‡4 Ancient Planet Tables' Long-Cycle Ancestries

Almajest Planet Mean Motions All Based on Period-Relations

by Dionysios[1]

A Almajest Planet-Tables' False & True Mathematical Bases

A1 For each of the five planets, Ptolemy provides mean motion tables (*Almajest* 9.4), based upon the daily mean tropical motion provided in the tables' preface (*Almajest* 9.3), which in each case is given to six sexagesimal places: about 1/50 billionth of a degree/day (note fn 10). For each planet, he claims[2] that the mean motion was determined by dividing a centuries-long time interval t into the long angular arc a of the planet's motion during that interval. Cliquish O.Neugebauer-Muffiosi were lockstep in credulously accepting this. (Neugebauer himself even falsely[3] claimed to have mathematically checked it.) They did so unanimously right up until their communal hate-object R.Newton actually performed the five a/t divisions and found that Ptolemy was lying in all five cases. (The Muffia-R.Newton contrast here needs to be kept in mind because those who were completely wrong continue to promote the deliciously ironic delusion that the they are the ultimate experts on Ptolemy, while Newton&co are just incompetent cranks.)[4]

A2 Inspired by this finding, DR ran continued-fraction analyses upon all the planets' tabular motions and was amazed to find that for Mercury, Venus, & Saturn the solutions perfectly matched (not Ptolemy's centuries-long a/t ratios [fn 2], but) decades-long period-relations given (*Almajest* 9.3) for those same planets, right in the tables' preface!

B Almajest Planet-Tables' False & True Empirical Bases

B1 But the contentions of this paper will be that:

[a] Each of *Almajest* 9.3's medium-length (decades or less in length) tropical period-relations (e.g., the three cited above at §A2), upon which the *Almajest* 9.4 tables were founded (as shown below), is in truth a *centuries-long* period-relation — made artificially much shorter through mere (pure math) division by an integer. (To a scientific readership, this realization should come as no surprise. Indeed, the elementary point that long temporal baselines ensure higher accuracy than short-period data has always been obvious to all astronomers & investigators [except a few present-day cultists: see, e.g., Rawlins 2002A p.26 fn 2] — consistently including Ptolemy himself, *all eight* of whose *Almajest* celestial mean motions were defended [§J1] by resort to centuries-long comparisons of alleged observations [see general point at ‡2 §H], and whose final work [*PlanHyp*] displays exactly the type [eq.1][5] of centuries-long integral sidereal period-relation proposed below [§B2] as the basis of all the *Almajest* 9.4 planet mean motion tables. Indeed, the *PlanHyp* Saturn period-relation [fn 38] is identical to our eq.48, below.)

[1] Dionysios is Greek for Dennis (a circumstance which might be related to the name of this journal). It was van der Waerden's nickname for Dennis Rawlins, *DIO*'s publisher.

[2] *Almajest* 9.10 (Mercury), 10.4 (Venus), 10.9 (Mars), 11.3 (Jupiter), 11.7 (Saturn).

[3] See *DIO 7.1* ‡5 fn 12. (Also *DIO 1.3* ‡10 fn 10.)

[4] See, e.g., *DIO 1.1* ‡1 §C7 & *DIO 4.3* ‡15 §G9. (Our contrasting attitude: *DIO 1.3* ‡10 3rd-last paragr.) Such opinions are still the norm in Muffia circles — though they are no longer unanimous, I am glad to report.

[5] The specific sidereal equations producing the *Almajest* planet mean motions are found below; we list the equations here (with Neugebauer 1975 page [citing ancient attestation] given in parentheses): eq.9 (p.466), eq.17 (p.605 n.11), eq.25 (no attestation, but see §F1), eq.40 (pp.390&605), & eq.48 (pp.390&906).

[b] And each of said long tropical relations is a slightly adjusted version of a nearby purely *integral* long-period sidereal relation based (in quite elementary fashion: fn 6) upon secular observations of planet stationary points (of which many were available to astronomers of Ptolemy's era, as he reports at *Almajest* 9.2). [Note p.33 #8: an independent case of a **sidereal** foundation to a Ptolemy **tropical** equation.]

B2 Integral periodic returns have the advantage (Rawlins 2002B §A3) that they automatically and directly produce the most accurate possible values for mean motion based on naked eye data. The approach that produces such relations in practice is well explained by Neugebauer (see fn 6). To put the method into concrete (and historically most-likely-version) terms: if a stationary point occurs near any star, and if that same type of stationary point repeats Y sidereal years (Earth's sidereal revolutions) later (at the same star) after G retrograde loops (synodic revolutions), then we know that the planet has travelled F sidereal heliocentric revolutions, where:

$$\mathrm{G} = |F - Y| \tag{1}$$

More specifically: for inferior planets eq.1 is $F = Y + \mathrm{G}$; for superior planets, it's $F = Y - \mathrm{G}$. (Again, keep in mind that F, G, & Y are all integers. Note also that sidereal variables are in italics — and that G, being differential, is neither tropical nor sidereal, and is thus invariant under transformation between these frames.)

B3 Ironically, Ptolemy himself (*Almajest* 9.2) claimed he'd had nothing at all to do with such methods (calling them "fraught with uncertainty"). This misreport evidences several spectacular innocences on the part of Neugebauer-Gingerich's "Greatest Astronomer of Antiquity" (Rawlins 2002V fnn 10&55):

[a] As we are about to learn below (§J5), Ptolemy's own planet mean motions unquestionably were based upon the very period-return approach he damned!

[b] Ptolemy didn't have enough outdoor experience (likewise regarding the Moon: *DIO 6* ‡1 fn 51) to know that integral periodic returns were the best not the worst way to find mean motions in his day.

[c] His specific *Almajest* 9.2 concern about stationary-point data (namely, that station-times are uncertain) is worse than a gaffe; it's an astounding inversion. For, it is this method's wonderful insensitivity to such exact-time difficulties that is precisely its *strength*,[6] not its weakness. To exploit the method, all one needs to know is: [i] what is the year (not day) of each stationary point (the difference in years becomes Y in eq.1); [ii] which star the turning point is at (so that a replica-station there [Y years later] can be looked for); [iii] how many retrograde loops (integer G in eq.1) occurred in-between.

B4 A point in passing: superficially, it seems incredible that empirically-determined ratios of mere *3-digit integers* could approximate real mean motions to 1 part in ordmag a million. Yet this is not only mathematically possible (see Rawlins 1984A n.27), but was unquestionably accomplished in antiquity for Mars and probably (Rawlins 2002V §C3) for Venus as well.

C Finding the Ancestor Period-Relations

C1 In the *Almajest* 9.3 preface to his mean motion tables, Ptolemy provides for each planet several tropical relations that are nearly equivalent. In this paper, we will analyse & develop primarily the versions which are effectively of the following form (applicable to either geocentric or heliocentric viewpoint), using subscript J to denote *Almajest*, and

[6] Neugebauer 1975 p.390: the method works "without the use of any instruments and without needing to determine accurately the moment when the planet is stationary since . . . the number of elapsed years must be an integer." It is good to see that Neugebauer understood this key point — and was perhaps the 1st to bring it to a wide modern audience.

finding the time-interval d via the *Almajest* 3.1-2 yearlength (eq.4, below):

$$\mathrm{F_J} + r_\mathrm{J} = \mathrm{G_J} = \mathrm{Y_J} + r_\mathrm{J} = d_\mathrm{J} \qquad (2)$$

where F = tropical helioc revs, G = synodic revs, r = a remainder expressed in degrees, and Y = tropical[7] years. (Henceforth, below, we will use corresponding lower-case superscripts in an obvious fashion, to signify revolutions [and still using italics to denote sidereal periodicity], e.g., superscript f = sidereal heliocentric revs, y = sidereal yrs; also f = tropical heliocentric revs, y = tropical yrs, and g = synodic revs.) For Ptolemy, tropical years Y are related to sidereal years Y by a well-known standard ancient unit-ratio[8] which we just call H, a ratio which implies the standard ancient 1°/century precession rate:

$$H = 36000^\mathrm{y}/35999^{y} = 1 \qquad (3)$$

We will also need the (very inaccurate) Hipparchos-Ptolemy tropical year-length:

$$1^\mathrm{y} = 365^\mathrm{d}1/4 - 1/300 = 365^\mathrm{d};15,48 = 54787^\mathrm{d}/150 = 365^\mathrm{d}.246666\ldots \qquad (4)$$

Multiplying eqs.3&4 yields the (very accurate) H-P sidereal year (*DIO 2.1* ‡3 fn 18):

$$1^{y} = 13148880^\mathrm{d}/35999 = 365^\mathrm{d};15,24,31,32,27,9 = 365^\mathrm{d}.25681268924 \qquad (5)$$

C2 To extract each planet's ancestor period-relation, we will simply apply continued-fraction analysis (truncating before Y exceeds 1000^{y}) to find the best integer-ratio G/Y that satisfies eq.2:

$$\mathrm{G}/Y \approx H \cdot \mathrm{G_J}/(\mathrm{Y_J} + r_\mathrm{J}) \qquad (6)$$

D Mercury

D1 For Mercury, the *Almajest* 9.3 prefatory period-relation (in the format of eq.2) is:

$$145^\mathrm{g} = 46^\mathrm{y} + 1^\circ = 46^\mathrm{y} + 1^\mathrm{d}1/30 = 16802^\mathrm{d}24' = 16802^\mathrm{d}2/5 \qquad (7)$$

We next apply eq.6 (& eq.3), according to the procedure of §C2, using the data of eq.7:

$$\mathrm{G}/Y = H \cdot 145^\mathrm{g}/(46^\mathrm{y} + 1^\circ) \approx 3^{\mathrm{g}/y} + \cfrac{1}{7 - \cfrac{1}{2 + \cfrac{1}{3 - \cfrac{1}{5}}}} = 684^\mathrm{g}/217^{y} \qquad (8)$$

Which recovers Mercury's integral ancestor:

$$901^{f} = 684^\mathrm{g} = 217^{y} \qquad (9)$$

— an anciently-attested (fn 5) sidereal Mercury period-relation.

D2 Now we reconstruct (see constraints: §J4) the probable history of how tropical eq.7 evolved from sidereal eq.9. First, we must convert eq.9 from the sidereal to the tropical frame of reference by multiplying its 217^{y} times eq.3, yielding the tropical equation:

$$684^\mathrm{g} \approx 217^\mathrm{y} + 2^\circ 1/6 \qquad (10)$$

(We have omitted the F [leftmost] term of eq.10 — since eq.1 renders it superfluous.) [Note added 2004. General ignoring of F turned out to be quite unwise: see §§F2&H3.] We now adjust eq.10 slightly by adding a small equation, whose integral G is found[9]

[7] See Rawlins 2002A fn 12.

[8] Sources (and several associated ancient developments) at *DIO 6* ‡1 eq.26.

[9] By truncating eq.8 at the 2nd fractional term.

through continued-fraction analysis to ensure near-integrality for Y in this adjustment equation (eq.11), which is arranged to be almost exactly proportional to eq.10 — and so chosen that adding it to eq.10 will ensure that, in the ensuing equation (eq.12), G and the integral part of Y will be simple multiples of the corresponding variables in eq.7. The smallest equation satisfying these desiderata is:

$$41^\mathrm{g} \approx 13^\mathrm{y} + 2^\circ 3/4 \approx 13^\mathrm{y} + 3^\mathrm{d} \qquad (11)$$

Summing eqs.10&11 (and using eq.4 to compute the number of days d in the interval):

$$725^\mathrm{g} \approx 230^\mathrm{y} + 5^\circ = 230^\mathrm{y} + 1/72 \approx 84012^\mathrm{d} \approx 230^\mathrm{y} + 5^\mathrm{d}1/6 \qquad (12)$$

whose integral components are obviously quintuple those of eq.7, where we are reflecting ancients' penchant for unit fractions (§E2). (The rightmost term in eq.12 comes from above [sum of parallel terms in eqs.10&11], not from the left. Similar descent at extreme right will recur here&there throughout this analysis, e.g., eq.28→eq.29.) Dividing by 5, we find:

$$145^\mathrm{g} \approx 46^\mathrm{y} + 1^\circ \approx 16802^\mathrm{d}24' \approx 46^\mathrm{y} + 1^\mathrm{d}02' = 46^\mathrm{y} + 1^\mathrm{d}1/30 \qquad (13)$$

(The 3rd & 4th entries in eq.13 come from above [eq.12], not from the left.)

The mean daily synodic motion m was then found from eq.13 (though it could have come straight from long-period eq.12, just by dividing 725^g by $d = 84012^\mathrm{d}$; see §J1):

$$m = 145^\mathrm{g}/16802^\mathrm{d}24' = 21750^\circ/7001^\mathrm{d} = 3^\circ;06,24,06,59,35,50/d \qquad (14)$$

which is precisely the *Almajest* 9.3-4 value for Mercury's mean motion, i.e., an agreement to the precision Ptolemy gives (§A1), which is (fortunately[10] way overdone); the match is to 1 part in ordmag a trillion. Eqs.13&14 account for all the *Almajest* 9.3 Mercury data (e.g., eq.7).

E Venus

E1 Next, we analyse Venus, again starting (as for Mercury at §D1) with the appropriate *Almajest* 9.3 relation (i.e., the Venus version of eq.2):

$$5^\mathrm{g} = 8^\mathrm{y} - 2^\circ 1/4 = 8^\mathrm{y} - 2^\mathrm{d}18' = 2919^\mathrm{d}40' \qquad (15)$$

and then applying eqs.6&3 to it, thereby attaining:

$$\mathrm{G}/Y = H \cdot 5^\mathrm{g}/(8^\mathrm{y} - 2^\circ 1/4) \approx 0^{\mathrm{g}/y} + \cfrac{1}{2 - \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{31 - \cfrac{1}{2}}}}} = 309^\mathrm{g}/494^{y} \qquad (16)$$

(I thank D.Duke & H.Thurston for catching horrendous typos here [& there].)

Thus, the ancestor equation, underlying the *Almajest* 9.3 Venus mean motion (eq.15) was:

$$803^{f} = 309^\mathrm{g} = 494^{y} \qquad (17)$$

— a fragment of which is anciently attested (fn 5).

[10] Without the *Almajest* 9.4 planet mean motion tables' enormous overprecision, we couldn't be absolutely certain of finding the prime factors, etc, underlying the planet tabular mean motions.

E2 To reconstruct the historical path from eq.17 to eq.15, we start by converting the former via eq.3 to:

$$309^{\mathrm{g}} = 494^{\mathrm{y}} + 5^\circ - \qquad (18)$$

Proportionally:

$$1^{\mathrm{g}} = 2^{\mathrm{y}} - (144^\circ +) \qquad (19)$$

Adding eqs.18&19 and applying eq.4 (& using ancient unit-fraction expression for 140°):

$$310^{\mathrm{g}} \approx 496^{\mathrm{y}} - [1/3 + 1/18] \approx (3260922^{\mathrm{d}} - [2191^{\mathrm{d}} + 365^{\mathrm{d}}])/18 = 543061^{\mathrm{d}}/3 \qquad (20)$$

Dividing eq.20 by 62 and using eq.4:

$$5^{\mathrm{g}} \approx 8^{\mathrm{y}} - 2^\circ 1/4 \approx (543061^{\mathrm{d}}/62)/3 \approx 8759^{\mathrm{d}}/3 = 2919^{\mathrm{d}}40' \approx 8^{\mathrm{y}} - 2^{\mathrm{d}}18' \qquad (21)$$

Dividing eq.21's degrees by its days finds the synodic mean motion:

$$m = 5^{\mathrm{g}}/2919^{\mathrm{d}}40' = 5400^\circ/8759^{\mathrm{d}} = 0^\circ; 36, 59, 25, 53, 11, 28/d \qquad (22)$$

Comparison of eqs.21&22, with the Venus numbers of Ptolemy's tabular preface (*Almajest* 9.3), shows that we have recovered every digit.[11]

E3 Venus' long-period-relation (eq.20) cannot directly produce the corresponding *Almajest* 9.3 tabular mean motion (eq.22). (This disjunct is more flagrant than for any other planet.) Two important and related[12] oddities: [a] The Venus Y is an ordmag less than that for any of the other four planets. [Note added 2003. Venus' Y is so small that not even a Muffioso will claim that it could serve as empirical foundation for tables which ancients could've thought reliable. If in extenuation it is proposed that the 8^{y} period-relation is obviously a larger relation divided by an integer — well, thanks: that's exactly the central contention of this paper. (See front cover & §B1.) Further (crucially): any integer less than about 30 — corresponding to a period of nearly 2 1/2 *centuries* (*far* larger than any of the Y listed by Ptolemy at *Almajest* 9.3) — is not going to permit an acceptably small remainder r (fn 12). This consideration alone supports our contention that Ptolemy's Y are descended from (masked) centuries-long empirical cycles. [b] The relative size of Venus' r is an ordmag greater than the other planets'.] Further, presumably because Y is so small, the rounding process in eq.21 is exceptionally untrivial[13] (§J1) and contributes to the curious poorness of m — for a planet which should have had (and perhaps did: Rawlins 2002V ‡6 §C3) one of the best values for m (because of the bright & sharp observability of Venus stations, as well as absence of the large secular inequalities that affect Jupiter & Saturn).

[11] But note the key original discovery (regarding Venus) by Toomer 1984 p.425 n.29 (see Rawlins 2002V §C4).

[12] Relatively, Venus has the biggest r of the *Almajest* 9.3 five — an ordmag bigger. But Venus is not a case where Ptolemy (or source) had any wiggle-room: a period-relation of less than 100^{y} (evident upper-limit for the five *Almajest* 9.3 period-relations) is mathematically barred, in the resonance-constrained case of Venus, from having a small relative r. (Note the causative factor here: the only continued-fraction expression [of those we generated for the five planets] which early-on suddenly hits a big number is Venus: the 31 in eq.16, an integer which postpones [fn 40] any small-r Venus return for over 2 centuries.) That is probably why *Almajest* 9.3's Venusian Y seems peculiarly small [merely 8^{y}]: if Venus' Y were anything like the size of the other four planets' *Almajest* 9.3-attested Y, it would display an embarrassingly large r, which would then raise a troublesome question: why even bother to provide (at *Almajest* 9.3) a short period-relation for a planet where only a centuries-long one can have an r small enough to come anywhere near competing with those of the other four planets — a question which might suggest the true huge dimensions of the cycles that were actually behind the *Almajest* 9.3 mean motions. See §J1.

[13] This evaluation assumes that the eq.16 approach is correct for Venus. By contrast, another interpretation might assume that the denominator there should be $8^{\mathrm{y}} - 2^\circ 18'$, which would (by analysis similar to eq.16) lead to sidereal cycle $613^{\mathrm{g}} = 980^{y}$, very near that of *PlanHyp* (fn 38) — though not consistent with it. I doubt that this is significant. (If such a long cycle were the ancestor of eq.22, our proposal at §L4 could not be valid.) But the discrepancy is presented here anyway (emulating Toomer's helpfulness: fn 16) so that others may pursue alternate theories if so inclined. Note: even though *Almajest* 9.3 consistently mentions day-remainders first, we have here throughout used (as primary) the *Almajest* 9.3 degree-remainders (since most of them [Saturn being the (perhaps slight: §J3) exception] are significantly rounder; in any case, using day-remainders instead would produce a different sidereal relation only for Venus, where rounding and Y's smallness combine to create uncertainty as to whether eq.18 is precisely that planet-motion's ancestor-relation. (The true Venus ancestor-relation might easily be 8^{y} either side of eq.18, an uncertainty corresponding to one's realization that the bottom part of eq.16 might be 30 or 31 instead of 30 1/2.)

F Mars

F1 Ptolemy's *Almajest* 9.3 period-relation for Mars is (note fn 16):

$$42^{\mathrm{f}} + 3^\circ 1/6 = 37^{\mathrm{g}} = 79^{\mathrm{y}} + 3^\circ 1/6 = 79^{\mathrm{y}} + 3^{\mathrm{d}}13' = 28857^{\mathrm{d}}43' \qquad (23)$$

Applying eqs.6&3 to eq.23, we have:

$$G/Y = H\cdot 37^{\mathrm{g}}/(79^{\mathrm{y}} + 3^\circ 1/6) \approx 0^{\mathrm{g}/y} + \cfrac{1}{2 + \cfrac{1}{7 + \cfrac{1}{3 - \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{2}}}}}} = 303^{\mathrm{g}}/647^{y} \qquad (24)$$

So the integral sidereal ancestor of the *Almajest* Mars mean motion tables was:

$$344^{f} = 303^{\mathrm{g}} = 647^{y} \qquad (25)$$

This is the only one of this paper's five deduced ancestor relations which is not attested in extant ancient materials. However:

[a] Eq.25 is the most accurate (§G4) integral sub-1000^{y} sidereal Mars period-relation.

[b] The continued-fraction development for Mars (eq.24) produced the most precise hit (of the five planets) upon such an ancestor relation. (I.e., the final digit [the 2] in eq.24 is exact to ordmag a percent.)[14] So we may take ancient use of eq.25 as highly probable, based upon Ptolemy's own numbers: putting eq.23's *Almajest* 9.3-4 Mars synodic motion into eq.6 (& his eq.3) yields eq.25. [A parallel eqs.6&3 approach via eq.31's *Almajest* 9.3-4 Mars longitudinal motion: truncating the cont'd-frac development $H\cdot(42^{\mathrm{f}} + 3^\circ 1/6)/(79^{\mathrm{y}} + 3^\circ 1/6)$ or $(42^{f} + 3^\circ 1/6 - 0^\circ.79)/(79^{y} + 3^\circ 1/6 - 0^\circ.79)$ will produce $344^{f} = 647^{y}$ (eq.25) to extremely high precision.]

F2 Now we look into how Ptolemy or a source could have gotten eq.23 from eq.25. First, applying precession (eq.3) to eq.25 produces:

$$344^{\mathrm{f}} + 6^\circ 28' = 303^{\mathrm{g}} \approx 647^{\mathrm{y}} + 6^\circ 28' \approx 236321^{\mathrm{d}}1/6 \qquad (26)$$

Then one can find an almost exactly[15] proportional version of the shortest near-return relation (the well known 15^{y} Mars cycle):

$$8^{\mathrm{f}} - 1/19 \approx 7^{\mathrm{g}} \approx 15^{\mathrm{y}} - 1/19 \approx 5459^{\mathrm{d}}1/2 \qquad (27)$$

Subtracting eq.27 from eq.26:

$$336^{\mathrm{f}} + 25^\circ 2/5 \approx 296^{\mathrm{g}} \approx 632^{\mathrm{y}} + 25^\circ 2/5 \approx 230861^{\mathrm{d}}2/3 \qquad (28)$$

[14] Actually 1/10 of a percent, but such extreme closeness is largely an accident of rounding.

[15] Regarding what is done to 19° in eq.27 (eq.52): we have earlier (e.g., *DIO 4.1* ‡3 §F3) exploited our induction that ancients habitually took advantage of the fact that 8′ is close to 1/8 of a degree (because $8 \approx 60^{0.5}$) to substitute 1°/8 for 8′ (or 7′: eq.29 here & *DIO 1.3* §M10, fn 251). But 19° is relatively an ordmag closer than this, to 1/19 of 360°. Since $360^{0.5}$ is within 1 part in ordmag 1000 of exactly 19, we are here assuming (see also eq.52) that ancient mathematicians, traditionally attached to unit fractions (see §E2), routinely used 1/19 of a circle interchangably with 19°.

Dividing by 8, one obtains exactly the *Almajest* 9.3 Mars digits[16] the synodic components of which are falsely claimed to be consistent with the *Almajest* tables.

$$42^{\mathrm{f}} + 3^\circ 1/6 \approx 37^{\mathrm{g}} \approx 79^{\mathrm{y}} + 3^\circ 1/6 \approx 79^{\mathrm{y}} + 3^{\mathrm{d}}13' \approx 28857^{\mathrm{d}}43' \tag{29}$$

Eq.29 (which, we note, can be gotten directly from eq.26 by proportions) yields mean daily synodic motion:

$$m = 37^{\mathrm{g}}/28857^{\mathrm{d}}43' = 799200^\circ/1731463^{\mathrm{d}} = 0^\circ;27,41,40,11,44,38/d. \tag{30}$$

which comes nowhere near to matching the actual *Almajest* 9.3-4 tabular Mars mean motion (eq.37). But, in a recent daring and brilliant hit-success, Alex Jones has (by defying the false perceptions of both Ptolemy [§F3] and DR [§G]) shown that using the *longitudinal* (leftmost) component of eq.29 (instead of the synodic part) produces:

$$m = (42^{\mathrm{f}} + 3^\circ 1/6)/(28857^{\mathrm{d}}43') = 907390^\circ/1731463^{\mathrm{d}} = 0^\circ;31,26,36,53,51,33/d \tag{31}$$

which matches the *Almajest* 9.3-4 Mars-longitude tabular motion, digit-for-digit.

F3 Thus, Ptolemy did not know the calculational basis of "his" *Almajest* 9.4 Mars tables, which he published without any credit at all to the actual skilled scientist who genuinely did the reasoning that produced them.

[Note added 2003. To be quite clear about the current culminating state of the Ptolemy Controversy: Ptolemy committed *three* deceptions *each* for Mars & Jupiter:

[1] Ptolemy claims (fn 2) to have based the tabular mean motion of all planets (including Mars & Jupiter) upon not only observational data (centuries-long-arc-divided-by-time) but

[2] specifically upon the *synodic* arc of motion: Mars *Almajest* 10.9, Jupiter *Almajest* 11.3. (Yet Jones found [eqs.31&45] that the Mars & Jupiter tables are strictly based upon *longitudinal* data — and that's *period-relation* data [not Ptolemy's claimed arc/time].)

[3] See also the clear *Almajest* 9.3 statement (Toomer 1984 p.425 final sentence) that all the outer-planet longitudinal-motion tables *came from subtracting the synodic motion from the solar motion*.

So it is now realized on both sides of the Ptolemy Controversy that all three statements ([1]-[3]) are false. Thus, Jones' exploratory verve has firmly established one of the most devastating series of proofs ever produced against Ptolemy's pretensions. These reversals sink for good any hope of ever restoring the formerly-orthodox belief that the *Almajest* planet mean motion tables were Ptolemy's own or that they were computationally based upon (whoever's) "observations" reported by him (fn 2) as their basis. (Independent proof of same point for Mercury [and now Saturn]: fn 34.)]

[16] See Toomer 1984 p.424 n.26 for his now-solidly (eq.31) Jones-redeemed proposal that 53′ was a scribal error for 43′: M for N. (Toomer *loc cit* had fairly noted mss' unanimity against his theory.) Odds on an arcmin-tens-place change in d hitting by chance right on Mars' *Almajest* 9.3 longitude-motion: 1/140000.

G DR's Neat, High-Odds, Lovely, & Utterly Unhistorical Mars Fit

G1 An astonishingly integral and perfectly-fitting DR speculation follows.

[It is now absolutely eliminated by A.Jones' wonderful recent discovery: eq.31. Jones' solution is based upon virtually fully-attested data (while DR's is not), additionally paralleling (fn 21) Jones' successful Jupiter solution — i.e., it exhibits *fruitfulness*. (Note delicious irony in the inversion here of an earlier DR-vs-Jones situation: noted at Rawlins 1991W fn 85.) DR's Mars solution (eq.37) found an integral-longitudinal source perfectly producing the *Almajest* 9.3-4 motion which (unlike DR's Jupiter solution: fn 26) hit the mark at seductively above-chance odds. [Evidently thousands-to-1 for Mars: fn 21.] Jones' Mars solution unquestionably must take precedence over DR's, as DR owned by fax to Jones 2003/9/17, an hour after learning of the solution via D.Duke. We retain the prior (false) DR solution here as a curiosity which may be instructive to future scholars in several ways, e.g., regarding complex-speculation-overextension vs caution & humility. See *DIO 2.1* ‡3 fn 26.]

G2 Our reconstruction here is mathematically equivalent to that of Rawlins 1987 p.237 (but eliminating the overapparent circularity noted by Thurston 2002S p.62). First, round[17] eq.26's d to the nearest day:

$$344^{\mathrm{f}} + 6^\circ 28' = 303^{\mathrm{g}} = 647^{\mathrm{y}} + 6^\circ 28' \approx 236321^{\mathrm{d}} \tag{32}$$

Next, eq.32 is adjusted by a different[18] proportional relation:

$$17^{\mathrm{f}} + 6^\circ 28' \approx 15^{\mathrm{g}} - 4^\circ 00' \approx (15 - 1/90)\cdot(236321^{\mathrm{d}}/303) \approx 32^{\mathrm{y}} + 2^\circ 28' \tag{33}$$

The computation of eq.33's time-interval in days (next-rightmost term) is then performed by a clever ancient-style means (extracting & holding sexagesimal factors until final division: Rawlins 2002A §A8), to ensure a round, non-infinite-sexagesimal result:

$$(15 - 1/90)\cdot(236321^{\mathrm{d}}/303) = (318797029^{\mathrm{d}}/101)/270 \approx 11690^{\mathrm{d}}53/135 \tag{34}$$

After subtracting eqs.33-34 from eq.32, we possess two numbers whose ratio will yield exactly the *Almajest* Mars tables' mean motion.

$$327^{\mathrm{f}} = 288^{\mathrm{g}} + 1/90 = 615^{\mathrm{y}} + 1/90 \approx 224630^{\mathrm{d}}82/135 \tag{35}$$

One could divide by 3 and have instead:

$$109^{\mathrm{f}} = 96^{\mathrm{g}} + 1/270 = 205^{\mathrm{y}} + 1/270 \approx 74876^{\mathrm{d}}352/405 \tag{36}$$

Simple division from the numbers in either eq.35 or eq.36 produces the same mean motion. Choosing eq.35, we find:

$$m = (288^{\mathrm{g}} + 4^\circ)/(224630^{\mathrm{d}}82/135) = 152145^\circ/329621^{\mathrm{d}} = 0^\circ;27,41,40,19,20,58/d \tag{37}$$

which matches perfectly (i.e., to full given precision) the *Almajest* 9.3&4 Mars tables. The agreement is to 1 part in more than 100 billion.

G3 In the foregoing Mars development, the integer in eq.35 [or eq.36] is the number of *heliocentric* revolutions, not the number of synodic revolutions (which is the integral term for all the other tropical ancestors here: eqs.12, 20, 28, 43, 52). Whereas one can (and always should)[19] look for alternate theories (e.g., the remainder's near-integrality in

[17] Integral d is less than the precise product (of 647 and eq.5) by barely 2 parts in 3 million.

[18] Vs. eq.27, that is. Both eq.27 and eq.33 are based on periodicities familiar to all astronomers: Mars close approaches occur every 15^{y} (eq.27) to 17^{y}, so one can use either, or (in eq.33) their sum [which is superior to either].

[19] The wisdom of entertaining alternate explanations [DR surely didn't do so enough for eq.37] isn't to be confused with a tactic of using such considerations to submerge unorthodoxy. [Note added 2003. Others must judge whether Gingerich (now redeemed on eq.37) was justified in his 1983 suppression of not just eq.37 but of the entire canvas (see, e.g., *DIO 11.1* fn 1) constituting the present *DIO 11.2*.]

eq.35 and-or smallness[20] in eq.36), DR's longtime interpretation[21] was that the calculator was a heliocentrist. [Note added 2003. Now that it is revealed (by Jones' intelligence: eqs.31&45) that the actual Mars & Jupiter tables were computed via heliocentric (longitudinal) revolutions, it might possibly be argued (§G4) that Jones' great discovery has doubled (from one to two) the number of *Almajest* 9.4 planetary motions whose hidden basis was heliocentric. However, as DR did not discover either of these motions' longitudinal basis (correctly anyway: fn 21), he shouldn't intrude as arbiter — which is why mention of heliocentrism has been eliminated from this paper's title in the present (2003) re-do, though we will later (§G4) remark some provocative differences (vs other planets') in the longitudinally-based developments of the Mars & Jupiter tabular motions.] In any case: for an as-yet-unrefuted *simple* (by contrast to DR's unhistorical Mars development: eqs.32-37) — and double — indication of subterranean heliocentrists lurking in an ancient geocentrist milieu, see Rawlins 1991W eqs.23&24. Evidence for ancient heliocentrism is nicely summarized (and forcefully augmented) by Thurston 2002S pp.60&62. For an array of further indications of ancient-heliocentrist influence, see: here at §§G4 (item [i]) & L8, fn 22, Rawlins 1987 p.238 & nn.34-38, and Rawlins 1991W §§N4, N17, Q1, and (above-cited) eqs.23&24, fn 284, *DIO 4.2* ‡9 §K13; see also Rawlins 1991P (esp. §§B-C&F) — an article which argues that, despite the public banning of heliocentricity, all competent ancient mathematical astronomers (at least from Aristarchos on) knew that the Earth circuited the Sun — a public-vs-private sociological schizuation which was to be repeated later in pre-Revolution 18th-century France, since the infallible Roman Church was still proscribing Copernican geomobility, continuing this banishment all the way up to

[20] The remainder in eq.36 is much less than that in eq.29. But in §G2, using $15^{\mathrm{g}} = 32^{\mathrm{y}} + 11^{\circ}$ (instead of eq.33) would have produced a remainder in the 205^{y} Mars finale (analogous to eq.36) of just $-1^{\circ}1/2$, hardly a difference (vs our eq.36's $1^{\circ}1/3$) worth seriously upsetting basic procedure for.

[21] [Note added 2003. The fact that eq.37 is a synodic division should have pushed DR to look more closely at the Mars situation. (As also the fact that his Mars & Jupiter solutions are disparate. By contrast, one of the several great strengths of Jones' eqs.31&45 is: they solve the Mars&Jupiter mean motions **the same way**.) A perverse irony: DR's false solution was a longitudinally-integral synodic quotient, whereas Jones' true solution was a synodically-integral longitudinal quotient. DR longly&wrongly regarded it as a meaningful coincidence that a plain integral multiple (92) of the only 6 digit-component ratio ($152145^{\circ}/329621^{\mathrm{d}}$) satisfying the *Almajest* 9.3 motion (eq.37), whose numerator's factors ensured sexagesimalesque d, had (2^{y} before DR knew it: *DIO 2.1* ‡3 fn 14) hit within 1 part in 69 million (eqs.32-35) upon not just an integral number (327) of revs but one which is a peculiarly Martian cycle of longitudinal (helioc) Metonic-tropical revs, namely (eq.35): 327^{f} per $615^{\mathrm{y}} + 1/90$, nextdoor (§J4) to $344^{f} = 647^{y}$, genuine ancestor (eq.24) of *Almajest* 9.3's Mars m. I.e., dividing eq.37's ratio by all its sexagesimal factors (product: 135) yields $1127^{\circ}/(329621^{\mathrm{d}}/135)$. But $92\cdot1127^{\circ} = 103684^{\circ} = 288^{\mathrm{g}} + 1/90$, and (via eqs.3, 25, & 26) this, subtracted from $647^{\mathrm{f}}H/303^{\mathrm{g}}$ times itself, equals $327^{\mathrm{f}} - 0'.102$. Given 216000 arcmin-tenths/circle and 2 possible integralities (longitudinal revs [unprescribed option] vs synodic revs), we can crudely estimate odds against accidentally fitting so closely as (precision divided by choices): $327\cdot216000/(2\cdot92) \approx$ 400,000-to-1. Regardless of these high computed odds (somewhat ameliorable by factoring further options into the denominator), DR's Mars solution is not at all historically significant. (Other huge meaningless coincidences: *DIO 8* ‡5 §§F&G, & P.Lowell's Pluto [Rawlins 1968].) In 2003 Sept, A.Jones (see p.30 fn 1) found what is surely the correct Mars solution: eq.31. Happily, a few DR redemptions follow from this: [i] Despite DR's **Gongggggggggggg** on Mars&Jupiter (failing to check [all] data right in the very preface where he'd found the other 3 solutions!), he is undisputed discoverer (p.31 fn 1) of the numbers behind 3 of the 5 planets: Mercury (a hit to within 1-part-in-a-**trillion**), Venus, & Saturn. [ii] DR is also (*idem*) the 1980 discoverer of the hitherto universally-rejected (Rawlins 1987 n.30) general theory, now naildown-vindicated (§J5) by Jones' finds, that **period-relations** (not Ptolemy's alleged arc/time ratios: fn 2) were the immediate computational fathers of the *Almajest* 9.3&4 planet motions. [iii] All 5 of DR's huge integral-sidereal grandfather period-relation solutions (§C2 or *DIO 2.1* ‡3 fn 17) still stand (Saturn's explicitly attested as Ptolemy's: §B1), unshaken by the elsewhere-reverberating implications of Jones' 2003 findings, findings which also [iv] ice (§F3) the long-banished RNewton&*DIO* position on Ptolemy's truthfulness.]

1835! (See *DIO 9.3* ‡6 fn 75; also Darwin *Voyage of the Beagle* 1845 Chap.3 [1832].)

G4 Note that the developments of Mars and (as we shall see in §H) Jupiter seem on several grounds to be by a different (possibly heliocentrist) hand than most or all of the rest: [a] The Ptolemy-attested Mars&Jupiter period-relations directly generating their mean motion tables are the longest. [b] The precision of these 2 planets' day-denominators d is greater than the other developments' (see §F1), causing greater computation-precision: the Mars & Jupiter tables are computed from integral quotients bearing 7-digit denominators d (eqs.31&45), rather than 6-digit d (Mercury, Venus, Saturn). [Note: within the Me-V-Sat-suggested 6-digit bound, DR was correct that eq.35 was the only reasonable Mars solution. Happily, Jones' mental sword cut through this Gordian constraint to find eq.31.] The ancient computer *cared* to go this precise. [c] He also *cared* enough about founding his Mars&Jupiter calculations on longitudinal (heliocentrist) revolutions that he chose to handle a cumbersome numerator (similar to false eq.37; irony: fn 21). [See §G3 regarding Jones' doubling the number of planets whose *Almajest* 9.3-4 mean motions are based on longitudinal cycles. So, depending on modern interpretation, the demise of DR's theory either weakens or doubles the mean-motion-tables' support for ancient heliocentrist influence. (See perversity cited in fn 21.) Main factors weakening case for ancient heliocentrist influence [in the planet tables]: [1] DR [a top modern proponent of said influence] carelessly misconstrued [1980-2003] the precise way in which longitudinal [heliocentric] motion related to the *Almajest* 9.4 tables of Mars. [2] For outer planets, geocentrist Ptolemy [*Almajest* 9.3 (Toomer 1984 p.426)] was not reluctant to provide explicit longitudinal [heliocentric] rates of motion [though unaware of their direct calculational rôle].)]
[d] Mars' eq.25 is (§F1) the only unattested grandfather period-relation. [e] The resulting Mars motion is the most accurate of the five planets: so well-founded that the *Almajest* 9.4 Mars tables will give a mean synodic position even *today* that is off by only $0^{\circ}.4$, which (as remarked with suspicion in Rawlins 1987 p.237) is better than the average accuracy of Ptolemy's alleged Mars observations *in his own time*. ("Observations" upon which he claimed he founded the *Almajest* Mars orbit! I.e., standard Ptolemaic truthfulness.) Incredibly, the *Almajest* 9.4 Mars mean synodic motion tables' long-term accuracy is comparable to that of the Mars tables of the modern AmPhilSoc tables (Tuckerman 1962&64), which were also off by ordmag $0^{\circ}.1$/millennium (Houlden & Stephenson 1986 p.ii Fig.1: Mars). One notes (Rawlins 1991W §S1) the coincidences that:
[i] By far the most accurate of the m is (as Jones proved) based on longitudinal (heliocentric) revolutions. The only other (though see §B4: Venus) accurate[22] *Almajest* celestial motions are the solar (terrestrial) sidereal speed and the three lunar motions. Again there are heliocentrist connexions: the earliest highly accurate sidereal year (Rawlins 2002A fn 14) and monthlength (*ibid* eq.6 or 19/235 [*ibid* eq.9] times *ibid* eq.12) both have Aristarchan associations; same for Hipparchos' lunar-model dimensions (Rawlins 1991W eqs.23&24).
[ii] Antiquity's Dr.Geocentricity (C.Ptolemy) ignorantly plagiarized all "his" m values (e.g., fn 34).
Lesson: whatever Muffiosi may wish to believe (in order to sanctify Dr.G), competency is not unrelated to proper perspective on the universe.

[22] For balance, we should note that Ptolemy's false precession (based upon the Metonic equation that 235 months equals 19 years) came to him via heliocentrists (Rawlins 1999). However, we do not know whether Aristarchos really believed that 235/19 months was a true tropical year — or whether he was aware that this was just a calendaric convenience (still surviving today, in the formula for determining Easter).

H Jupiter

H1 To find the *Almajest* 9.3 Jupiter tabular mean synodic motion's ancestor, we start with the planet's *Almajest* 9.3 short period-relation:

$$6^{\rm f} - 4°5/6 = 65^{\rm g} = 71^{\rm y} - 4°5/6 = 71^{\rm y} - 4^{\rm d}9/10 = 25927^{\rm d}37' \tag{38}$$

and, just as we have throughout, apply §C's method: using eqs.6&3, we have:

$$G/Y = H \cdot 65^{\rm g}/(71^{\rm y} - 4°5/6) \approx 0^{{\rm g}/y} + \cfrac{1}{1 + \cfrac{1}{11 - \cfrac{1}{7 + \cfrac{1}{5}}}} = 391^{\rm g}/427^{y} \tag{39}$$

thus revealing the well-known (fn 5) Jupiter centuries-long sidereal period-relation

$$36^{f} = 391^{\rm g} = 427^{y} \tag{40}$$

to have been eq.38's ancestor. [Parallel, via eq.45's *Almajest* 9.3-4 Jupiter longitudinal motion: truncating the cont'd-frac development $H\cdot(6^{\rm f} - 4°5/6)/(71^{\rm y} - 4°5/6)$ or $(6^{f} - 4°5/6 - 0°.71)/(71^{y} - 4°5/6 - 0°.71)$ will produce $36^{f} = 427^{y}$ (eq.40) to extremely high precision.]

H2 The descent from eq.40 starts with conversion to tropical frame by eq.3:

$$391^{\rm g} \approx 427^{\rm y} + 4° \tag{41}$$

and the development from this point (summarized at Rawlins 1987 n.27) is suggested by the denominator of eq.38's degree-based r, namely 6. (Similarly, d's fractional ending in eq.7 denominators [2/5 for Mercury][23] suggest the integers divided into the long tropical ancestor in those cases.) Presuming the ancient computer looked for a relation near eq.40 & divisible by an integer, he would find that a tiny proportional adjustment

$$1^{\rm g} \approx 1^{\rm y} + 33° \tag{42}$$

would shift all integral terms in eq.41 into multiples of 6. Subtracting eq.42 from eq.41 yields, using eq.4:

$$36^{\rm f} - 29° \approx 390^{\rm g} \approx 426^{\rm y} - 29° \approx 155565^{\rm d}2/3. \tag{43}$$

Dividing by 6, we soon come into possession (eq.44) of all the Jupiter digits of *Almajest* 9.3:

$$6^{\rm f} - 4°5/6 = 65^{\rm g} \approx 71^{\rm y} - 4°5/6 \approx 71^{\rm y} - 4^{\rm d}9/10 \approx 25927^{\rm d}37' \tag{44}$$

H3 But the synodic data do not produce the *Almajest* 9.4 Jupiter tables. So this gets Ptolemy into unambiguous authorship problems yet again. We know to a certainty that he plagiarized the Jupiter tables because the equation[24] he specifies in *Almajest* 9.3 does not[25] give his tabular motion,[26] while the longitudinal equation (eq.44's data) does so precisely,

[23] See also r fractional ending at §J3: 5/7 [alternate Saturn evolution]. (If the Mars d's ending is a remnant of discarded earlier work, not a scribal-error [fn 16], we note 53′ ≈ 7/8.)

[24] As also with Mars. [See §F3 bracket.]

[25] If we perform the synodic division from Ptolemy's explicit *Almajest* 9.3 equation, we get $m = 1404000°/1555657^{\rm d} = 0°;54,09,02,42,55,53/{\rm d}$ — disagreeing with the *Almajest* 9.3-4 Jupiter mean motion already by the 4th place.

[26] In his first astronomical work, *CanInscr*, Ptolemy merely expresses Jupiter's m to 5 places, which misled DR (who should have considered that the chances of one of five planets' motions ending in a zero was 1/12, hardly significant): *DIO 2.1* ‡3 fn 23. Though DR's Mars solution was remarkable (§G1), his carelessly-conceived Jupiter solution (*DIO 2.1* ‡3 §C3) has been found (upon DR's own [disgracefully belated] investigation) to have chance unlikelihood of merely c.10%, statistically insignificant. (But both Jones solutions' chance-odds are [even without attestion] ordmag 1/10000. See fn 16.)

as recently revealed by A.Jones' investigative intelligence:[27]

$$m = (6^{\rm f} - 4°5/6)/25927^{\rm d}37' = 129310°/1555657^{\rm d} = 0°;04,59,14,26,46,31/d \tag{45}$$

H4 The Jupiter short tropical period-relation (eq.38) is the *Almajest* 9.3 case which is most-obviously derived from a long-period ancestor (eq.40), as we will here show. It's already been noted (§H2) that the degree-fraction (5/6) in the r of eq.38 hints at division by the integer 6. But a whole lot stronger than a hint is the glaring fact that a $71^{\rm y}$ cycle is such a dreadfully poor short-tropical period-relation choice that it's nearly impossible to explain — other than by the process laid out in §H2. Neugebauer 1975 p.605 attempts to connect to Babylonian Goal-year-texts the *Almajest* 9.3 choices of short period-relation durations: Mercury $46^{\rm y}$ (eq.7), Venus $8^{\rm y}$ (eq.15), Mars $79^{\rm y}$ (eq.23), Jupiter $71^{\rm y}$ (eq.38), Saturn $59^{\rm y}$ (eq.46). But Neugebauer 1975 p.605 n.6 admits that these texts also cited the Jupiter $83^{\rm y}$ period-relation. So, why didn't Ptolemy choose that excellent period? He was supposedly looking for near-perfect returns, but the Jupiter $71^{\rm y}$ remainder $r = -4°5/6$ is the worst in absolute terms (though see fn 12) — needlessly, since adopting the $83^{\rm y}$ Jupiter period-relation (instead of the $71^{\rm y}$ one) would ensure a remainder r more than *fifty times* smaller: $76^{\rm g} = 83^{\rm y} - 0^{\rm d}05'$. Conversely, what *was* the attraction of the $71^{\rm y}$ cycle for the ancient constructor of the Jupiter numbers of *Almajest* 9.3? Simple: to draw the $83^{\rm y}$ cycle from the 427^{y} ancestor cycle which we found (eq.39) had bred the *Almajest* 9.3-4 Jupiter tabular motion, one would require subtraction of a $12^{\rm y}$ adjustment equation ($11^{\rm g} \approx 12^{\rm y} + 4°43'$) — an adjustment more than ten times larger than the $1^{\rm y}$ one (eq.42) which we watched (eq.40→eq.44) elicit the *Almajest* 9.3 Jupiter $71^{\rm y}$ period-relation. This contrast supports the theory (§J4) used throughout this paper's reconstructions of the *Almajest* 9.3 computer's developments of short cycles from long ones: he preferred a small adjustment-equation.

I Saturn

I1 We begin with the *Almajest* 9.3 Saturn tropical data:

$$2^{\rm f} + 1°43' = 57^{\rm g} = 59^{\rm y} + 1°43' = 59^{\rm y} + 1^{\rm d}3/4 = 21551^{\rm d}18' \tag{46}$$

As previously for the other planets, we apply to this equation the method of §C:

$$G/Y = H \cdot 57^{\rm g}/(59^{\rm y} + 1°43') \approx 0^{{\rm g}/y} + \cfrac{1}{1 + \cfrac{1}{28 + \cfrac{1}{2 + \cfrac{1}{5}}}} = 313^{\rm g}/324^{y} \tag{47}$$

Which gives us (Ptolemy's [fn 38]) Saturn ancestor sidereal long period-relation:

$$11^{f} = 313^{\rm g} = 324^{y} \tag{48}$$

[27] All scientific historians should be grateful that Jones fortunately ignored DR's overconfident and (fn 26) quite unfounded surety that his own Jupiter solution was valid. Jones' fit is perfect to all 6 places, as comparison of eq.45 to the longitudinal motion at *Almajest* 9.3&4 shows. (DR's solution was good only to 5 places.) While DR's check on his Jupiter solution was inexcusably sloppy, his Mars solution was perfectly fitting, and (§G1) at formal odds of thousands-to-1. (Is the reason no one [during two decades!] tried Jones' elementary solution-approach simply: Muffiosi — most-motivated to [a] boost Ptolemy & [b] denigrate DR's work — was [a] still hoping Mars&Jup solutions were via arc/time, and-or [b] actually believed all along that DR was right?) In any case, this situation has happily resulted in the truth behind the *Almajest* 9.4 Mars & Jupiter tables being discovered by an ever-adventurous scholar whose work is so frequently original & central.

I2 We will later (§L) hypothesize that this was based on a time-span of 6 1/2 centuries (like Mars [eq.25] & maybe Mercury), so we'll start our reconstruction by using double eq.48:

$$22^f = 626^g = 648^y \tag{49}$$

Transforming to the tropical frame:

$$626^g = 648^y + 6°+ \tag{50}$$

Small proportional adjustment:

$$1^g = 1^y + 13°- \tag{51}$$

Add:[28]

$$627^g \approx 649^y + 1/19 \approx 237045^d 05' + 19^d 13' = 237064^d 3/10 \tag{52}$$

Dividing this equation by 11 yields[29] (using also eq.4):

$$57^g \approx 59^y + 1°43' \approx 21549^d 33' + 1^d 3/4 = 21551^d 18' \tag{53}$$

— which are the *Almajest* 9.3 numbers displayed at eq.46. (Note §J3's alternate route from eq.48 to eq.53.) Division of this equation's (or longer eq.52's) outer elements yields:

$$m = 57^g / 21551^d 18' = 205200° / 215513^d = 0°;57,07,43,41,43,40/d \tag{54}$$

which is the Saturn tables' *Almajest* 9.3-4 synodic tropical mean motion.

J What the Tables Reveal

J1 The question has been raised: why did *Almajest* 9.3 even bother to disguise long cycles as short ones? (A step which has so durably misled certain astronomically-naïve historians — who've extrapolated their planetary delusion to luney extremes: Rawlins 2002B §A2.) Neugebauer as always sought (§H4) a Babylonian explanation for the *Almajest* 9.3 cycle-durations. Other possibilities: short cycles had greater utility (than long ones) in practical calculation; and-or it was merely desired to make slight corrections to canonical (long-since-thoroughly-tabulated) sub-100^y cycles, a reasonable position that is more or less that of both Neugebauer & Ptolemy. (True, it's also possible [fn 12] that the disguising was a deception. If so, it cannot be laid to Ptolemy, who [a] didn't create this material, [b] didn't pretend *any* of his mean motions [Sun, Moon, all five planets, & stars] came from subcentury-separated data, since he cited *long* time-bases for all eight [i.e., he had more common sense than his modern worshippers: Rawlins 2002B §A2, Rawlins 2002H §E5].) And the disguise was not entirely formal: we have noted (§E3) that for Venus, the *Almajest* 9.4 tables could not be exactly based upon any larger period we encountered during their developments here. (Indeed, in the case of Venus, we learned [*idem*] that a rounding [connected to the shortening of the apparent base-cycle] sensibly affected the tables' accuracy.) By contrast, we found that the centuries-long cycles of Mercury (eq.12) & perhaps Saturn (eq.52) will directly produce their respective tables.

J2 Regardless of our speculations on reasons for disguise: the origin of each of the short tropical *Almajest* 9.4 planet tables has been revealed to be an empirical (stations-based) long sidereal cycle. We have already discussed the evidence for this (above at §§B1&H4). The most obvious point is simply that no intelligent ancient would[30] found highly precise tables upon anything but the sort of accurate base which only a long period-relation can provide. (Note analogy to central reasoning at Rawlins 2002A §A11.)

[28] Regarding the rounding of 19° to 1/19 of a circle in eq.52: see fn 15.

[29] Despite the approximation-signs of eq.53, its d descends exactly from eq.52's rounded d.

[30] Exception to procedure (or intelligence) in *Almajest* 9.3: §E3.

J3 The point has not only been proved here in general — in addition, all five previously un-connected ancestors have been uncovered by the method of §C, and developed individually as we went along. Parts of the developments of the Mercury,[31] Venus, & Saturn cases are of course not guaranteed to be unique (though the Mars & Jupiter evolutions [big-ancestor→small-*Almajest* 9.3 period-relation] are each essentially unambiguous, and each of the 5 planet-developments' final step [producing the five m of *Almajest* 9.3] is effectively unique). E.g., for the Saturn case, one could take $313^g = 324^y + 3°+$ (eq.48 transformed to tropical, i.e., eq.50 halved) & add proportional adjustment $86^g = 89^y + 9°-$, resulting in $399^g \approx 413^y + 1/30$; division by 7 then produces $57^g = 59^y + 1°43' = 21551^d 18' = 59^y + 1^d 45'$, the *Almajest* 9.3 numbers (eq.46). (Or subtract proportional $113^g = 117^y - 9°$ from eq.50 for $513^g = 531^y + 15° 1/2$; division by 9 gives eq.53.)

J4 However, this paper's developments, from empirical long integral sidereal period-relations to *Almajest* 9.3's short tropical period-relations, have proceeded according to a consistent theory (powerfully recommended by the case of Jupiter: §H4): that all proportional adjustment equations were chosen to be as small as possible.[32] (Perhaps the ancient computers believed that such procedure was less likely [than large adjustments] to degrade the accuracy of the original empirical sidereal period-relations.) Indeed, for the cases of Saturn (eq.51), Jupiter (eq.42), and Venus (eq.19), the adjustments were barely 1^y.

J5 Though there are indeed variations in the treatments for each planet, the differences are felicitously informative (and made writing this paper — fitfully over a period of more than 20^y — more of an adventure than otherwise). The treatment of Venus seems exceptionally crude (this was somewhat forced by factors noted at fn 12) and actually was so in one respect (§E3). The sources of the tables of Jupiter and of Mars were not even known to "author" Ptolemy (who just blythely & blindly rode-sourceback through the creation of both planets' tables: §K1). Whether there is any hint of heliocentrism in the longitudinal basis (§G3) for these two planets' tables must be left to others to decide. In any case, Jones' grand discovery of the true (longitudinal) sources of the *Almajest* 9.4 Mars & Jupiter mean motion tables now establishes beyond any doubt the truth of the revolutionary (contra-Muffia) contention first propounded in DR's 1980/4/13 & 9/2 letters to Gingerich (*DIO 1.2* fn 55) and 1st published in Rawlins 1985K & Rawlins 1987: all 5 planets' *Almajest* mean motions are based not upon Ptolemy's alleged arc/time ratios but upon period-relations. [From the letter cited, Gingerich knows DR was additionally discoverer of the precise solutions for the majority (3/5) of the *Almajest* 9.4 planet mean-motions. So far, neither he nor the Muffia has explicitly acknowledged this. Standard cult-generosity toward the heathen.]

J6 That these ancestral relations' Y were centuries long is strongly indicated by the particular cases of Venus [§E3's 2003 bracket], Mars (§F1 [b]), & Jupiter (§H4) — but, then, we have already noted in passing (§B1) that the point is obvious anyway both from extant testimony to ancient methodology and from elementary empirical considerations.

[31] An example of arbitrary rounding occurs in the Mercury development at eqs.10&11, where the former d is kept more exact than the latter. Which is a reminder that this paper is a study in inducing not the *best* procedures but the most plausible reconstructions of the *actual* procedures of the *Almajest* 9.3-4 material's understandably shortcut-happy ancient computers, working in a pre-electronic (pre-blackboard!) era. In seeking out most-likely explanations of the *Almajest* numbers, the required moderate flexibility added spice to the inductive process — though ever in mind was an Occamite realization that too much nonuniformity of approach can end up fertilizing *ad hoc* fantasies. (We have discussed excesses in this connection several times previously, e.g., *DIO 1.2* §F4, *DIO 4.3* ‡15 §F5. [And note our *own* excess: see §G1 bracket & fn 21.]) Those who read this paper carefully will see that we have operated with an accent on coherency (e.g., §§B1&J4) while nonetheless showing how one can recover all the actual extant numbers.

[32] It was that key criterion which discouraged the (otherwise attractive) alternate Saturn evolution, provided for contrast at §J3. Another common thread throughout these developments (except for Mars): use of unit fractions in the ancestor tropical relations.

References

Almajest. Compiled Ptolemy c.160 AD. Eds: Manitius 1912-3; Toomer 1984.
J.Berggren & B.Goldstein 1987, Eds. *From Ancient Omens to Stat Mech*, Copenhagen.
Canobic Inscription. Compiled Ptolemy 146-147 AD. Ed: Heiberg 1907.
J.Evans 1987. JHA *18*:155 & 233.
O.Gingerich 1980. QJRAS *21*:253.
O.Gingerich 1988. JHA *19*:142. Review of Berggren & Goldstein 1987.
O.Gingerich 2002. Isis *93.1*:70.
B.B.Goldstein 1967. *Arabic Version of Ptolemy's PlanHyp*, AmPhilosSocTrans *57.4*.
B.B.Goldstein 2002. JHA *33*:1.
J.Heiberg 1907, Ed. *Claudii Ptolemaei Opera Astronomica Minora*, Leipzig.
M.Houlden & F.R.Stephenson 1986. *Suppl* [Tuckerman 1962&64], APS Mem *170*.
Karl Manitius 1912-3, Ed. *Handbuch der Astronomie* [*Almajest*], Leipzig.
O.Neugebauer 1955. *Astronomical Cuneiform Texts*, London.
Planetary Hypotheses. Comp. Ptolemy c.170 AD. Eds: Heiberg 1907; B.Goldstein 1967.
D.Rawlins 1968. Sky&Tel *35*:160.
D.Rawlins 1984A. Queen's Quarterly *91*:969.
D.Rawlins 1985H. BullAmerAstronSoc *17*:583.
D.Rawlins 1985K. BullAmerAstronSoc *17*:852.
D.Rawlins 1987. American Journal of Physics *55*:235. [Note §G & fn 26.]
D.Rawlins 1991H. DIO *1.1* ‡6.
D.Rawlins 1991P. Journal for Hysterical Astronomy *1.1* ‡7.
D.Rawlins 1991W. DIO&Journal for Hysterical Astronomy *1.2-3* ‡9.
D.Rawlins 1996C. DIO&Journal for Hysterical Astronomy *6* ‡1.
D.Rawlins 1999. DIO *9.1* ‡3. (Accepted JHA 1981, but suppressed by livid M.Hoskin.)
D.Rawlins 2002A. DIO *11.1* ‡1.
D.Rawlins 2002B. DIO *11.1* ‡2.
D.Rawlins 2002H. DIO *11.1* ‡3.
D.Rawlins 2002V. DIO *11.3* ‡6.
B.Schaefer 2001. JHA *32*:1.
Hugh Thurston 2002S. Isis *93.1*:58.
Gerald Toomer 1977. ArchivesIntHistSci *27*:137. Review of Pedersen 1974.
Gerald Toomer 1984, Ed. *Ptolemy's Almagest*, NYC.
B.Tuckerman 1962&64. *Planetary, Lunar, & Solar Pos*, AmPhilosSocMem *54*&*56*.
B.van der Waerden 1988. *Astronomie der Griechen*, Darmstadt.

K Appendix 1. Ptolemy's Plagiarisms

K1 Who computed the *Almajest* 9.4 planet mean motion tables? In §L, we will suggest a possible author. But it is harder to prove who did the tables than to prove (§F3) Ptolemy didn't. As noted at §J5: with Mars (eq.31) & Jupiter (eq.45), Ptolemy isn't aware of the numerical basis of "his" own tables. [Note capper consideration in §F3 bracket.] He *blindly* plagiarized them — *and the numbers underlying them*. He doubtless did likewise for all 5 planets — but got flatout caught in the cases of Mars & Jupiter, where his alleged synodic numbers unquestionably do not match the tables, while (as A.Jones has found: eqs.31&45) the longitudinal numbers do perfectly produce those very tables. (Deceptions fully summarized at §F3.)

K2 And do not fail to appreciate just how screwed-up the Muffia's "Greatest Astronomer of Antiquity" was. Recapping §F3:
[a] He claimed (fn 2) that the tables (*Almajest* 9.4) were based upon the observed arc/time ratios he elaborates for each planet; but all 5 motions are instead based upon period-relations right in the tables' preface (*Almajest* 9.3) — a preface which appears to have been as thoroughly plagiarized as the tables themselves.
[b] Further, we have found that for 2 planets, the tabular mean motion cannot (§F3) be calculated even from the synodic period-relation numbers he cites at *Almajest* 9.3 — so Ptolemy cannot have known where the Mars & Jupiter tables came from.

K3 Plagiarizing all the planet mean-motion tables is perfectly consistent with Ptolemy's thoroughly-established ethics.[33] Indeed, we've already for some time possessed independent proof of embarrassingly clumsy theft for, e.g., Mercury's[34] mean motion and the Ancient Star Catalog.[35]

L Appendix 2. Implications of Greek-Exploited Records' Antiquity

L1 I had previously doubted that Ptolemy's mean planetary motions could be stolen from Hipparchos (c.140 BC) or Apollonios (c.200 BC) because, e.g., the Mars mean motion (based upon a 647^y cycle) would have required Apollonios to have used data from the 9th century BC, much earlier than the up-to-now-accepted date for the oldest Babylonian records available to classical-era astronomers. But the block to such extended investigation has now (fn 37) been lifted.

L2 Once we come to realize that there is previously[36] unperceived evidence[37] for Greek use of pre-747 BC astronomical observations, our vision is broadened and led into new paths from which the flamekeepers of orthodoxy are sadly self-barred (*DIO 10* ⊙ 20).

L3 In pondering this point, the context is best kept in mind. Every other precise *Almajest* mean motion was effectively finalized in the period between Aristarchos (c.280 BC) and Hipparchos (c.140 BC): the Sun (Aristarchos-Hipparchos [Rawlins 2002A fn 17]), the Moon (Aristarchos [Rawlins 2002A] & Hipparchos [Rawlins 2002H]), and the stars's precession (Aristarchos-Hipparchos [Rawlins 1999]). So, we are naturally inspired to ask whether the planet mean motions (if not some detailed model-features as well, e.g., equant) are also from astronomers of the period 280-140 BC. We will now discuss indications that happen to lend independent support to this idea.

L4 As detailed in the foregoing paper, the ancients determined and tabulated (sometimes remarkably accurate) tropical planetary mean motions by transforming integral sidereal cycles obtained by comparing stationary point data centuries apart. (See fn 6, Rawlins 2002B fn 4, Rawlins 1987 n.28.) So each planet-cycle's length (subtracted from the cycle-discoverer's date) has the potential to lead us to the date of that planet's earlier stationary-point observation(s). The cycles behind the *Almajest* planet mean motions (*Almajest* 9.3-4) have been given earlier here: eqs.9, 17, 25, 40, & 48. (See also *DIO 2.1* ‡3 fn 17.) The longest is 647^y. But a later Ptolemy work, *PlanHyp*, gives cycles (Neugebauer 1975 p.906 Table 15) which are mostly about 3 1/2 centuries longer[38] than that, e.g., Mars 1010^y, Mercury 993^y, Venus 964^y. Many Ptolemy loyalists believe (and Ptolemy claims) that this late Ptolemy work's alterations reflect contemporary-data-based improvements of

[33] Ptolemy's ethics? At the 1997 History of Science convention (Lѐge), the bright & lovable authoress of the recent book on this oxymoronic theme gave a talk on the subject. DR naturally offered from the floor that the only question he has about Ptolemy's ethics is: who'd he steal them from?

[34] See Rawlins 1987 pp.236-237 item (5) and van der Waerden's appreciative reaction (Rawlins 1991H fn 37). [Note added 2003. Alex Jones has recently made the original observation that the same type of argument applies somewhat to Saturn as well.]

[35] See *DIO 12* ‡1 Fig.1 (K.Pickering) & *ibid* ‡2 pp.33-34 (D.Duke). For other key new evidence, see *DIO 10* fn 177.

[36] See Rawlins 2002B fn 7.

[37] Rawlins 1996C §§D-E, Rawlins 2002B §§A5&G2, Rawlins 2002H §C.

[38] The *PlanHyp* cycles (tabulated at Neugebauer 1975 p.906) are: Mercury $3130^{\mathrm{g}} = 993^y$, Venus $603^{\mathrm{g}} = 964^y$, Mars $473^{\mathrm{g}} = 1010^y$, Jupiter $706^{\mathrm{g}} = 771^y$, Saturn $313^{\mathrm{g}} = 324^y$.

Ptolemy's parameters. (See Neugebauer 1975 p.907.) If so, then most *Almajest* mean planet motions are indicated to have originated over 3 centuries before those of the *PlanHyp*, i.e., c.200 BC. We will next explore some more detailed and thus more tentative speculations.

L5 Take the specific case of Mars: assuming the same rare 9th century BC Mars stationary point data were used to found both *PlanHyp* (fn 38) & *Almajest* (eqs.24-25) cycles: subtracting their 363^y difference ($1010^y - 647^y$) from 160-170 AD (*PlanHyp* date) yields c.200 BC. For Jupiter (where both cycles are shorter),[39] the similarly-computed date (for creation of *Almajest* 9.3) would be: 165 AD $- (771^y - 427^y) =$ c.180 BC.

L6 Another provocative set of coincidences: the Y for Mars' eq.25, Saturn's eq.49, and triple Mercury's eq.9 are all virtually equal: all three Y values fall within a 1/2 percent of 650^y. This suggests a hypothesis. Suppose Mercury's 217^y *Almajest* period was based upon 651^y-separated pairs of stations; then comparison (as in §L5) with the *PlanHyp* period (993^y) leads us to c.180 BC for the origin of the *Almajest* 9.3 material. If the *Almajest* Saturn base was eq.49's 648^y (double eq.48) vs *PlanHyp* 972^y (triple eq.48), then c.160 BC is indicated[40] as the rough date of its establishment. So we have speculatively induced c.180 BC$\pm 20^y$ as an approximate date for the originator of the ancestor relations behind the *Almajest* planet mean motions.

L7 Planet-theorist Apollonios[41] was late 3rd century BC. So: was he (or a rough contemporary) the true source of some or all of the planet mean motions of the *Almajest*? Another possibility, curiously unexplored [especially inexplicable, considering that he first went to Alexandria out of admiration for Aristarchos of Samos]: was Apollonios a heliocentrist?

L8 For anyone who senses the reality behind ancient astrologers' geocentrist façade (Rawlins 1991P), it should not be at all unthinkable that such a knowledgeable mathematician & astronomer as Apollonios knew (perhaps just privately: §G3) the true pattern of the Solar System — presumably including precession, the 1°/cy estimate of which is used throughout the foregoing reconstruction of geocentrist Ptolemy's planet mean motions, since 1°/cy was adopted by the ancient geocentrist tradition. This heliocentrist value (going back at least to Aristarchos, 280 BC: Rawlins 1999) was poor [cause: Rawlins 1985H]; but the very idea of precession was revolutionary, and (Rawlins 2002A fnn 14&16) it was only natural that realization of the Earth's gyroscopic precession should come from those who understood that our planet was not terra-firma but terra-turner. See Rawlins 1999 for the rather simple evidence for Aristarchos' priority: he alone is cited for *two* disparate yearlengths on the ancient Greek lists (*ibid* Table 1), and their difference *is* precession.

[39] This could indicate that the early-remote stationary-point observations for Jupiter were about 2-plus centuries later than those for Mars and (according to the theory of §L6) Mercury & Saturn. Thus, its data are not indicated to be 6 1/2 centuries before the *Almajest* 9.3 period-relations (perhaps from c.200 BC or a little later: §§L5&L7), as are perhaps the cycles for those three planets, though Jupiter's cycles do fit the 3 1/2-century differential pattern broached at §L4 and discussed here.

[40] Venus *PlanHyp* $Y = 964^y$ fits §L4's idea that most *PlanHyp* planet cycles use 9th century BC data; but long Venus cycles are blocked from having the same *Almajest*-vs-*PlanHyp* 3 1/2-century interval which can work for the other 4 planets. Eq.16's huge 31 (fn 12) (reflecting 8-5 resonance exactness), locks large Venus-returns into 1/4 millennium steps. [Venus-transits gaps: 1/2 that.] So it's irrelevant (& see Rawlins 2002V §C5) that Venus doesn't fit our 3 1/2 century gap proposal.

[41] Apollonios [who even bore a lunar nickname] probably published mean motion tables for the Moon (Rawlins 1991W fn 242), and he was famous in antiquity for his knowledgeability regarding planetary motion: *Almajest* 12.1. (Hipparchos is a less likely source; *Almajest* 9.2 speaks of the very limited nature of Hipparchos' then-extant planet investigations and mentions contemporaries' work, which he checked. And see Toomer 1984 p.421 n.10 on Hipparchos.)