Swerdlow's intensely religious revelation: Ptolemy used accurate unreported observations to secretly found his theories, but then reported these same theories in the Almajest as if they were based upon highly inaccurate observations. See further extreme irony noted at Rawlins 1987 p. 237 (item 3 \& final sentence of item 2).
${ }^{53}$ Note: Swerdlow 1979 p. 524 affected horror at R.Newton's attack on Ptolemy's "reputation for integrity" - yet when Swerdlow finally, a decade later, agreed (fn 52) to the Ptolemy frauds we're discussing, one item was found inexplicably missing from Swerdlow 1989 (especially from its n.1, which specifically cites his earlier attack on RN): namely, NS' appropriately contrite retraction of his earlier repulsive abuse of R.Newton for saying the same things. (See fn 12.) But, OG\&NS share a common problem: how can an upper-case Authority retract? The surest mark of a privately-insecure fake expert is his palpable terror of ever being viewed as seriously mistaken. The resultant rigidity explains why high-society academe (increasingly crowded by creatures who frantically spend too much energy at brainkissing to leave much over for serious research: DIO $2.3 \ddagger 6 \S \mathrm{~F} 2$ ) ends up weighed down by functional cranks - since a crank's most characteristic feature is his inability to change his opinion in response to incoming evidence
${ }_{54}^{54}$ See R.Newton 1977 Table XI. 7 and here at $\S$ C 3 \& fn 30.
${ }^{55}$ Linguistic manipulation ("approximations" for deliberate forgeries) is a classic Orwellian technique for avoiding admission of controversy-loss; in like (if less perverse) spirit, Swerdlow 1989 p. 35 repeatedly calls Ptolemy's faked Venus places "required" positions. (Gingerich 2002 p. 72 [pushing same sham]: "desired" positions.) Translation: Ptolemy's forgeries were computed from pre-known theory and brought forth by him as evidence for that very same theory (fn 57), a swindle (note related [and even more spectacular] archonal flipflop cited at Rawlins 1991 W fn 99) which O.Gingerich has been telling us for decades is the best - "the Greatest" - type of ancient science! (For several unevadable proofs that Ptolemy possessed parameters before constructing his purported proofs of them, see: fn 57, §B3, Thurston 1994P, Rawlins 1987 pp.236-237 item (5) \& n.25.) In brief: between slanders of those who condemn Ptolemy, OG is now admitting (Gingerich 2002) that Ptolemy indeed (as all OG's slandees realized decades before OG) used indoor computations to fake outdoor observational data. Thus, OG has undergone a crucial position-shift on the evidence, but (revealingly) it has not caused him to retreat one micron from his pre-admission (Gingerich 1976) position - as he exhibits wondrously ineducable constancy (fn 10) in verbatim-echoing Neugebauer 1975 p. 931 's rating of a plagiarist as "the greatest astronomer of antiquity": see Gingerich 1976, Gingerich 1980 p.264, \& Gingerich 2002 p.70. The religious adherence to preconception, entirely regardless of incoming evidence, is downright Ptolemaic. See fn 13, and especially fn 61 \& DIO $2.3 \ddagger 8$ §§C25 \& C31-33.
of journals \& foundations that repeatedly promote such amateur reveries (unrefereed by anyone truly capable in the area) as the judicious inductions of a genuine expert? ${ }^{56}$
I4 Such repulsive corruption of the ideal of scientific creativity is sufficiently remarkable that the entire academic community ought to be alerted to Ptolemy-defenders' pioneering efforts at rolling back the frontiers of archonal apologia.
I5 With respect to Ptolemy's Venus doubledating-pratfall (fn 52): while NS \& OG funtasize that Ptolemy was forced ${ }^{57}$ into cheating, they slyly fail to face the question of what the hell this speculation has to do with explaining Ptolemy's fakery getting so fouled up? Is our lovably impervious defenders' next trick going to be: convincing the world that Ptolemy needed to create hilariously and ( $\S \S B 2 \& B 3$ ) enormously self-contradictory fakes? - capped off by his $140 / 2 / 18$ claim that he saw (at virtually the correct date) Venus' elongation from the mean Sun at $48^{\circ} 1 / 3$, even though Venus never got beyond $48^{\circ}$ at any time during its entire 140 AD evening-star cycle, a discrepancy larger than the lunar semidiameter, yet another genuine \& impressive Ptolemy first (deserving rank almost up there with double-dating a celestial event): The Greatest's elongation was greater than the greatest elongation. One can similarly describe Ptolemy's 136/11/18 "observation": how can he at Almajest 10.2 call $47^{\circ} 1 / 3$ the greatest 136 AD evening Venus elongation when - by his own massively disparate (fn 24) but immediately-adjacent Almajest 10.1 account - the greatest 136 AD evening Venus elongation was $1^{\circ} / 5$ higher $\left(47^{\circ} 8 / 15\right)$ on $136 / 12 / 25$ ? Thus, The-Greatest racks up one more ${ }^{58}$ greater-than-greatest elongation. (This

[^21]time greater than his own [rather than reality's] greatest.)
I6 Familiarly, we are here confronted with two quite different Ptolemy superexcesses: fraud and bungling. [Nobody's perfect.] R.Newton's low evaluation of Ptolemy coherently solves both. (By contrast, Ptolemy's Occam-defying, Osgoodly-impenetrable defenders continue [see similarly at DIO $4.3 \ddagger 15$ §F5, DIO 10 §L9 \& fn 109] to have to concoct \& juggle multiple distinct and disjunct alibis: fnn 20\&61.) E.g., for one of his two initial Venus proofs [either Almajest 10.1 or 10.2], why couldn't a smart faker have just used the nearly identical Venus 128/12/16 greatest evening elongation instead of the 136/12/14 one? This would have to earn fewer snickers than Ptolemy's actual procedure: using the very SAME Venus 136 AD greatest elongation in two conflicting proofs, megacontradictorily.
I7 NS-OG's sober alibiing of Ptolemy's Venus fumblefarces is akin to a defense-lawyer going into court to get-off a counterfeiter who was so stupidly careless that he accidentally printed Ben Franklin on both sides of his attempts at faking hundred-dollar bills. What lawyer (outside innermost $J H A d u m$ ) would try to excuse such inept criminality by claiming that the bungled bucks showed immortal, greatest-technician-of-the-era BRILLIANCE?
I8 A penchant for such almost-perfectly-inverted judgements seems to be bubonically infectious in the Muffiose circle. (See also, e.g., fnn 7\&35, DIO 11.1 p.2.) Note that these cranial warps issue not from the Flat Earthers or the Scientologists but from highly placed professors at Harvard and the University of Chicago, and are regarded as utterances-ofgenius by no less than the MacArthur Foundation.
I9 We know ( $\ddagger 5$ § C2, Rawlins 1991 W fn 123, or Swerdlow 1989 p.32) that Ptolemy got his Venus $r$ from Pliny ( 77 AD ) or his sources, and we found hints of Ptolemy's $e$ and $A$ at eq. $30 \& \mathrm{fn} 48$, respectively. However, the more important point is: his $e$ is not only wrong but the correct value could have been estimated in his own time - and with just two years of real elongation data ( $\S \mathrm{H} 2$ ) - by techniques (explained in our present paper and that [ $\ddagger 7]$ following) comparable to those he said he used for the outer planets (fn 27).
I10 So it is no longer relevant to claim ${ }^{59}$ that R.Newton was unhistorical in using modern math and computers to get his best-fit Venus elements (R.Newton 1977 p. 311 Table XI.2). For, we have already demonstrated here by quantitative test ${ }^{60}$ that the ancient-style triobased method of the present paper would have gotten virtually the same results as R.Newton, and from just a few years of careful observations - taken outdoors.
I11 But it's that italicized last condition that was always a problem with Ptolemy. For, as we know on other and even simpler evidence, "Ptolemy doesn't seem to have allowed his eyeballs out at night." ${ }^{\text {6 }}$

[^22]
## $\ddagger 7$ Unveiling Venus

## by Hugh Thurston

Although we may never know for certain, it is interesting to speculate how early people, the Greeks in particular, came to deal with the motion of Venus, their extant efforts culminating in the theory described in Ptolemy's Syntaxis.

## A Early Days

A1 Early people will certainly have seen a bright planet (which the Greeks called Eosphoros) visible in the morning for several months at a time, and a bright planet (Phosphoros) visible in the evening, but not during the time when Eosphoros was visible. Quite early, before the time of Eudoxus, the Greeks realized that the two were in fact one planet (our Venus). The Babylonians, the Chinese, and even the relatively unsophisticated Mayas, also realized this.
A2 Also quite obvious were the retrogressions of the three visible outer planets. Early on it became clear that their motion was to-and-fro round a steadily moving "mean planet". (Literally so called by the Hindus. The Sanskrit is madhya graha.) Eudoxus had the planet moving in a figure-of-eight; and the Chinese had it moving in the shape of a willow-leaf. A good quantitative theory became possible when some genius thought of having it move round on an epicycle.
A3 The change from evening star to morning star is a retrogression (past the sun) so the motion of Venus can also be dealt with by using an epicycle. The center of Venus' epicyle keeps in line with the mean Sun.

## B Epicycles and Eccentrics

B1 Ptolemy used the mean longitude, not the true longitude, of the sun, and earlier astronomers may well have done the same: the doctrine of regular circular motion, which probably underlay this choice, is older than Ptolemy.
B2 If the speed of the planet round C , the speed of C round the earth, the radius of the epicycle and the distance of C from the earth were all constant, retrogressions would all be equally long. They are not, so something has to change.
B3 I suggest that the first modification to the theory was to make the centre O of the orbit of C a point distinct from the earth T, used by Hipparchus for the sun with considerable success.
B4 We need to find the direction of O from T , the distance $e$ of O from T , the radius $r$ of the epicycle and the radius $R$ of the orbit of C . The ancients could not measure astronomical distances, so they could find only the mutual ratios of $e, r$, and $R$.
B5 The astronomers started with a list of timed longitudes of Venus. For each they could calculate the mean longitude of the Sun, MLS. Its difference from the longitude of Venus is the mean elongation. They worked with the maximum mean elongation: MME.

[^23]

Figure 1: See $\S$ D2.

B6 There are two things that this could mean. If, starting just before the middle of a period of visibility, they measured the longitude of Venus each night and picked the greatest of the mean elongations, that would give a maximum with respect to time.
B7 Alternatively, if they had a list of mean elongations they could look at all the mean elongations for a given MLS and pick the greatest. This gives the greatest ME for a given position of the epicycle. Clearly, when Venus is at the greatest ME for a given position of the epicycle, the line of sight from the earth to Venus is an exact ${ }^{2}$ tangent to the epicycle. From now on, this version is what I mean by MME. It is the version used by Dennis Duke in $\ddagger 5$ of this issue of $D I O$ and is the version that appears in Ptolemy's calculations. For each MLS there will be two MMEs, one to the east and one to the west. ${ }^{3}$ Their sum gives the apparent size of the epicycle. The first step is to find the longitude of O . There are at least three ways to do this.
B8 Method 1: Find the position of C for which the apparent size of the epicycle is greatest and the position for which it is least. To do this the astronomer would tabulate the MMEs in the order of their MLSs, for each western MME find by interpolation the eastern MME for the same MLS, add the two MMEs together, note how the sum varies, and find by interpolation the MLSs giving the greatest and least values. (If they do not differ by $180^{\circ}$, there is a problem.)

[^24]B9 Method 2: Find an eastern MME equal to a western MME. A moderately large list of MMEs would give a reasonable chance of finding such a pair. Ptolemy said that he used this method, and, assuming that the line OT runs halfway between the two equal MLSs, found that it extends along the diameter $55^{\circ}$-to- $235^{\circ}$. Comparing the apparent size of the epicycle when the MLS is near to $55^{\circ}$ with the size when it is near to $235^{\circ}$ showed that the longitude of O is $55^{\circ}$.
B10 The trouble here is that the equality of the two MMEs does not guarantee that OT runs halfway between the MLSs. This is easily seen from Dennis Duke's graph [1] of eastern MME against MLS [Figure 1 of article $\ddagger 5$ in this DIO].
B11 For each eastern MME (other than the greatest and least ones) there are two MLSs. If comparing one with an equal western MME (as graphed at $\ddagger 5$ Figure 2) gives the right direction to OT, comparing the other one with the same western MME will not. Ptolemy's result is roughly correct, so if he actually used this method he must have been lucky in picking a well-matched pair.
B12 Method 3: Use three MMEs. The appendix (§D) shows how.
B13 Admittedly, we have no direct evidence that any ancient astronomer used method 3, but perhaps we should not expect any.
B14 The next step is to find $e$ and $r$ in terms of $R$. If we have used method 3 there is no problem. If we have used method 1 or 2 the most straightforward way is to use two MMEs when C is on the line OT. Ptolemy claimed to have used this method and explained it in chapter 2 of book 10 of the Syntaxis.
B15 The trouble here lies in finding MMEs when C is in the right position. Suppose that when C is on the line, say at $55^{\circ}$, Venus is at the point V of the epicycle. C will reach $55^{\circ}$ at yearly intervals. The synodic period of Venus is very close to $8 / 5$ of a year. Let V , $\mathrm{W}, \mathrm{X}, \mathrm{Y}$, and Z divide the epicycle into five equal arcs. The next five times that C is at $55^{\circ}$ Venus will be at Y, W, Z, X, and back at V; not exactly but close. The positions of Venus when C is at $55^{\circ}$ will be close to these five points for a long time. Unless we are lucky and one of them yields a MME we will not find what we want. Ptolemy said that he found it (on May 20th, 129 AD).

## C The Equant

C1 Ptolemy's final theory used an equant. Dennis Duke has shown [ $\ddagger 5 \S \S E 1-E 4]$ how Ptolemy might have been led to this. Duke's reasoning can easily be restated in ancient Greek fashion. Duke also explained [ $\ddagger 5 \S \mathbf{C} 2$ ] how the ancients might have found the radius of the epicycle.
C2 Several writers have suggested that Ptolemy adjusted (or even fabricated) his observations. In particular, Swerdlow wrote that Ptolemy "could not have observed some of the reported elongations" (he certainly could not have observed two MMEs 37 days apart) and "Ptolemy's adjustments of whatever he observed were of the order of $1^{\circ}$ " [2, page 54].

## D Appendix

D1 Figure 1 shows an occasion when the sight-line from the earth T to Venus V is a tangent to the epicycle. OQ points in the direction of zero longitude. Angle COQ is the mean longitude of the sun. The angle between TV and OQ is the longitude $\beta$ of Venus. CVT is a right angle.
D2 Drop perpendiculars OF to CV, OM to TV, TZ to OQ, and ZH to OM. Angle COF is the elongation $\gamma$. (OT is the eccentricity, and TO points towards the apogee.) Angle ZTS is $90^{\circ}-\beta$. Then $\mathrm{HS}=(\mathrm{ZT} / 120) \cdot \operatorname{chd}\left(180^{\circ}-2 \beta\right)$. Angle $\mathrm{HZO}=\beta$. Then HO $=(\mathrm{OZ} / 120) \cdot \operatorname{chd}(2 \beta)$. Thus, $r=\mathrm{CV}=\mathrm{CF}+\mathrm{FV}=\mathrm{CF}+\mathrm{OM}=\mathrm{CF}+\mathrm{HM}-\mathrm{HO}=$ $(1 / 2) \cdot \operatorname{chd}(2 \gamma)+(\mathrm{ZT} / 120) \cdot \operatorname{chd}\left(180^{\circ}-2 \beta\right)-(\mathrm{OZ} / 120) \cdot \operatorname{chd}(2 \beta)$.

D3 Given three such occasions, we have three expressions for $r$, giving [by equating the three expressions] two linear equations in the two unknowns ZT and OZ, which the Greeks could solve. That gives both the distance OT [eccentricity $e$ ] and the angle between OZ and OT and hence the direction [apogee $A$ ] of O from T. Then any one of the three expressions gives $r$. This method has been applied to real data by Dennis Rawlins [3] using modern mathematics (coordinate geometry) and by Dennis Duke, using vectors (unpublished).

## References

[1] Fax [ $\ddagger 5$ Figure 1] on July 3rd, 2002, from Dennis Duke, Florida State University, [2] Noel M. Swerdlow. Ptolemy's theory of the inferior planets, Journal for the History of Astronomy volume 20 (1989) pages 29 to 60.
[3] Dennis Rawlins, The Crucial-Test, DIO volume 11 pages 54 and 70 to 90 [paper $\ddagger 6$ : that preceding the present one].

## E Program for Thurston Geocentric Method [2003 DR Addendum]

Hugh Thurston's geocentric method ${ }^{4}$ can be programmed just as readily as DR's heliocentric method (p.54). To save space - and highlight differences in the two approaches - we below provide merely a dozen lines of the geocentric program, since all the other lines are identical to those of p .54 's program. Thus, a simple 12 -line replacement-operation (merge-command) upon the 35 -line heliocentric program on p. 54 produces the full 35 -line geocentric program. Despite the methods' differing math steps and reference-frames, the results of the two programs are the same, both for the equant ${ }^{5}$ (where every loop's results are absolutely identical: e.g., $\ddagger 6 \S \mathbf{G} 9$ ) and the eccentric model.
$130 \mathrm{~N}(\mathrm{I})=1$ :NEXT I
$150 \mathrm{H}=\mathrm{E} 0 * \mathrm{SIN}(\mathrm{G}): \mathrm{G} 0=\mathrm{G}: \mathrm{G}=\mathrm{G}-\mathrm{ATN}\left(\mathrm{H} / \mathrm{SQR}\left(1-\mathrm{H}^{\wedge} 2\right)\right): \mathrm{N}(\mathrm{I})=\mathrm{SIN}(\mathrm{G}) / \mathrm{SIN}(\mathrm{G} 0)$
190 NEXT I
220 IF $\operatorname{SGN}(\mathrm{G}(\mathrm{I}))=\mathrm{SGN}(\mathrm{G}(\mathrm{J}))$ THEN SI=1 ELSE SI $=-1$
$230 \mathrm{~J}(\mathrm{I})=\operatorname{COS}(\mathrm{V}(\mathrm{J}))-\mathrm{SI} * \operatorname{COS}(\mathrm{~V}(\mathrm{I}))$
$240 \mathrm{~K}(\mathrm{I})=-(\operatorname{SIN}(\mathrm{V}(\mathrm{J}))-\mathrm{SI} * \operatorname{SIN}(\mathrm{~V}(\mathrm{I})))$
$250 \mathrm{Z}(\mathrm{I})=\mathrm{SIN}(\mathrm{G}(\mathrm{J})) * \mathrm{~N}(\mathrm{~J})-\mathrm{SI} * \operatorname{SIN}(\mathrm{G}(\mathrm{I})) * \mathrm{~N}(\mathrm{I}):$ NEXT I
$270 \mathrm{~W}(\mathrm{I})=\mathrm{J}(\mathrm{J}) * \mathrm{~K}(\mathrm{I})-\mathrm{J}(\mathrm{I}) * \mathrm{~K}(\mathrm{~J})$
$280 \mathrm{~A}(\mathrm{I})=(\mathrm{J}(\mathrm{J}) * \mathrm{Z}(\mathrm{I})-\mathrm{J}(\mathrm{I}) * \mathrm{Z}(\mathrm{J})) / \mathrm{W}(\mathrm{I})$
$290 \mathrm{O}(\mathrm{I})=(\mathrm{Z}(\mathrm{J}) * \mathrm{~K}(\mathrm{I})-\mathrm{Z}(\mathrm{I}) * \mathrm{~K}(\mathrm{~J})) / \mathrm{W}(\mathrm{I})$
$300 \mathrm{R}(\mathrm{I})=\mathrm{ABS}(\mathrm{N}(\mathrm{I}) * \operatorname{SIN}(\mathrm{G}(\mathrm{I}))-\mathrm{O}(\mathrm{I}) * \mathrm{COS}(\mathrm{V}(\mathrm{I}))+\mathrm{A}(\mathrm{I}) * \mathrm{SIN}(\mathrm{V}(\mathrm{I}))): \mathrm{NEXT} \mathrm{I}$
$310 \mathrm{X} 0=\mathrm{A}(3) / \mathrm{F}: \mathrm{Y} 0=\mathrm{O}(3) / \mathrm{F}$

[^25]DIO: The International Journal of Scientific History [www.dioi.org] is published by DIO, Box 19935, Baltimore, MD 21211-0935, USA.
Telephone (answering machine always on): 410-889-1414.
[Email: dioi@mail.com.]
Research \& university libraries may request permanent free subscription to DIO. Each issue of DIO will be printed on paper which is certified acid-free. The ink isn't.
Editor: Keith Pickering, address below. Publisher: Dennis Rawlins (DR), address above.
$D I O$ is primarily a journal of scientific history \& principle. However, high scholarship and-or original analytical writing (not necessarily scientific or historical), from any quarter or faction, will be gladly received and considered for publication. Each author has final editorial say over his own article. If refereeing occurs, the usual handsome-journal anonymity will not, unless in reverse. No page charges. Each author receives 50 free offprints.

The circumstance that most $D I O$ articles are written by scholars of international repute need not discourage other potential authors, since one of DIO's purposes is the discovery \& launching of fresh scholarly talent. Except for equity\&charity reply-space material, submissions will be evaluated without regard to the writer's status or identity. We welcome papers too original, intelligent, and-or blunt for certain handsome journals. (Dissent \& controversy are per se obviously no bar to consideration for DIO publication; but, please: spare us the creationist-level junk. I.e., non-establishment cranks need not apply.)

Other journals may reprint excerpts (edited or no) from any issue of DIO to date, whether for enlightenment or criticism or both. Indeed, excepting DIO vols. $3 \& 5$, other journals may entirely republish $D I O$ articles (preferably after open, nonanonymous refereeing), so long as DIO's name, address, \& phone \# are printed adjacent to the published material - and to all comments thereon (then or later), noting that said commentary may well be first replied to (if reply occurs at all) in DIO's pages, not the quoting journal's.

DIO invites communication of readers' comments, analyses, attacks, and-or advice.
Written contributions are especially encouraged for the columns: Unpublished Letters, Referees Refereed, and regular Correspondence (incl. free errtime for opponents). Contributor-anonymity granted on request. Deftly or daftly crafted reports, on apt candidates for recognition in our occasional satirical J. for Hysterical Astronomy will of course also be considered for publication.

Free spirits will presumably be pleased (and certain archons will not be surprised) to learn that: at DIO, there is not the slightest fixed standard for writing style.

Contributors should send (expendable photocopies of) papers to one of the following DIO referees - and then inquire of him by phone in 40 days:
Dennis Duke [ancient astronomy, data analysis], Physics Dep't, Florida State University Tallahassee, FL 32306-4052; tel 850-644-0175.
Robert Headland [polar research \& exploration], Scott Polar Research Institute, University of Cambridge, Lensfield Road, Cambridge CB2 1ER, UK; tel (44) 1223-336540.
Charles Kowal [celestial discovery, asteroids], Johns Hopkins University Applied Physics Laboratory, Johns Hopkins Road, Laurel, MD 20707; tel 410-792-6000.
Keith Pickering [navigation, exploration, computers, photography, science ethics],
10085 County Road 24, Watertown, MN 55388; tel 952-955-3179; fax 952-955-2398
E. Myles Standish [positional \& dynamical astronomy], Jet Propulsion Laboratory 301-150, Cal Tech, 4800 Oak Grove Drive, Pasadena, CA 91109-8099; tel 818-354-3959.
F. Richard Stephenson [ancient eclipses, $\Delta T$ secular behavior], Department of Physics, University of Durham, Durham DH1 3LE, UK; tel (44) 191-374-2153.
Hugh Thurston [early astronomy, WW2 cryptography] (Univ. Brit. Columbia, Prof. Emer. Math), Unit 3 12951, 17 ${ }^{\text {th }}$ Ave, S.Surrey, B.C., Canada V4A 8T7; tel 604-531-8716.
Christopher B. F. Walker [Mesopotamian astronomy], Dep't of Western Asiatic Antiquities, British Museum, Great Russell Street, London WC1B 3DG, UK; tel (44) 171-323-8382.
© 2002 DIO Inc.
ISSN 1041-5440.
This printing: $2010 \backslash 10 \backslash 16$.

# A Fresh Science-History Journal: Cost-Free to Major Libraries 

Telephone 410-889-1414

## DIO - The International Journal of Scientific History. Deeply funded. Mail costs fully covered. No page charges. Offprints free.

- Since 1991 inception, has gone without fee to leading scholars \& libraries.
- Contributors include world authorities in their respective fields, experts at, e.g., Johns

Hopkins University, Cal Tech, Cambridge University, University of London.

- Publisher \& journal cited (1996 May 9) in New York Times p. 1 analysis of his discovery of data exploding Richard Byrd's 1926 North Pole fraud. [DIO vol.4.] Full report co-published by University of Cambridge (2000) and DIO [vol.10], triggering History Channel 2000\&2001 recognition of Amundsen's double pole-priority. New photographic proof ending Mt.McKinley fake [DIO vol.7]: cited basis of 1998/11/26 New York Times p. 1 announcement. Nature 2000/11/16 cover article pyramid-orientation theory: DIO-corrected-recomputed, Nature 2001/8/16. Vindicating DR longtime Neptune-affair charges of planet-theft \& file-theft: Scientific American 2004 December credits DIO [vols.2-9].
- Journal is published primarily for universities' and scientific institutions' collections; among subscribers by request are libraries at: US Naval Observatory, Cal Tech, Cornell, Johns Hopkins, Oxford \& Cambridge, Royal Astronomical Society, British Museum, Royal Observatory (Scotland), the Russian State Library, the International Centre for Theoretical Physics (Trieste), and the universities of Chicago, Toronto, London, Munich, Göttingen, Copenhagen, Stockholm, Tartu, Amsterdam, Liège, Ljubljana, Bologna, Canterbury (NZ).
- New findings on ancient heliocentrists, pre-Hipparchos precession, Mayan eclipse math, Columbus' landfall, Comet Halley apparitions, Peary's fictional Crocker Land.
- Entire DIO vol. 3 devoted to $1^{\text {st }}$ critical edition of Tycho's legendary 1004-star catalog.
- Investigations of science hoaxes of the $-1^{\text {st }},+2^{\text {nd }}, 16^{\text {th }}, 19^{\text {th }}$, and $20^{\text {th }}$ centuries.

Paul Forman (History of Physics, Smithsonian Institution): "DIO is delightful!"
E. Myles Standish (prime creator of the solar, lunar, \& planetary ephemerides for the preeminent annual Astronomical Almanac of the US Naval Observatory \& Royal Greenwich Observatory; recent Chair of American Astronomical Society's Division on Dynamical Astronomy): "a truly intriguing forum, dealing with a variety of subjects, presented often with [its] unique brand of humor, but always with strict adherence to a rigid code of scientific ethics. . . . [and] without pre-conceived biases . . . . [an] ambitious and valuable journal."
B. L. van der Waerden (world-renowned University of Zürich mathematician), on DIO's, demonstration that Babylonian tablet BM 55555 ( 100 BC ) used Greek data: "marvellous." (Explicitly due to this theory, BM 55555 has gone on permanent British Museum display.)

Rob't Headland (Scott Polar Research Institute, Cambridge University): Byrd's 1926 latitude-exaggeration has long been suspected, but DIO's 1996 find "has clinched it."

Hugh Thurston (MA, PhD mathematics, Cambridge University; author of highly acclaimed Early Astronomy, Springer-Verlag 1994): "DIO is fascinating. With . . . mathematical competence, $\ldots$. judicious historical perspective, [\&] inductive ingenuity, ... [DIO] has solved . . . problems in early astronomy that have resisted attack for centuries . . . "

Annals of Science ( 1996 July), reviewing DIO vol. 3 (Tycho star catalog): "a thorough work . . . . extensive [least-squares] error analysis . . . demonstrates [Tycho star-position] accuracy . . . much better than is generally assumed . . . . excellent investigation".

British Society for the History of Mathematics (Newsletter 1993 Spring): "fearless . . [on] the operation of structures of [academic] power \& influence . . . much recommended to [readers] bored with . . . the more prominent public journals, or open to the possibility of scholars being motivated by other considerations than the pursuit of objective truth."


[^0]:    ${ }^{1}$ [Note by DIO: Supercomputer-specialist Professor Dennis Duke (Florida State University Physics Department: www.csit.fsu.edu/ /dduke) has lately produced a series of refined \& original technical analyses bearing upon ancient astronomical issues. More of his current work appears in Archive for History of Exact Sciences, Journal for the History of Astronomy, and DIO 12 \& DIO 13.]
    ${ }^{2}$ Ptolemy's Almagest, transl. by G. J. Toomer (London, 1984).
    ${ }^{3}$ C. Wilson, "The Inner Planets and the Keplerian revolution", Centaurus, 17 (1972) 205-248; R. R. Newton, The crime of Claudius Ptolemy (Baltimore, 1977); D.Rawlins, "Ancient Heliocentrists, Ptolemy, and the equant", American Journal of Physics, 55 (1987) 235-239; N. M. Swerdlow, "Ptolemy's Theory of the Inferior Planets", Journal for the history of astronomy, 20 (1989) 29-60; H.Thurston, "Ptolemy's Backwardness", DIO, 4 (1994) 58-60

[^1]:    ${ }^{4}$ Pierre Bretagnon and Jean-Louis Simon, Planetary Programs and Tables from -4000 to +2800 (Richmond, 1986).
    ${ }^{5}$ The greatest elongations are assembled using a simple tabular method, much as the ancient analyst might have done. Start with an empty table of 360 rows, one row for each rounded value of the longitude of the mean Sun. When an elongation is determined, look into the corresponding row of the table and enter the value of the elongation if it is larger than the value already in the table. One might also save the date and any other pertinent information.
    ${ }^{6}$ Almagest IX. 2 refers to models consisting of eccentric circles, concentric circles carrying epicycles, and both combined. Also, the iterative process of finding the equant and apogee for the outer planets using three oppositions begins by assuming an eccentric deferent carrying an epicycle.

[^2]:    ${ }^{7}$ This is discussed in detail in Wilson, op. cit. (fn 3) p. 211.

[^3]:    ${ }^{8}$ C. Wilson, op. cit. (fn 3) p. 234.
    ${ }^{9}$ The data in these charts are collected daily. [The dots in Fig. 10 represent greatest sums (see rules in fn 5) of Fig.9's morning \& evening Venus elongations at a given solar longitude during 127-141 AD. The Venus curves' maxima in Fig. 10 very nearly correspond to greatest-elongation-pairs, due to the helpful accident that a Venus evening greatest-elongation is always followed about two years later by a morning greatest-elongation at virtually the same solar longitude. (See, e.g., lines 70\&90 in program at p.54: mean solar longitudes $105^{\circ} \& 111^{\circ}$, resp.) For Mercury, such neat matches require a longer wait, which may help explain why ancient astronomers were more confused by the innermost planet's motion.]
    ${ }^{10}$ However, the discussion of this paragraph must be tempered by the recent demonstrations by Rawlins (in a heliocentric presentation: $\ddagger 6$ ) and Thurston (in an equivalent geocentric presentation: $\ddagger 7$ ) that a straightforward geometrical analysis of one trio of elongations can yield an estimate of the eccentricity (or equant), the apogee direction, and the radius of the epicycle.

[^4]:    ${ }^{11}$ An early mention of this is in the Almagest itself, chapter XIII.8, and it was also mentioned by the Babylonians. The point is also sometimes raised in our time, e. g. in Owen Gingerich, "Ptolemy and the Maverick Motion of Mercury", Sky \& Telescope, 66 (1983) 11-13.
    ${ }^{12}$ These are the visibility conditions used by Jean Meeus, More Mathematical Astronomy Morsels (Richmond, 2002) p. 347.
    ${ }^{13}$ N. M. Swerdlow, The Babylonian Theory of the Planets (Princeton, 1998), p. 107.
    ${ }^{14}$ A. J. Sachs and H. Hunger, "Astronomical Diaries and related Texts from Babylonia. I-III", Denkschriften / Österreichische Akademie der Wissenschaften, Philosophisch-Historische Klasse 195 (1988), 210 (1989), 246 (1996).
    ${ }^{15}$ H. Hunger, "A 3456: eine Sammlung von Merkurbeobachtungen", in E. Leichty et al., A Scientific Humanist, Studies in Memory of Abraham Sachs. Occasional Papers of the Samuel Noah Kramer Humanist, Studies in Memory
    Fund 9 (Philadelphia
    ${ }^{16}$ James Evans, The History and Practice of Ancient Astronomy (New York, 1998), pp.362-368.

[^5]:    ${ }^{17}$ James Evans, "On the Function and the Probable Origin of Ptolemy's Equant", American Journal of Physics, 52 (1984) 1080-1089.

[^6]:    ${ }^{1}$ DIO $2.3 \ddagger 8$ §fn 46.
    ${ }^{2}$ See p. 54 .
    ${ }^{3}$ See, e.g., DIO $2.3 \ddagger 8$ §C33, DIO 12, \& www.dioi.org.
    ${ }^{4}$ See Thurston 1998A.
    ${ }^{5}$ Later published in Rawlins 1987 p. 236 item (4). See also R.Newton 1985 pp.13-19.
    ${ }^{6}$ See $\S$ I11. The only existing antique statue of Ptolemy (photo at Pedersen 1974 p.2) is not from the ancient era; but, through luck or wisdom (or just DIOnysian bad taste) the sculptor rendered Ptolemy's eyes nearly completely shut - just the way one would depict a blind man. This charming little wooden statue may be found in Europe's tallest cathedral, that in Ulm, Germany.

[^7]:    Those who have sales-pitched Ptolemy as The Greatest ancient astronomer (fn 10) have been increasingly exposed as having persistently and ineducably (fn 35 \& Rawlins 2002B fn 3) confused secondary work with primary - thus promoting derivative, plagiarized, and laughably bungled work as the central pinnacle of immortal original ancient science. (Ptolemists are adamantly convinced that no one whose work included the high math analyses in the Almajest could be a mere plagiarist. They seem utterly unable to handle the following just-too-complicated-for-archons alternate-hypothesis: if Ptolemy plagiarized hundreds of admirable star-data [a point now universally regarded as at least probable], then: mightn't he also have plagiarized the admirable math analyses?) The eminent institutions involved in this Upper-Deadwood-inspired crusade - which has entailed systematic suppressions, slander, \& (creditably noted by Schaefer 2002 p.40) shunnings of heretics - comprise a Who's-Who of academe: Harvard (my own school), Yale, Princeton, Brown, UChicago, Cambridge, Oxford U Press - all implicated in a mass-imposition that continues to make little sense but (fn 20) lots of dollars.
    ${ }^{8}$ DIO $9.3 \ddagger 6$ § N ticks off bovine-mentality cultists' standard sweet-to-vicious three-step evolutionhistory: sacred-cowing $\rightarrow$ money-cowing $\rightarrow$ critic-cowing
    ${ }^{9}$ While claiming that Ptolemy had to forge data to find Venus' orbital elements, both apologists (Swerdlow 1989 p. $31 \&$ Gingerich 2002 p. 73 Fig. 1 caption) of course have to admit (since faking positions requires elements) that Ptolemy (secretly) got these elements some other way - which contradicts their entire he-had-to-use-fake-data-to-establish-elements premis. (Further details at fn 22 \& fn 52.) With appropriately deep gratitude, we must ask: outside of Muffiose literature or Saturday Night Live's ornately-pratfalling Gerry Ford, how often does one have the jolly good fortune to encounter stuff like this? (One recalls the SNL-Prez-Ford's puzzled reaction, when D.Hamill's 1976 Olympic performance was worshipfully described to him as culminating in a double sitzspin \& triple somersault; he replied: so what? - I've done that getting out of a cab.) Perhaps the most extraordinary part of the spectacle: neither the authors, nor the publishers, nor the MacArthur Foundation detected this imposition before launching it as the prizewinning pinnacle of (centrist) Hist.sci scholarship.
    ${ }^{10}$ This point is gently made by Hugh Thurston in his 2002 Isis paper (p.65). In Gingerich's reply immediately following, OG unfortunately continues to project that he has been absolutely right about everything throughout his three-decade crusade, during which he has prominently, repeatedly (fn 55), and adamantly defended astronomy's biggest faker and plagiarist as "the greatest astronomer of antiquity". (Thurston 2002S p. 68 gave Gingerich the chance to retreat from this hilariously upsidedown judgement. Instead, Gingerich 2002 p. 70 obligingly updates his explicit insistence upon it; again, do not miss fn 55.) So, it should not particularly surprise us that OG could additionally conclude (fn 52) that deliberate scientific fraud is The-Greatest.

[^8]:    ${ }^{16}$ The figures of Swerdlow 1989 p. 42 and DR agree on this, to the arcmin.
    ${ }^{17}$ DIO $1.1 \ddagger 1 \mathrm{fn} 18 \& \ddagger 3$ §D2. OG also (fn 23) calls RN's views "offensive" \& "absurd".
    ${ }^{18}$ Note clearly a key point here: if Ptolemy had said he calculated his Venus data, there would be no controversy. But instead, he claimed (fnn 12\&24) that he visually observed, in the outdoor sky, Venus positions which all parties now agree were computed indoors.
    ${ }^{19}$ See $\S \S$ B1\&I5.
    ${ }^{20}$ Swerdlow 1989 p. 35 (emph added): the $8^{\text {y }}$ Venus-Earth resonance "makes the problem of finding greatest elongations close to the required positions . . . even more difficult." And p.36: Ptolemy's observation-dates' disagreement with ("departure from") the truth "is not an error, but a compromise necessitated by the positions of the mean Sun required for the demonstrations." Thus, poor Ptolemy $H A D$ to forge reality into the positions he wanted. . . . In this MacArthur-grant-subsidized paper (published by Gingerich's JHA), Swerdlow (loc cit) also alibis that since (near maximum) Venus' elongation changes merely $1^{\circ} / 12$ in $6^{\mathrm{d}}$, "in no way could Ptolemy estimate the time" of greatest [maximum] elongation more accurately. (Gingerich 2002's incomparable p. 72 goes even further into legalblindnessland, claiming that one-degree-accuracy in observation "is what Ptolemy typically worked with" - a sleight which neatly confounds ordmag $0^{\circ} .1$ ancient observational accuracy with the ordmag $1^{\circ}$ enormity of the most delicious Ptolemy fudge.) We have already previously (DIO $1.1 \ddagger 5$ fn 20) dealt with the tragic pre-highschool mentalblindnessland adventure of Swerdlow 1979 pp .526 527 (in the journal of PhiBetaKappa!), regarding estimation of maxima-times (solstices in that case), so I won't reprise the pathetic details here merely because he later repeated the folly under the MacArthur Foundation's aegis. But I will comment that none of this excuses inaccuracies of several weeks in Venus observations, leading to dishonestly-reported "observational" figures which are off by way over a degree. Moreover, what has uncertainty in time of greatest elongation (an error which can only reduce the elongation) to do with a faked "observation" which (§I5 [a]) exceeds the greatest elongation? It is on the basis of such then-politically-correct apologia that Swerdlow 1989 p. 35 tractably concludes: "the selection of a particular date for true greatest elongation would be arbitrary in any case." This

[^9]:    ${ }^{24}$ Those who have never consulted this ultimate Ptolemy blunder ought to look it up: Almajest 10.1 dates the 136 greatest evening elongation of Venus to $136 / 12 / 25$, while on the very next page, at Almajest 10.2, he dates it to $136 / 11 / 18$. See, e.g., Toomer 1984 p. 470 vs p. 471 . Perhaps hitherto unhighlighted: Ptolemy not only gives contradictory dates\&locations but also can't even get his story straight as to what this "greatest" elongation itself was. Almajest 10.1 makes it $47^{\circ} 32^{\prime}(136 / 12 / 25)$, while Almajest 10.2 makes it $47^{\circ} 20^{\prime}(136 / 11 / 18)$. See $\S$ I5 [a].
    [Note added 2003. Hugh Thurston emphasizes that the 136 double-date disaster (reality 136/12/14 [Table 2 row 2] vs $136 / 12 / 25$ [Almajest 10.1] again-vs 136/11/18 [Almajest 10.2]) is not atypical in giving wildly false Venus dates \& positions for greatest elongations. E.g., Ptolemy alleges he observed the 129 greatest morning elongation at (Almajest 10.2) $t=129 / 5 / 20, L=55^{\circ} 2 / 5, V=10^{\circ} 3 / 5$; compare to real data of Table 1 row 1: errors about 2 weeks \& well over $10^{\circ}$. And Ptolemy alleges he observed the 127 greatest morning elongation at (Almajest 10.1) $t=127 / 10 / 12, L=197^{\circ} 13 / 15$, $V=150^{\circ} 1 / 3$; compare to real data of Table 1 row 2 : errors nearly 3 weeks \& c. $20^{\circ}$.]
    ${ }^{25}$ That's 5 stars out of 1025 in the Ancient Star Catalog.
    ${ }^{26}$ The bases of the Almajest Venus \& Mars synodic motions were off by merely ordmag $1^{\prime} /$ century. The Mars motion fortunately came through without confusion, so that the Almajest 9.3-4 tabular mean synodic position of Mars is still today (2002) accurate to about $0^{\circ} .4$.

[^10]:    ${ }^{27}$ The iterative geometric proof that forms the heart (§G) of this paper has some similarities to the Almajest's for the outer planets. (See Thurston 1994P.) So: why didn't Ptolemy know this? Suggestion: others' proofs for the outer planets were available to him, but the proof of Venus' elements was not - which deprivation forced Ptolemy into inventing (or perhaps grabbing from some other bungler) the off-the-scale-funniest fumble of his entire career of hoaxery: see $\S$ B1. Another possibility: the Venus situation is not so clear [though see $\ddagger 7$ ] when viewed geocentrically (see, e.g., Pedersen 1974 p. 300 Fig.10.1), as against our choice here to (likewise Gingerich 2002 n.6) view it heliocentrically, which holds Venus' orbit near one convenient place (§D4); so, did Ptolemy's false (geocentric) general view of the universe help cause his specific Venus-botch embarrassment?
    ${ }^{28}$ This happens because Ptolemy is busy with more than three "observations", so that he can pretend he proved the equant's validity from Venus. See fn 35 .
    ${ }^{29}$ See Rawlins 1987 p. 238 \& nn.34-38, Rawlins 1991P, Rawlins 1991W. The point has always been self-evident to any investigator who is inductively inclined, mathematically able, and openminded: see, e.g., van der Waerden 1970 or Thurston 2002S p. 62

[^11]:    ${ }^{30}$ See fn 50, and R.Newton 1977 Tables XI. 2 \& XI.7-8; also pp.336-339. And note Gingerich 1980
    Fig. 4 \& Fig. 4 \& caption.

[^12]:    ${ }^{31}$ We are expressing our result far more precisely than could be supported by naked-eye input data, but for effect we are noting that the three answers of course agree mathematically on the nose.

[^13]:    ${ }^{32}$ All the foregoing numerical developments were done to better than 1 part in a million, though presented here several ordmags more crudely. That is why a given calculation here\&there may appear slightly inconsistent in the last place.

[^14]:    ${ }^{33}$ To be convinced of this at-first-surprising truth, one need only ponder an extreme case: a $99.9999 \%$ -eccentric-orbit (direct-moving) comet just inside the Earth's orbit. If we start with a neat (nil-elongation) situation with the Earth at the longitude of the comet's perihelion and the comet at aphelion, it is physically obvious that between then and comet-perihelion time (a little over 2 months after the time of the initial situation) - by which time the elongation is eastern - Earth observers have seen the comet at a greatest western elongation, though all western orbit-tangents during this time have been to the (long symmetric) half of the comet-orbit which the comet has not occupied. For other DIO examples of the heuristic utility of resorting to extreme cases, see: [a] DIO 10 fn 91 , [b] DIO $2.3 \ddagger 8$ $\S \S A 5 \& A 7$, [c] DIO $4.3 \ddagger 13 \mathrm{fn} 13$. As for the time $t_{\mathrm{F}}$ it takes a body (comet or whatever, starting from null relative velocity) to fall into the Sun from a distance of 1 AU : this question has appeared on freshman astronomy tests - and without providing the neat answer (the simplicity of which hints at how it can be instantly obtained): $t_{\mathrm{F}}=2^{-2.5}$ years.
    ${ }^{34}$ Which is fortunate for our analyses of Venus, the planet with the smallest $e$ in the Solar System. And, though bigger, Earth's $e$ is also ordmag $1^{\circ}$. For approximately Bodean $r$ and modest $e$, greatest elongations will on average (rms) be distant from tangency by an angle equal to ordmag the rss (root-sum-square) of the two planets' $e$; so, even for the unextreme Venus-Earth case, the errors are ordmag $1^{\circ}$, obviously contributing to roughness in our results for $A$ (\& etc).

[^15]:    ${ }^{35}$ See $\S E 1$. It might be objected that the reason we can here find our three Venus' elements from three greatest-elongations (while Ptolemy needs over twice as many to fake his Venus derivation: fn 28) is that we're assuming the equant model at the outset. Comments: [a] The equant was obviously (see Evans 1984) discovered via Mars (e 12 times Venus’ \& 5 times Earth’s), so Ptolemy too was assuming the equant model before he did his Venus math. (See NS' \& OG's admissions that Ptolemy knew at least some of his Venus orbital parameters ahead of time: fn 52 here \& Gingerich 2002 Fig. 1 caption.) [b] For either planet, the discovery of the equant could ( $\S \mathrm{H} 2$ ) have been based upon the fact that the equant model gives a much neater and more stable fit to observational data (than the eccentric model). For Venus, anyone will quickly see this by comparing results from testing the eccentric ( $\S \mathrm{E}$ ) vs equant ( $\S \mathrm{G}$ ) models upon several randomly chosen real greatest-elongation trios, using our program (p.54): eccentric-model scatter evaluated at $\S \mathrm{H} 1$; equant-model scatter, at fn 48 , the latter quite clearly an ordmag smaller. This dramatic contrast (not Ptolemy's disastrously faked derivation) is the probable historical origin of the equant's discovery, a point correctly perceived long ago by Toomer 1984 (p. 474 n .12 ) and Evans 1984 (see also the latter's well-written 1998 book). Over the years, various scholars have suggested that Ptolemy personally invented the equant to solve huge problems presented by nontrivially-eccentric planetary motion. Question: if he did, then why did he "prove" it (Almajest 10.1-3) via the most trivially-eccentric planet (Venus), and using a method which was so infantile and impossible that it required faking "observational" data into obviously nonexistent symmetries and other ridiculously pat arrangements? More broadly, Gingerich 2002 p. 71 (confusing mere street-astrologers with the real astronomers that Ptolemy leeched off of [fortunately for our ability to reconstruct genuine ancient astronomy]) opines that "It really does appear that Ptolemy's work fundamentally changed the way planetary astronomy was done." Hmmm. As mathematician Jerry Wolf recently $(02 / 10 / 18)$ commented: do such folk also imagine Euclid invented the Elements?! Secondary-writer Gingerich's ready confusion (fn 7) of mere secondary-sources with deep thinkers and intellectual pioneers is typical of the sort of elementary mis-step that can lead theoretically limited politician-scholars \& clonies down blind-alleys (and repeated embarrassment-in-controversy disappointments) for wasted decades. On this almost-too-naïve-to-be-believed archonal-tradition, see DIO $6 \ddagger 1 \mathrm{fn} 106$

[^16]:    ${ }^{36}$ For a genuine ancient heliocentrist, this entire procedure would be inherently flawed by its Ptolemaic presumption that the Venus epicycle is exactly uniform-circular - when one of the advantages (see Rawlins 1987 pp.237-238) of the heliocentrist scheme is that it permits Venus \& Earth to each have orbital eccentricity. Thus, if this paper's trio-method was actually used in antiquity, it was probably as just a heliocentrist's first step (viewing from an already-known Earth orbit) towards determining Venus' orbital elements. The actual historical evolution was probably: heliocentrists developed a full set of planetary elements - which were later corrupted by geoocentrists twisting them to pretend that all epicyclic motion was uniform-noneccentric. See idem.

[^17]:    ${ }^{37}$ It is possible that Ptolemy's Venus woes were initiated by his unawareness of this step (and its importance to efficient iteration here), which is consistent with (fn 35) our contention that he did not discover the equant (our halving) from original analysis of Venus' motion (a once-common belief).
    ${ }^{38}$ Of course, the precision here is misleading, given the input's empirical uncertainty and the limitations (see $\delta \mathrm{F}$ ) of the iterative procedure. However, the stability of the solution is not illusory. A measure of that: we have ignored apsidal precession (Earth and-or Venus) entirely in these analyses. (After all, the $\S$ G development is based upon data covering only $12^{y} .6$.) If we include it, eq.46's results all shift by less than one part in 1000 . So, since the effect is trivial and since we do not in any case know exactly what (if anything) real ancient scientists would have done about precession during this math, it seemed pointless to introduce such speculation into these investigations. A note in passing: if one injects reasonable noise into the input data [lines 70-90 at p.54] for honest computation of elements, the effects on induced $A$ and $r$ are trivial; on $e$, temperate.

[^18]:    ${ }^{39}$ See the Amer Astr Soc-affiliated HAD's H.A.D.News, which regularly showers Gingerich with HAD-board praise so fulsome that the HAD should have been megaretchedly embarrassed even before the present revelations.
    ${ }^{40}$ See discussion at DIO 1.2 §I10. Same problem here: $x$ equations of condition will determine $x$ unknowns. What scholar posing as an authority in orbital matters would not know that?! The ancient computer of the "Ptolemy" proofs of the outer planets' elements was obviously familiar with this ultra-elementary point.
    ${ }^{41}$ JHA 26.2:164 (1995 May). (See also DIO $6 \ddagger 3$ §H.)
    ${ }^{42}$ See DIO $4.3 \ddagger 15$ §H6 \& Gingerich’s 2000 referee report on Thurston 2002S for Isis.
    ${ }^{43}$ In the present context (stubborn \& technically-innocent insistence upon clinging to one's central cult-tenet), the spectacle of watching (e.g., DIO $1.1 \ddagger 3 \S \mathrm{D} 2$ ) Swerdlow accuse another scholar of being crank sets new records in the irony dep't.
    ${ }^{44}$ See, e.g., DIO $1.1 \ddagger 3 \S \S$ D2-D3 \& $\ddagger 6$ fn 5, DIO 1.2-3 fnn 92 \& 252.
    ${ }^{45}$ However, regarding likely historical chronology, See fn 35 item [a].
    ${ }^{46}$ See fn 35 item [b]. Also $\ddagger 5$ §E4.
    ${ }^{47}$ In the Dionysian era: for evening greatest elongations, the mean of results for $e$ is $0^{\mathrm{p}} 51^{\prime}$ in a range of $\pm 0^{\mathrm{P}} 02^{\prime}$; for morning greatest elongations, mean is $0^{\mathrm{P}} 57^{\prime}$ in a range of $\pm 0^{\mathrm{P}} 03^{\prime}$. [The program at p. 54 will also automatically handle a mixed trio (morning\&evening data): sample at lines 70-90.]
    ${ }^{48}$ Sample-testing of series of well-spaced evening and morning trios in the Dionysian era finds that the average for $A$ is $57^{\circ}$ : evening $60^{\circ}$, morning $54^{\circ}$ (an asymmetry that varies according to era), with scatter-ranges under $4^{\circ}$. If one uses 3 consecutive evening greatest elongations or 3 consecutive morning greatest elongations, the configurations are less well-spaced - but the mean and scatter (of induced elements) are virtually identical to those for well-spaced elongations. Since numerous Dionysios-era trios produce $A$ very close to $55^{\circ}$, it is tempting to wonder if this result came through

[^19]:    to Ptolemy without correction for precession.
    ${ }^{49}$ That is, the last four Venus elongations that occurred before the final "observation" (141/2/2) recorded in the Almajest (9.7).
    ${ }^{50}$ R.Newton 1977 p. 311 showed (see also fn 30) that the equant should not use symmetric eccentricities: for Venus, the ratio of the two eccentricities should (R.Newton 1977 Tables XI.2\&8) have been about 8:7 instead of equal (as Ptolemy makes them: Almajest 10.3). But here we simply sum the two equant-style eccentricities he found, in order to allow a crude comparison to our eccentric-model values. Of course, we should use half this sum (when comparing Newton's equant results to ours: $\S \mathrm{G}$ ), namely, $0^{\mathrm{p}} 50^{\prime}$.
    ${ }^{51}$ As hinted in §E12: the poorness of his $e$ may be due to his having accidentally gotten it from someone's eccentric-model analysis - see eq. 30 .

[^20]:    ${ }^{52}$ Top-public-apologist O.Gingerich is now going about exclaiming his ecstasy in discovering that Ptolemy systematically "shaded" his purported Venus observations to make up for the inconvenient reality that Venus wasn't actually at the greatest elongation point when \& where his fraud required it to be. (Crucial-test-on-crucial-testing: does OG's belated recognition of Ptolemy's Venus frauds decrease [Rawlins 2003J fn 21] or increase or leave-fixed OG's superlative rank of his ancient idol? See fn 55.) Likewise Swerdlow 1989 p.35: "the observation has been adjusted to the position the planet would have had if it were at greatest elongation"; p.42: data were "arranged"; p.54: Ptolemy "could not have observed some of the reported elongations"; p.54: "Ptolemy's adjustments of whatever he observed were of the order of $1^{\circ}$ ". But the strangest part of MacArthur-funded Swerdlow 1989 is (p.31): "Ptolemy must earlier have carried out analyses by quite different means . . . [to find] at least preliminary parameters". (See fn 9.) Since the Almajest proofs of celestial parameters are patently phony, these numbers clearly had a different origin, but we continue to look in vain for the data (other

[^21]:    ${ }^{56}$ The (sometimes) illogical outgassings of Ptolemy's (mostly) math-challenged defenders are taken seriously because some are at major universities. Readers who may on this basis have supposed that these gentlemen have some special understanding of orbital matters should notice the embarrassingly one-sided ratio of each side's original research publications on dynamical astronomy in professional astronomical journals. (One would normally be very reluctant to bring up such a point - but if Muffiosi somehow find it self-reassuring to spend decades slandering [as nuts, fools, etc] those who merely disagree with them, then coldwater-reminders on genuine expertise are occasionally going to happen. Note that DIO has always held that such imbalances do not prove that we are righter than Muffiosi regarding the contended points between us. [See DIO $1.3 \ddagger 10$ (3rd-last paragraph).] Our position is merely that the Muffia's incessant and curiously aggressive we-are-The-authorities-aroundhere posturing just might perhaps be a slightly less than completely perfect guide to the truth. Unless we are seeking transparent indicia of which side is seething with inward insecurity.)
    ${ }^{57}$ Gingerich 2002 p.73: "there is no other way to establish this geometry" (see also §B4) so Ptolemy "silently [vs fn 12] increased elongation angles, which makes his solution for the final parameters considerably more accurate." Gingerich's comment on parameter-accuracy seems intended to justify \& defend Ptolemy's Venus frauds. But the point is that Ptolemy already knew roughly-correct parameters (as OG realizes on the very same page) "from many unspecified observations". (Actually, the Venus $e$ and [Almajest 10.4] mean motion weren't very good.) So Ptolemy simply fudged his proof's data in order to get the "right" answers. (Historians may not know what getting the right answers from wrong data means, but scientists certainly do. See, e.g., Rawlins 1987 p.237.) This outrageous perversion of inductive science, which (fn 55) pretends to find parameters from observations when the reverse is the case, lifts OG to praise which is so ethics-bending that it can serve as lawyeresque inspiration to forgers everywhere. (See DIO $2.1 \ddagger 2 \S \S H 31-32$.) Henceforth, all freshman-lab cheaters who skip the day's experiment and instead "silently" fake data (from the very theory they're supposed to be testing) can just pass off such dishonesty as what a well-known Harvard prof has, after all, prominently deemed "a brilliant tour-de-force of theoretical and observational INTERACTION." (See Gingerich 2002 p.73; emph \& caps added.) One can joke about it (will successful street-mugging now be renamed "brilliant provider and providee interaction"?) - but, as I once asked (DIO 1.2 §E4, in related exasperation \& context): "how does one satirize material that already reads like satire?"
    ${ }^{58}$ Extra oddity: at Almajest 10.4, Ptolemy correctly says that his $138 / 12 / 16$ position for Venus is "after" greatest elongation - even though it is closer (off by $19^{\text {d }}-$ ) to the actual event ( $138 / 11 / 27$ ) than are two of the dates $(127 / 10 / 12 \& 136 / 11 / 18)$ which the Almajest nearby gives for unqualified "greatest elongations"! This is evidence that genuine accurate ancient data were available to Ptolemy - but it is also evidence that he knew enough to realize how dishonest it was to invent "better" data whenever he wished to promote the calculated deception that his models truly represented the heavens.

[^22]:    ${ }^{59}$ Based upon this cocky delusion, HamSwerdlow 1981 pp. $62-63$ heaps superior-airs scorn upon R.Newton: "[his] arguments make anachronistic demands on what someone in antiquity should have been able to do, e.g., Ptolemy should have found the elements for the planets that Newton had his computer find in making a best-fit model for a series of accurate planetary positions in all orbita configurations." Yet the present paper's simple method could produce Venus elements very near those of R.Newton's analysis, from $2^{y}$ of accurate gr.elong data ( $\S \mathrm{H} 2$ ) plus a few minutes of math ( $\S \mathrm{E} 13$ ).
    ${ }^{60} \S \mathrm{H} 2$ (\& see fn 48) successfully compared elements gotten from a mere trio (eqs.46\&47) to the result (eq.48) of R.Newton's Gaussian statistical fit (based upon dozens of Venus positions).
    ${ }^{61}$ See Rawlins 1985 G p. 266 \& fn 6 [ \& fn 6 here], where we show: just as Ptolemy gives 2 contradictory data-sets for the same Venus elongations (fn 14), he also gives 2 contradictory latitudes each for Alexandria (his own city!), Heliopolis, \& Syene. (And his Mercury contradiction [Rawlins 1987 pp.236-237, Rawlins 2003J fn 34] is worse.) In closing, we recall: when in 1976, O.Gingerich plunged into defending Ptolemy (fn 12), he didn't yet know of the Venus double-date contradiction. So, when, in 1983, Venus-disaster hit Ptolemy's biggest fan (fn 5), OG could only react with an elaborately complex rationalization (§A3) to defend his original position. Which suggests a challenge: for Ptolemists' next unOccamite trick, they should concoct another \& equally fancy ad-hoc singular theory, sculpted specially to explain-away Ptolemy's equally contradictory double city-latitudes, so as to again (like §I6 [\& see DIO $6 \ddagger 1 \mathrm{fn} 47$ ]) avoid accepting a simple but hated common theory. For the latitude \& Venus contradictions, the common explanation is plain\&coherent: plagiarist Ptolemy was Dr.Sloppy-Copy.

[^23]:    ${ }^{1}$ Professor Emeritus of Mathematics, University of British Columbia. Address: Unit 3 12951, 17th Avenue, South Surrey, BC, Canada V4A-8T7; phone 604-531-8716.

[^24]:    ${ }^{2}$ The fact that the non-tangency rule noted at $\ddagger 6 \S$ F applies to greatest MEs with respect to time but not to those with respect to MLS was first mentioned in 1977 by F.W.Sawyer (ПPILMATA, pages 169 to 181).
    ${ }^{3}$ The difference between the two kinds of MME can be seen beautifully clearly in Fig. 1 of Dennis Duke's article [ $\ddagger 5$ ] (Eastern elongations are seen in the evening). The dotted lines are graphs of ME against MLS. MLS increases steadily with time, so the highest point on each gives a maximum with respect to time. The points on the outer envelope give the greatest ME for each position of the epicycle.

[^25]:    ${ }^{4}$ The Thurston method is mathematically quicker than the DR method ( $\ddagger 6$ or p.54); though the latter solution, if performed graphically, could hardly be more elementary.
    ${ }^{5}$ Adjusting the Thurston method for the equant model (to produce the present versatile BASIC program) requires only that in Fig.1, the epicycle-center (still moving at constant angular velocity around O , now the equant point) is at unit distance from OT's midpoint (instead of from O), which will be $e$ (instead of $e / 2$ ) distant from O and from T. Thus (using the law of sines $\& \ddagger 6 \mathrm{eq} .33$ ), OC equals $\sin u / \sin g$ (instead of unity). The appropriate adjustments (to equant model) of the $\S D 3$ method are built into lines $310(\& 10), 250 \& 300(\& 150)$ of the geocentric program.

