‡2 Ancient Solstices

Ancient Solstice-Determiners' Delicate Voyage 'Twixt Random Error's Scylla and Systematic's Charybdis

Tihon Finds Hipparchos' −157/6/26 18^{h} Solstice Its Significance and Neat Surprise-Solution

New Light on Hipparchos' Calendar, Solar Elements, & Year-Length

A Summary & Unwelcome Shock-Confirmation of *DIO* Prescience

In 2010, Anne Tihon (www.springer.com/us/book/9789048127870) meticulously analysed a recently recognized papyrus (*P.Fouad* 267A, fortunately recommended to her expert examination by Jean-Luc Fournet) bearing: Hipparchos' −157/6/26 Summer Solstice, use of his hitherto-unknown 500^{y} solar longitude tables (one of them Kallippic), a new precession rate for the tropical points, also a new ancient yearlength which D.Duke soon correctly reasoned was based on comparison to Meton's −431 S.Solstice. Below, we show how ancient solstices were determined outdoors — as well as detailing the problems Hellenistic scientists had to balance, to achieve an accurate estimate of a solstice's hour. We also examine why the best ancient scientists preferred solstices to equinoxes as bedrocks for their calendars; and we consider the newly-available −157 solstice's implications for dating some of Hipparchos' astronomy. Curiously, no commentator on the papyrus' −157 solstice has yet remarked that the 1st and only prior paper to propose (§K) Hipparchos sought a −157 solstice & used Kallippic mean solar motion is Rawlins 1991W. Do non-citers believe *DIO* happened only by blind luck to improbably [a] hit upon the now-papyrus-confirmed date of Hipparchos' 1st try at a solstice (& orbit), [b] induce his Kallippic solar speed?!

B *Journal for the History of Astronomy* Biggies' 4 Solstice Adventures Sending History-of-Astron's MacArthur Genius Up to 9th Grade

The laugh-crying need for a competent article on mathematical and historical matters regarding ancient solstices may perhaps be brought home to the reader by a swift foray here into the wisdom on the subject that's been emanating from academe's two most highly-placed and expensive Experts[1] in the field of ancient astronomy. [It would not be necessary to highlight the weird stuff that follows here, except that — typically for cohesive, wagon-circling cults — despite years of opportunity (and *DIO* nudges), the perps ***have not retracted on-the-record a single one of the strange-science adventures we enumerate below***.]

B1 A.Jones, sometime Princetitutee, now at NYU's hugely endowed Inst. for the Study of the Ancient World, Boardmember-for-Life at history of astronomy's "premier" (Schaefer 2002 p.40) *Journal for the History of Astronomy* (& *JHA*'s discoverer of the Winter Equinox: Jones 1991H p.119), has added to *JHA*'s rep (www.dioi.org/jha.htm#kqlz) for meticulous refereeing by rejecting in its pages the reliable standard ancient method (*Almajest* 1.12) for finding latitude&obliquity via solstices, using equinoxes instead: Jones 2002E, a paper taken rather too seriously by PU's history of early trig, van Brummelen 2009, p.65 n.76, though with fair citation of *DIO 4.2* p.56's [or Rawlins 2009S p.20's] stark Table 1. (Unlike Jones, who persists in nonciting this Diller-DR table's *perfect* data-fit, to fake Jones 2002E's viability, not even producing his own table! *Do not miss fn 10 below*.)

[1] Despite their here-appreciated screwball gags, each of our roastees has made solid contributions to knowledge, as seen at, e.g., §B4, *DIO 4.3* ‡13 §D8, *DIO 11.2* cover [owed to Duke&Jones], *DIO 12* ‡2.

B2 Noel Swerdlow (Yale, UChi, CalTech), History-of-astronomy's MacArthur-Genius & *Journal for the History of Astronomy* Boardmember-for-Life, has, like *JHA* Assoc.Ed. J.Evans 1998 p.206, spent decades misunderstanding the 9th-grade-level method used by ancients to measure solstices, an achievement recognized by DR at R.Newton 1991 fn 20:

> One of the more amusing moments in [HamSwerdlow 1981], which RRN is too polite to note, is [HamSwerdlow 1981]'s sarcastic mock astonishment while commenting upon a key RRN discrimination: "most remarkable of all, that solstices could be observed with more accuracy than equinoxes." That RRN is correct (in the very judgement which *HS* attack as "remarkable" folly) is obvious to any unprejudiced scientist familiar with the instrumental problems involved. (See the lucid discussion at R.Newton 1977 pp.81-82 [or §G1 here].) . . . *all* known ancient astronomical-observer-calendarists (excluding [indoor] Ptolemy . . .) depended primarily upon solstices for gauging the year's length: Meton, Euktemon, Kallippos, Aristarchos, Hipparchos. (Hipparchos observed numerous equinoxes [§O]; but even *his* year-lengths were based upon solstices: see, e.g., [Rawlins 1991H] eq.8 [& below eqs.32&34].) However, Swerdlow, an historian [then] with the official rank of professor [at U.Chicago's Astron.Dep't] cannot understand this elementary point: during a gloriously delirious passage (p.527) in his prominent 1979 attack on Newton (in *American Scholar* [Phi Beta Kappa!] *48*:523 . . .), Swerdlow argues:
>
> > At the time of the solstice, the meridian altitude of the sun changes by less than fourteen seconds of arc per day, and measuring this quantity, let alone any fraction of it, was obviously ridiculous.
>
> The only ridiculous aspect of this astounding piece of reasoning is that a member of *the University of Chicago's Dep't of Astronomy* should so conspicuously exhibit his touching innocence of the implications of 1st-year calculus and of the standard technique known[2] as "equal altitudes". It is easy to see that Hist.sci archon Swerdlow's reasoning is essentially equivalent to insisting that the time a vertically oscillating body reaches maximum altitude cannot be determined since at that moment it lacks vertical motion!

Or, to reduce this to around junior-high: our nay-jerk R.Newton-hater (*DIO 1.1* ‡3 §§D2-D3) is essentially claiming that if you toss a ball upwards at t_1 and catch it at t_2, it is "ridiculous" to suppose that its height maxed at $(t_2 + t_1)/2$.

B3 Far from admitting his elementary misunderstanding, invincibly-ineducable Swerdlow keeps promoting the same reasoning's validity a decade later,[3] in the very *Journal for the History of Astronomy* paper (Swerdlow 1989 p.36) which got him his MacArthur!

[2] The Bowditch *American Practical Navigator* 1981 ed. 2:799 defines equal altitudes thusly: "Two altitudes numerically the same. The expression applies particularly to the practice, essentially obsolete, of determining the instant of local apparent noon by observing the altitude of the sun a short time before it reaches the meridian and again at the same altitude after transit, the time of local apparent noon being midway between the times of the two observations, if the second is corrected as necessary for the run of the ship. [*DIO*: And solar δ-shift.] Also called DOUBLE ALTITUDES." See fn 6 here.

[3] Rawlins 2002V fn 20: "In this MacArthur-grant-subsidized paper (published by Gingerich's *JHA*), [Swerdlow 1989 p.36] . . . alibis that since (near maximum) Venus' elongation changes merely 1°/12 in 6$^{\text{d}}$, 'in no way could Ptolemy estimate the time' of greatest [maximum] elongation more accurately. (Gingerich 2002's incomparable p.72 goes even further into legalblindnessland, claiming that ***one-degree***-accuracy in observation 'is what Ptolemy typically worked with' — a sleight which neatly confounds ordmag 0°.1 ancient observational accuracy [§B4 & Rawlins 2009E] with the ordmag 1° enormity of the most delicious Ptolemy fudge.) We have already previously ([R.Newton 1991] fn 20) dealt with the tragic pre-highschool mentalblindnessland adventure of Swerdlow 1979 pp.526-527 (in the journal of PhiBetaKappa), regarding estimation of maxima-times (solstices in that case), so I won't reprise the pathetic details here merely because he later repeated the folly under the MacArthur Foundation's aegis. But . . . none of this excuses inaccuracies of several weeks in Venus observations, leading to *dishonestly-reported* 'observational' [V elongations] which are off by way over *a degree*."

Such obstinate-incredible semi-numerate escapades by ultra-archons, serve the useful purpose (additionally to funnybone-exercise) of measuring the hist.astron community's relative skills at science vs six-figure-profitable careerist politics. See similarly at Rawlins 2009S. Dozens more such Premier-journal larfs are chronicled at www.dioi.org/jha.htm#lmvl.

B4 [Section added 2012/12/29.] *JHA*'s 2008 Aug Pb paper Duke 2008W is yet another snooze-refereed solstice-study, claiming Greek solar-declination δ data had *random* error of standard deviation $\sigma_\delta = 15'$ (vs actual c.1′: eq.14, below), citing irrelevant (*DIO 16* ‡3 fn 36) Greek star $\sigma_\delta = 10'$ (vs real median 5′: *DIO 4.1* ‡3 Table 3), & speculating Greeks found solstices via vast melds of motley-weight equal-altitude pairs with ere&aft intervals d ranging from 20$^{\text{d}}$ to 55$^{\text{d}}$; result unquantified beyond meld's $\sigma_{\text{SS}} = 5^{\text{h}}$. But ancient Greek $d = 55^{\text{d}}$ would cause S.Solstice systematic error -8^{h}. If σ_δ really were 15′ (nearly the span from solar center to limb!), random error would've been half a day for $d = 55^{\text{d}}$ and *over a day* for $d = 20^{\text{d}}$. Actual Greek solstices' errors: 0$^{\text{h}}$-3$^{\text{h}}$ (§§H-J & Table 3). Hipparchos' Rhodos 147-128BC equinox data (excellent list Duke 2008W Table 1): *systematic* δ error $+6'.5 \pm 0'.4$ [pioneer John Britton 1967 p.24 got 7′; R.Newton 1977 p.78 same] (7$^{\text{h}}$−), accounted-for (within c.1′) by $+4'+$ correction to noon altitude h for Hipparchos' fantasy 7′ solar parallax (Swerdlow's neat discovery: Rawlins 1991W fn 280), plus non-correction for sunlight's 0′.7 atmospheric refraction, plus transit-circle 1′1/3 mis-set via refracted polestar-light ($\sigma \doteq 1'$, like Hipparchos' geographical latitude σ from stellar δ data: Rawlins 1994L Table 3); *random* error $\sigma_\delta = 1'3/4$, near-same as rms 1′.7 scatter from 1/4-day rounding (constraining raw empirical random error to c.1′). Rounding cardinal-point solar data so was calendaric-tradition and-or Aristyllan-modest overcaution (‡1 §C1; *Alm* 3.1) against being responsible for unreliable data. (Note: Hipparchos' 162-158BC equinoxes' systematic error may've been mostly from asymmetric gnomon.)

C Precisely Determined Ancient Solstices

C1 Given the hist.astron center's continuing problems in the area of solstices (e.g., Rawlins 2009S §F3), it will help if we cite (and later list: Table 3) what we have hitherto possessed of outdoor ancient solstices where *the hour not merely the date is known*. After discounting those (Table 1) truncated to day-epoch — Meton's (−431) & Aristarchos' (−279) and the faked solst (Table 2) of Ptolemy (+140) — we find that we have just four so far, most only by modern reconstruction, not direct attestation. (The exception is −146, confirmed by *P.Fouad* 267A: §M4.) The −329 S.Solstice launching Kallippos' famous calendar is reconstructable by realizing (§J4; Rawlins 1985H) that his pioneering yearlength (nearly 3 centuries before Julius Caesar's Sosigenes), $Y_{\text{K}} = 365^{\text{d}}1/4$, arose from his comparison of his own S.Solstice observation to Meton's famous Athens −431/6/27 S.Solstice, which was typically (for calendarists) truncated to the beginning (sunset for Athens) *of the 24*$^{\text{h}}$ *period containing the event*. So add $102Y_{\text{K}}$ or $37255^{\text{d}}1/2$ to the start of Meton's calendar to find the solstitial moment of the Kallippic calendar's start:

$$-431/6/27\ 3/4 + 102 \cdot (365^{\text{d}}1/4) = -329/6/28\ 1/4 \qquad (1)$$

(+3$^{\text{h}}$ error). Like logic allows reconstruction of Aristarchos' −279 solstitial observation, using his Saros-cycle-fitting (Rawlins 2002A fn 14; Rawlins 2018C §G6) yearlength, Y_{A} to go 152$^{\text{y}}$ (8 Metonic cycles) beyond Meton:

$$-431/6/27\ 3/4 + 152 \cdot (365^{\text{d}}1/4 - 15/4868) \doteq -279/6/27\ 1/4 \qquad (2)$$

(0$^{\text{h}}$ error), which was truncated to −279/6/26 1/2 for the Dionysios calendar's day-epoch, which was *noon* (Table 1), as 1st computed by Rawlins 1985H (& Rawlins 1991H eq.8). Similarly, starting with Babylonian Astronomical Cuneiform Text 210 (BM55555), whose Greek-based year-length is

$$Y_{\text{U-M}} = 365^{\text{d}}14'44''51''' \doteq 365^{\text{d}}73/297, \qquad (3)$$

we know from Rawlins 1991H that Hipparchos' 135BC solstice (which he used to found his final "UH" solar orbit: *ibid* §C) occurred 297 of ACT 210's years $Y_{\text{U-M}}$ or 108478$^{\text{d}}$ (§P6

Table 1: Ancient Calendarists' Truncated Solstices

Solstice Observer	Truncated Time	Real Time	Error
Meton	−431/6/27 3/4	6/28 11^{h}	-17^{h}
Aristarchos	−279/6/26 1/2	6/27 06^{h}	-18^{h}

Table 2: Ancient Astrologers' Indoor-Calculated Solstices

Solstice Calculator	Computed Time	Real Time	Error
Hipparchos	−157/6/28 1/4	6/26 18^{h}	$+36^{\mathrm{h}}$
Hipparchos	−157/6/26 3/4	6/26 18^{h}	$+00^{\mathrm{h}}$
Ptolemy	140/6/25 1/12	6/23 14^{h}	$+36^{\mathrm{h}}$

Table 3: Ancient Astronomers' Firm & Precise Outdoor Solstices

Solstice Observer	Observed Time	Real Time	O−C Error
Kallippos	−329/6/28 1/4	3^{h}	$+3^{\mathrm{h}}$
Aristarchos	−279/6/27 1/4	6^{h}	0^{h}
Hipparchos	−146/6/26 1/2	10^{h}	$+2^{\mathrm{h}}$
Hipparchos	−134/6/26 1/4	7^{h}	-1^{h}

below) after Meton's Solstice as misunderstood by Hipparchos. (Who interpreted Meton's start-of-day as dawn instead of Athenian sunset. Perhaps just to find or force a fit to the overlong Metonic lunisolar scheme? See below at §§P4-P5 & §Q1.) Rawlins 1991H eq.6:

$$-431/6/27\ 1/4 + 297 \cdot (365^{\mathrm{d}}73/297) = -134/6/26\ 1/4 \tag{4}$$

Zzzzz-reffed Duke 2008W Table 1 crucially (‡3 fn 8) misclaims *Almajest* 3.1 makes it noon.

C2 Incredibly, Kallippos', Aristarchos', & Hipparchos' reliable, precious, professionally-observed solstices' precise hours have never [been perceived outside of *DIO*] — much less deservedly highlighted at last by exclusive tabulation. We do the honors here (Table 3), as we discover a hitherto-unknown addition (eq.27) to the list.

D Hellenistic Astronomers' Outdoor Empiricism

The three accurate solstices cited (eqs.1-2&4) add to the accumulated evidence that Greek astronomers were anything but the dreamy, data-inventing critters that certain truly dreamy historians imagine. See, e.g., our comments (at Rawlins 2008R §A) on Muffia god-pop O.Neugebauer's strange vision. Other evidences of Greek empiricism's accuracy & primacy (Rawlins 2008Q §K4 & n.9) include the half-percent precision of Greeks' basis-measure for finding the Earth's radius (‡1 §B3) — and more spectacularly their three lunar periods (‡3 fn 27; www.dioi.org/thr.htm), ***each accurate to better than 1 part in a million***.

E Truncated Solstices

We list all extant day-start-truncated solstices (§§C1-C2&E1-E3) in Table 1.

E1 Meton's calendar started on −431/6/27 3/4 since Athens' day began at sunset. As late as a century after, Kallippos knew the original Meton calendar epoch and (eq.1) founded his year-length upon it — though (§§C1&P4) Hipparchos later misconstrued Meton's S.Solstice by -12^{h}, making its error -29^{h}, which caused (along with eq.31) huge systematic errors in later astronomers' yearlength estimates (§Q1; Rawlins 1999 §B6).

E2 Exactly 2 Kallippic cycles after Meton's S.Solst & less than 3^{d} before the −279/6/30 total lunar eclipse, Aristarchos' *observed* −279 S.Solstice (eq.2), as distinguished from his *calendaric* S.Solst, may've helped retro-firmup establishment of heliocentrists' epoch −284 Dionysios calendar (§C1, 1st reliably reconstructed by Böckh 1863 & van der Waerden 1984-5), which used Kallippos' yearlength Y_{K}), and generically differed little from his calendar, maybe adding embedment of Aristarchos' Great Year (Rawlins 2002A §A4).

E3 Aristarchos' day-epoch-truncated calendaric S.Solstice (Rawlins 1991H eq.8) is reconstructable from Hipparchos' −134 solstice, combined with the *Almajest* 3.1-attested interval. (See Rawlins 1991H eq.8.) Thus, in Table 1, each of the truncated solstices was later used by Hipparchos to find the length of the year, where the truncations contributed to results that were seriously too long — below eqs.32&34 — but (§P7) just about right for matching Metonic preconception's eq.31 (perhaps based on luni-solar politics: §P8).

F Equal-Altitudes: How the Ancients Determined Solstices

F1 As noted at §B2, *DIO* has for decades asserted (against Muffia-MacArthur geniusdum) that ancient solstices were observed via Equal-Altitudes. Understanding the method shouldn't challenge a high-schooler.

F2 Starting d days before the Solstice, as the Sun transits (culminates) at Local Apparent Noon (LAN), the observing astronomer records in degrees and arcminutes[4] the altitude h of the Sun's center (preferably just a few degrees below the h_{SS} of eventual solstitial culmination). This noon will be called t_1. By obvious symmetry, the LAN Sun's altitude will be back near[5] h at d days after Solstice, LAN-culminating at a time which will be called t_2. The midpoint between the two times is then taken as the Solstice-hour t_{MidPt}:

$$t_{\mathrm{MidPt}} = \frac{t_1 + t_2}{2} \tag{5}$$

Obviously, the 2 times' relation to d is (see further at §§G3&§§J1-J2 & eqs.19-21):

$$d = \frac{t_2 - t_1}{2} \tag{6}$$

[Meaning pair-means for several d (e.g., 19^{d}, 20^{d}, 21^{d}) can ensure reliably accurate solst.]

F3 The method is attractively[6] simple. But the Equal-Altitudes Method is subject to small errors (to be quantified below: §§H-I), which had to be carefully accounted for, by any ancient scientist intending to acquire maximally accurate naked-eye results using it.

G Solstice-Observation Technique: Going Beyond Naïve Eq.5

G1 The great accuracy-advantage of solstices vs equinoxes is this: if there is uncertainty in adopted solar parallax, atmospheric refraction, the transit-instrument's mounting or secular settling or arc-ruling-uniformity, then an equinox-timing is corrupted (§B4) by each's systematic error. But not a solstice, since all these errors' effects on t_1 & t_2 are nearly the same while of opposite sign, thus leaving eq.5 unaffected. Yet solstitial determination has its own problems, which are [a] lesser, but [b] serious and (except for random-error problems, which are smaller with equinoxes) completely different from the traditional bothers for equinox-observations.

[4] The ultimate new proof, that Hellenistic scientists had adopted Babylon's sexagesimal measure for angles as early as the 3rd century BC, is found here at ‡1: Archimedes' masked solar diameter brackets.

[5] Astronomers will see that we are merely using h to measure solar declination δ, in order to find two times on either side of Solstice when δ is the same. Generally, finding the exact time t_2 when the 2nd estimate of δ exactly matches (that which occurred at t_1) will require interpolation — since only by rare luck does the post-Solstice h_2 match the earlier noon h_1 almost exactly at noon.

[6] Our present annual version of the technique has a diurnal parallel (fn 2) often used by pre-GPS-era explorers. (Among others: the Isaac Hayes & Rob't Peary expeditions.) Secondary-school classes teach an analogous method for finding when a thrown ball reaches maximum height: §B2.

G2 If the Sun's motion were uniform, it is obvious from symmetry that LAN solar altitudes h_1 & h_2 measured at respective times t_1 & t_2 with an instrument set at constant solar altitude h so that

$$h_2 = h_1 \tag{7}$$

would ensure that the corresponding solar longitudes are each at the same angular distance S from the S.Solst point:

$$90^\circ - \phi_1 = \phi_2 - 90^\circ = S \tag{8}$$

Assuming symmetry, the average of the two times of eq.5 would be exactly the sought quantity: the time $t_{\rm SS}$ of the S.Solst.
G3 What very slightly but aggravatingly upsets the ideal eq.5 situation is the Earth's elliptical orbit. The vital elements were in -157 and thereabouts:

$$\text{Apogee } A = 66^\circ.1 \qquad \text{eccentricity } e = 0.0176 \tag{9}$$

The non-uniform solar motion entailed by the asymmetry[7] of the Sun's elliptic motion causes a *systematic error* that becomes quadratically **larger** (eq.13), the larger the number of days d on either side of the Solstice one chooses to take observations at — even while the process' *random* error becomes **smaller** for greater d (eq.18). So picking the ideal d is a delicate choice (§J), whose pitfalls we now examine. [Note: Many equations to follow here are approximations — though marked as equalities if the roughness is slight.]

H Charybdis

H1 An equation for the asymmetry-caused longitudinal systematic error q, of an Equal-Altitudes-obtained $t_{\rm SS}$, may not have been previously published; so we have derived (and have substituted eq.9 values into) the following simple formula for q as a function of S, the number of longitude-degrees on either side of the solstice one chooses to start & finish at:

$$q = -\frac{\pi e \cos A}{3} S^2 = -0'.0075 S^2 \tag{10}$$

with q in arc-minutes and (again) S in degrees.
H2 We all know the Sun moves about 1°/day, so obviously S is nearly equal to d — near enough for the difference to be largely ignorable here. Nonetheless, we supply useful approximations, relating q to the asymmetry-caused solstice-error H in hours, for solar motion near a Summer Solstice during Hipparchos' era,

$$H = \frac{24 \cdot 365^{\rm d}.2425}{60 \cdot 360^\circ} q/(1 - 2e \sin A) = 0.42q \tag{11}$$

and relating S to d:

$$d = \frac{365^{\rm d}.2425}{360^\circ} S/(1 - 2e \sin A) = 1.05S \tag{12}$$

H3 Combining eqs.10-12 yields our ultimate desired simple practical formula (valid for the range of d that knowledgeable ancients would wish to use) expressing systematic error in hours H as a function of the Equal-Altitudes symmetric (ere&aft) interval d in days:

$$H = \frac{-0.0075 \cdot 0.42}{1.05^2} d^2 = -0.0029 d^2 \tag{13}$$

where the minus-sign reflects that for -157 the error of the Equal-Altitudes Method will cause naïvely (eq.5) deduced $t_{\rm SS}$ to be too-early by H hours.
[Analysis simplified at www.dioi.org/cs.pdf, §E2.]

[7] Things were easy in 1245AD, when the solar apogee arrived at longitude 90°. Had this obtained in Hipparchos' era, our entire discussion of asymmetry here would be superfluous.

I Scylla

We now turn from systematic error to *random error*.
I1 If solar altitude h could be measured perfectly, the foregoing Charybdis section would be a complete error analysis. But the measure of S is from visual determination of altitude h, which can be measured to no better than 1/10000 of a radian (Rawlins 2002B eq.1), called here the Optimal standard-deviation $\sigma_{\rm Opt}$ for human vision — and contrasted with widely-assumed Ordinary visual discrimination (oft apt in-practice: §B4), $\sigma_{\rm Ord} \doteq 1'$:

$$\text{Optimal Discrim } \sigma_{\rm Opt} \doteq 1/10000 \text{ radian} \doteq 1'/3 \qquad \text{Ordinary Discrim } \sigma_{\rm Ord} \doteq 1' \tag{14}$$

Eq.14 causes an uncertainty ($\sigma_{\rm SS}$) in an Equal-Altitudes-determined S.Solst time $t_{\rm SS}$ which requires statistical evaluation. So, to find an accurate $t_{\rm SS}$, we initially need to know how strongly h-uncertainty produces uncertainty in hours of solar motion.
I2 Since LAN solar h and solar declination δ virtually differ by a constant, we start by gauging the statistical relation of longitude ϕ's uncertainty σ_ϕ to h's uncertainty $\sigma_{\rm h}$:

$$\frac{\sigma_{\rm h}}{\sigma_\phi} = \frac{\Delta h}{\Delta \phi} \doteq \frac{\Delta \delta}{\Delta \phi} = \tan\epsilon \sin S \doteq \frac{\pi \cdot \tan\epsilon}{180} S \tag{15}$$

(σ_ϕ & $\sigma_{\rm h}$ in arcmin), where obliquity ϵ was 23°.7 in Hipparchos' era. Also (a statistical parallel to eq.11), we find the effect of σ_ϕ upon observed $t_{\rm SS}$'s uncertainty $\sigma_{\rm SS}$ (in hours):

$$\frac{\sigma_{\rm SS}}{\sigma_\phi} = 0.42/\sqrt{2} \tag{16}$$

where the $\sqrt{2}$ reflects $t_{\rm SS}$'s dependence (eq.5) upon not one but two h measures, averaged.
I3 Combining eqs.15, 16, & 12 establishes standard-deviation ratios:

$$\sigma_{\rm SS} = \frac{0.42}{\sqrt{2}\tan\epsilon\sin S}\sigma_{\rm h} \doteq \frac{180 \cdot 0.42}{S\pi\sqrt{2}\tan\epsilon}\sigma_{\rm h} \doteq \frac{39}{S}\sigma_{\rm h} = \frac{41}{d}\sigma_{\rm h} \tag{17}$$

We note that when $S = 0$ (the Swerdlow-Moment: §B2), uncertainty ($\sigma_{\rm SS}$) in an Equal-Altitude-Method-obtained S.Solst-time is infinite — as it obviously should be.
I4 To evaluate $t_{\rm SS}$'s uncertainty $\sigma_{\rm SS}$ as a function of d for Optimal and Ordinary visual discrimination, we exploit eq.17 by substituting into it eq.14's respective values for $\sigma_{\rm h}$:

$$\text{Optimal } \sigma_{\rm Opt\ SS} \doteq \frac{14}{d} \qquad \text{Ordinary } \sigma_{\rm Ord\ SS} \doteq \frac{41}{d} \tag{18}$$

I5 The above considerations show that accuracy to well within the ancient-cited (*Almajest* 3.1) allowance of $6^{\rm h}$ error was possible, so it should be no surprise that all three of the firm outdoor solstices of Table 3 are accurate within the uncertainty-estimates of the present section: after all, for accurate data correctly rounded to $6^{\rm h}$ precision, the implicit error-range is $\pm 3^{\rm h}$.

J Balance

J1 We next weigh the tricky choice an ancient solstice-observer had to face. If chosen d is too small, he is prey to the quirky Scylla of corruption of his project by random error of indeterminate size and even sign. But if the ancient astronomer over-counters that danger by opting for too-large d, he leans too near Charybdis' tranverse swirl and thus intolerable systematic negative error. (Hartner 1977 & Thurston 2001 cite pre-telescopic observers' d ranging from $45^{\rm d}$ [eq.13: $-1^{\rm d}/4$ syst.error] to $8^{\rm d}$ [$1^{\rm d}/4$ random error for Ordinary eq.18, $2^{\rm h}$ for Optimal].) To estimate an ideal Balanced interval $d_{\rm BAL}$, we can combine eqs.13&18 to ensure that H and $\sigma_{\rm SS}$ are about the same size:

$$\text{Optimal } d_{\rm BAL} = \sqrt[3]{14/0.0029} \doteq 17^{\rm d} \qquad \text{Ordinary } d_{\rm BAL} = \sqrt[3]{41/0.0029} \doteq 24^{\rm d} \tag{19}$$

But we must not forget that: [1] The two errors (eqs.13&18) are of quite different type. [2] An ancient scientist would instinctively sense eq.18. [3] There's no evidence that any ancient (or modern?) knew of eq.13; if he had, he'd have compensated, either by correcting for it (thus positive errors in Table 3?) or suppressing its effect via modest-sized d.

J2 Eq.19 indicates that 20^{d} is about the best choice for d. Substituting into eq.18:

$$\text{Optimal } \sigma_{\mathrm{SS}} \doteq 0^{\mathrm{h}}.7 \qquad \text{Ordinary } \sigma_{\mathrm{SS}} \doteq 2^{\mathrm{h}} \tag{20}$$

Neither creates a problem. And eq.13 gives for $d = 20^{\mathrm{d}}$ a systematic error:

$$H \doteq -1^{\mathrm{h}}1/6 \tag{21}$$

Virtually negligible, and (since we listened to eq.19) roughly equal to eq.20's random errors Indicated net accuracy by combined effect of eqs.13&18 is easily within, indeed, a good deal better than, the $1^{\mathrm{d}}/4$ outer error-possibility (§I5) Hipparchos cites. This grants assurance the 3 firm solstices of Table 3 are validly non-accidental in their nearness to reality.
J3 Kallippos' non-trivial positive error ($+3^{\mathrm{h}}$) has the wrong sign (§H3) for serious systematic error, so he may have used $d \doteq 10^{\mathrm{d}}$ when measuring the -329 S.Solst, leaving him open to ordmag 1^{h} of random error. But it is hard to tell, since $1^{\mathrm{d}}/4$ rounding obscured the exact hour measured. It was natural for Kallippos to round thusly since Greek calendars always started on a quarter-day mark. This particular observation was superlatively calendaric, in that the S.Solst occurred closer to the New Moon (within ordmag 1^{h}) than any S.Solst for ordmag a century, which is presumably why Kallippos chose this moment to launch his famous *luni-solar* calendar. Also, the most recent eclipse (exceptionally cited by Pliny 2.72 & *GD* 1.4.2) visible to Kallippos was the $-330/9/20$ Gaugamela (Arbela) eclipse, the fame of which he — as Alexander's astronomer — may have enhanced.
J4 The time-interval, between Meton's truncated (Table 1) -431 dusk S.Solst and Kallippos' dawn S.Solstice turned out to be exactly divisible by $365^{\mathrm{d}}1/4$ (eq.1). Thus, Kallippos' calendar — evidently due to a truncation that caused a huge -17^{h} error (Table 1) in the earlier datum! — became the earliest known to have used the $365^{\mathrm{d}}1/4$ year (§C1). And, again, Kallippos was the 1st astronomical calendarist to (crucially) start a still-extant calendar without rounding his contemporary founding-S.Solst to conventional day-epoch (which we know he didn't do, because the day interval from Meton's day-epoch isn't integral: §C1), properly starting it instead at what he estimated to be the nearest quarter-day point, 6^{h} in the morn, his decision reinforced by that hour's proximity to an extremely rare close-confluence (§J3) of S.Solstice & New Moon (the latter nearer dawn than midnight). [Greeks defining New Moon by true-longitudinal syzygy, rather than following Babylon's crude First-Visibility definition, indicates who was ahead in math astronomy by 330 BC.]
J5 The error of Hipparchos' -134 S.Solst (eq.4) is trivially negative (Table 3), as it should be; though, again, $1^{\mathrm{d}}/4$ rounding muddies our evaluation. Regardless, we can say that Hipparchos achieved the most[8] accurate of all surviving ancient solstices.
(Unless we count the phantom solstice of $-157/6/26$: §M4 & Table 2.)

K Hipparchos' $-157/6/28$ Dawn Summer Solstice

[Thanks to *DIO* refereeing, albeit (uncharacteristically) late in this case, §§K-P have been rethought, recalculated, & rewritten (2018 Winter): prior mistakes fixed & new finds added.]
K1 Hipparchos c.-157 was using past records of eclipse-times to start building his famous 600^{y} eclipse canon (§M2; Rawlins 1991W §M7), a list which included Hipparchos-computed solar longitudes ϕ for each eclipse's historically known time. Later, these ϕ were brought in when he analysed eclipse-trios. In Rawlins 1991W §K9, we found that his math-analysis of eclipse-Trio B (‡3 fn 5) used ϕ computed (for each eclipse-time) from what we dubbed his "EH Orbit" (founded -157), which was afflicted with terrible apogee $A = 44°$ & eccentricity $e = 3^{\mathrm{p}}1/4$, by taking (Rawlins 1991W §K8) EH's S.Solst — via indoor math — from Kallippos' $-329/6/28$-epoch calendar (accumulated error $+1^{\mathrm{d}}.3$ in the 172^{y} interim), due to its over-long year-length, Y_{K} (§C1). From §C1 & eq.1 (see Tables 2&3):

$$\text{EH Summer Solst} = -329/6/28\ 1/4 + 172 \cdot 365^{\mathrm{d}}1/4 = -157/6/28\ 1/4 \tag{22}$$

[8] Again: this may merely be due to the actual -134 S.Solst being accidentally closer to a quarter-day mark than those of -329 & -146. See fn 22.

K2 Relative to the present analysis, the key point to notice is this: *Hipparchos in -157 would not have computed a solstice from a predecessor's calendar unless he didn't yet know how to observe a solstice reliably*. (The poetic irony here is that before his career was done, Hipparchos left us [§J5] THE most accurate outdoor-observed solstice that survives from antiquity, the error in which is merely about an hour. See Table 3.)
K3 But if the young Hipparchos needed to resort to an earlier astronomer's calendar to obtain his $-157/6/28$ dawn solstice (eq.22, used for constructing his EH solar orbit of -157: §K1 or Rawlins 1991W §§K8-K9), then where did his newly discovered (§A) $-157/6/26$ solstice come from? And when? Rigid impediment to casual thinking hereabouts: his calculational use of the EH orbit's tables as late as -145 (Rawlins 1991W §§M4-M6) shows that the 6/26 replacement-improvement *was not adopted immediately*.
K4 Duke's idea (people.scs.fsu.edu/~dduke/Duke-Neugebauer-2.pdf) that the $-157/6/26$ solstice was empirically determined to have occurred at 21^{h}, seems to be based upon his perceptive recognition of Meton's fingerprint: multiplying the papyrus' tropical yearlength (convincingly extracted from it by Tihon 2010 p.5)

$$Y_{\mathrm{p}} = 365^{\mathrm{d}}1/4 - 1^{\mathrm{d}}/309 = 365^{\mathrm{d}}.24676 \tag{23}$$

times the 274^{y} gap since Meton, and adding the product to Meton's S.Solst, as misunderstood by Hipparchos & Ptolemy (§C1), produces:

$$\text{S.Solst} = -431/6/27\ 1/4 + 274 \cdot Y_{\mathrm{p}} = -157/6/26\ 20^{\mathrm{h}}43^{\mathrm{m}} \doteq -157/6/26\ 21^{\mathrm{h}} \tag{24}$$

However: [a] All known Hipparchos cardinal point data are rounded to the quarter-day. [b] In reverse, eq.24's 21^{h} time produces yearlength about $365^{\mathrm{d}}1/4 - 1^{\mathrm{d}}/313$, not $1^{\mathrm{d}}/309$, and so doesn't solve eq.23's origin. [c] In -157, Hipparchos wasn't yet (§L4) sky-observing at a level likely to find an accurate solstice such as that proposed. [d] The papyrus says that the $-157/6/26$ solstice occurred at an unknown number of hours of the *day* not night.
K5 Potential resolution of [a]-[d]: if the papyrus said "12 hours of the day" (18^{h} or 6 PM), that would make the gap from Meton ($-431/6/27$ 1/4) to Hipparchos ($-157/6/26$ 3/4) equal to $100077^{\mathrm{d}}1/2$. But the ancient scholar who created eq.23 could have accounted for seasonal hours' solstitial day-lengthening, taking $14^{\mathrm{h}}3/4$ as the nearest klima (of *Almajest* 2.6's traditions) to a mean between Athens' & Nicaea's *GD* Book 8 longest days (Diller 1984):

$$Y_{\mathrm{s}} = (100077^{\mathrm{d}} + 14^{\mathrm{h}}3/4)/274 \doteq 365^{\mathrm{d}}1/4 - 1^{\mathrm{d}}/309 \tag{25}$$

(We here assume early Hipparchos didn't know of or ignored small longitude differences.) Had the $14^{\mathrm{h}}5/8$ Athens klima (*GD* 3.15.22) been used, the remainder would've been $-1^{\mathrm{d}}/308^{\mathrm{y}}$, perhaps an alternate value, as suggested by the *P.Fouad* 267A left column's remainder $+3^{\mathrm{d}}/308^{\mathrm{y}}$ (Tihon 2010 p.7). Either way *Fouad* Hipparchan precession appears (but note fn 16) exactly or nearly $4^{\mathrm{d}}/308^{\mathrm{y}} = 1^{\mathrm{d}}/77^{\mathrm{y}} \doteq$ (*ibid* pp.6-7) $1°/78^{\mathrm{y}}$ (vs actual $1°/72^{\mathrm{y}}$ then), hinted at *Almajest* 7.2 ("not less") but not explicitly relayed there (& a better figure than Ptolemy's $1°/100^{\mathrm{y}}$). [Or reverse? Prior $1^{\mathrm{d}}/77^{\mathrm{y}}$ precession estimate times $4 \rightarrow 308^{\mathrm{y}}$?]

L When Was the $-157/6/26$ 3/4 Solstice Observed?

L1 The seemingly odd title of this section is not meant facetiously. (Though it puts one in mind of humor at the level of what-was-the-color-of-George-Washington's-white-horse?) It is deliberate — because we are faced with a weird contradiction, *two different dates for the same event*, the -157 S.Solst: $-157/6/28$ 1/4 (§L2) vs $-157/6/26$ 3/4 (papyrus: §K5).
L2 Rawlins 1991W[9] (see §K1 above) has shown that Hipparchos' eclipse-trios A&B cannot closely enough fit *Almajest* 4.11's intervals for a solar eccentricity less than 3^{p}. (And

[9] Parts of Rawlins 1991W are written in an anti-tyrannical spirit which is bound to offend anyone unfamiliar with the cult that has for decades financially puppetized most of the history of ancient astronomy community, to its tragic cost in competence, refereeing, neutrality, and most importantly: valid history. If the History of Science Society can (fn 10) wince and stomach DR's idiosyncratic writing style [once upon a time! — see Rawlins 2018A], *in order to get at the truth* of Hipparchos' early observations & lunisolar elements, then fair-minded individual investigators ought to be able to manage same — for *P.Fouad* 267A will never be understood if §§K-O are discounted.

Trio B can't fit an apogee A above 50°. Both limits are grossly discrepant vs the standard *Almajest* PH orbit's $e = 2^{\mathrm{p}}1/2$, $A = 65°1/2$.) Rawlins 1991W found that this clash is neatly accounted-for by a huge error (over 1^{d}!) in solstice — and that the EH orbit satisfying[10] this glaring oddity is also consistent (like no later A.Eqx) with the quite erroneous −157 Autumn Equinox (off by 11^{h}, nearly half a day!) reported at *Almajest* 3.1 and is consistent with a −157 solstice at 6/28 1/4 (§L1; Table 2), exactly[11] where the Kallippic calendar has it. EH was used by Hipparchos until his adoption of the later-canonical PH orbit in −145, when EH's rôle in Trio A's Frankenstein-orbit solution proves Hipparchos anchored at S.Solst −157/6/28 1/4 right up until −145, not at the *Fouad* papyrus' −157/6/26 3/4.

L3 If *Fouad*'s −157/6/26 solst were outdoor-observed, why was its adoption and use (in founding PH) delayed until 12^{y} later? Rawlins 1991W §M reveals a lucky glimpse of Hipparchos' −145 solar math, as he semi-shifted into upcoming full adoption of PH. **NB**: *ibid* §M6 *must be read with care to appreciate the non-random split in Frankenstein orbit* (half its elements are EH; other half, PH) which Hipparchos used for computing Trio A, and ***the obvious explanation of this striking bifurcation***: he of course temporarily retained the tabular EH elements, since the 2 necessary PH tables would take awhile to create (eventual results: *Almajest* 3.2&6), while swiftly adopting non-tabular PH elements (constants A&ϵ_{o}), as carefully explained at Rawlins *loc cit*: when computing eclipse Trio A in −145 early Spring (§N5), Hipparchos ***was in mid-transition from EH to PH***.

L4 *Ibid* §§K2-3 noted several symptoms of roughness in Hipparchos' work from −161 to −157, and wondered if earlier he was yet even using vertical instruments, before[12] he by −146 gained or hired the scientific skills that ultimately made him justly famous. E.g., the *GD* latitudes of the sites near his Bithynian origins, Nicaea & Byzantion, latitudes which are too high by 1°1/2 & 2°, resp (Diller 1984 Table 15): astonishingly large errors, impossible for any transit instrument, esp. that which Hipparchos used from −146 on. (Rawlins *op cit* §K2 airs the possibility the errors arose not from vertical instruments but from use of horizon phenomena such as ortive amplitudes.) These latitudes almost certainly came into Ptolemy's *GD* from Hipparchos of Nicaea, who thus could not have been doing serious astronomy ere departing Bithynia (ultimately arriving at Rhodos by −146 at the latest). Again: if Hipparchos in −157 was fully conversant with instrumental astronomy, why did he need to indoor-obtain a −157 solst from Kallippos' calendar and use the resultant EH orbit (in whole or part) for years[13] (−157 to −145) to compute eclipses such as Trios A&B? The S.Solst used for final computation of the PH orbit was −145/6/26 3/4 (whether observed or extrapolated from observed −146/6/26 1/2), when the EH orbit gave way to PH — as we see from multiple coherent indicia: §L3, consecutive-triplet orbit-base (fn 13), −145 V.Eqx's capper PH-rôle (*idem*), and Physkon's −145 accession (§O3). Extra hint: eq.26's ultra(excessive!?)neatness. (Also: −145 minus −157 = $12^{\mathrm{y}} = 0^{\mathrm{y}}$ mod 4^{y}).

L5 From the −146/6/26 1/2 S.Solst, Hipparchos need only go back 11 Kallippic years, to create the "observed" −157/6/26 3/4 S.Solst of his *P.Fouad* 267A tables & could've even more easily extrapolated 1^{y} ahead to ensure a −145/6/26 3/4 S.Solst (if not confirmed by year-later outdoor re-observation) for establishing his ultimately canonical-regnal −145 PH orbit (for §L4). Pseudo-observed solstice-hours Kallippically extrapolated from his −146 solstice-hour for −157 & −145 would differ acceptably little from extrapolations based on Hipparchos' yearlength (eq.23 or eq.32): 53^{m} & 5^{m}, resp.

L6 As seen at Rawlins 1991W §§K4&8, Hipparchos was in −157 searching for a S.Solst not by outdoor observation but by indoor calculation. Which tells us that he at this time didn't know how to measure a solstice, nor even how to choose an expert who did. Perhaps it was just convenient (‡3 fn 6) to stick with the increasingly inaccurate Kallippic calendar, revered as that (too)long-standard among astrologers, most of whom ignore the outdoor sky, Hipparchos later becoming the 1st known major exception. This discussion occasions our tabulation of the indoor solstices we have from antiquity (Table 2, chronologically ordered according to date of creation), including Ptolemy's well-known 140 AD fraud at *Almajest* 3.1. The papyrus' Hipparchos solstice (2nd in Table 2) is only technically an indoor observation, as noted at fn 17: the accuracy of its outdoor procreator, the −146/6/26 1/2 S.Solstice (§M4), transferred faithfully (§L5) to the −157/6/26 3/4 extrapolation.

M Solving the −157 Double-Solstice Mystery

M1 The ultimate implication of the foregoing is weird but simultaneously satisfies the various[14] above-enumerated evidential features: following Hipparchos' outdoor capture of the −146/6/26 1/2 solstice, the papyrus' −157/6/26 3/4 solst was extrapolated from it

[10] Having *once* (*DIO 11.2* cover) been righter than *one* part of *one* (*non-DIO*) DR paper, bloodtasting Jones&Duke have for 10^{y} been on a knowledge-subtractive mission to trash (in unrefereed forums) as many *DIO* discoveries as possible by ***altering or condemning each's data-base*** (i.e., attacking Strabo, *Almajest*, etc, since DR's invulnerable math keeps leaving no other choice for fanatics kill-bent on denigration) *while for all 6 cases noncitng the very (reffed) DIO papers targetted*. Hideous details at Rawlins 2018A §§C-G: ***DO NOT MISS***. Similar 7th case: Rawlins 2009S fnn 54-55. Excepting one klima (*ibid* eq.3), the 6 data-sets DR used were standard. ***Until DR solved them***. [1] Duke 2005T pp.170f nakedly (‡3 §§K1&K4) alters Toomer 1973's work to rig matches to *Almajest* 4.11 data which Rawlins 1991W matched tamplessly. [2] The 14 Strabo klimata data, *perfectly-fit honestly* by Diller-DR (Rawlins 2009S Table 2), destroyer Jones 2002E alters by 100 stades but, *uniquely for this flap, displays no table*: such would show his insane (Rawlins 2009S §B6 & fn 55) theories don't fit ***even his own fudged data***. [3] Via cont'd fractions, Rawlins 1999 eqs.3&9 connected Vat. gr. 381's Aristarchan entry to 152^{y}, while [4] Rawlins 2002A eqs.12&13 connected Vat. gr. 191's Aristarchan entry to 4868^{y}, both known Aristarchan intervals. Jones 2010A (p.21 & n.27) reacts by forgery, deleting all accents so no solution is possible (sterility) & non-cites DR, though Jones read *DIO 9.1* on 1999/7/14. [5] Rawlins 1991H & [6] Rawlins 1991W undeniably-accurately recovered 3 Hipparchan orbits' elements from *Almajest* data; so, incredibly, Duke 2008W complains (‡3 fn 8) that, if (like [1] above) he rigs things, the results are too darned sensitive to his proposed *Almajest* re-writes! I.e., he couldn't (§N3) find non-*DIO* elements satisfying the *Almajest* 4.11&5.3&5 data WITH $1^{\mathrm{d}}/4$-rounded times for cardinal-pt ϕ, as valid Hipparchan orbits must, & as the UH, PH, & EH orbits all do. (On the evidence of ‡3 fn 22, Duke rummaged hard to find such.) More sterility. Comments: [a] Don't Duke-Jones know data-tampering is improper? [b] Ever heard of a *single* other case of data-trashing to refute heresy? Much less SEVEN cases, all aimed at the same #1 *JHA*-hate-object? Rawlins 1991W's *double* EH orbit inductions (Trio A & Trio B each lead us to EH) were math-checked & backed by Britton, Thurston 2002S pp.66-67 (Hist.Sci.Soc.) & Curtis Wilson (letter 1994/12/29). One would never know from *JHA*-HAD (JHAD) noncitation that world-class experts encouraged EH. The shocking & unique intercultural implication of eq.4 has long been displayed at the British Museum and accepted by, e.g., Dicks 1994 fn 37, Britton, Thurston 2002S p.62, even Jones 2005 (non-*JHA*) pp.23-24. Outside low-end JHAD, Duke 2008W Table 1 is alone in rejecting eq.4, mis-stating *Almajest* 3.1 contradicts it.

[11] This is not an *ad hoc* adjustment: Rawlins 1985H's novel finding that Kallippos' calendar-founding solstice was at dawn (−329/6/28 1/4) occurred 6^{y} before discovery of the EH orbit using it. Just one more of the many vindications (http://www.dioi.org/vin.htm) of *DIO* findings which keep exasperating our toadily-awesome phalanx of maid-boy *DIO*-assassin-wannabees.

[12] The huge errors in Hipparchos' early Bithynian data (§§L2&L4) could've been from use of ortive amplitudes and-or a mere gnomon (Rawlins 1991W ffn 186&195). Solar altitude errors, if steady, could allow an accurate solstice (§G1), but their sheer size (half a day!), and the proximity of their mean (12′) to the 16′ error characteristic of an asymmetric gnomon, suggest sufficient crudity as to cast doubt (independent of §L3) on whether he got an accurate outdoor S.Solstice ere Rhodos-arrival.

[13] No Hipparchos orbit until PH gibes with *Fouad*'s −157/6/26 3/4 S.Solst. ***But the PH orbit could not exist*** until the −145 V.Eqx. An orbit's 3 required empirical cardinal-pt bases were best arranged consecutively, and no Hipparchos Winter Solst was used for orbits. (Just for finding obliquity & latitude, as also 100^{y} earlier: Rawlins 1982G.) So the V.Eqx-S.Solst-A.Eqx triplet producing the final PH orbit used −145/6/26 3/4.

[14] Tihon 2010 p.7 proves col.3 adopted $-1^{\mathrm{d}}/309$. Col.3's −657-epoch table was completed (−145) *before* computation of his then-still-incompletely-calculated eventual PH f-table, which (eq.32) rounded to $-1^{\mathrm{d}}/300$ and used ϵ_{o} exactly fitting *Alm* 3.2 (§N1 item [5]) via the same PH yearlength. Did young Hipparchos use (§M2) epoch Phil 1 (−323) for astronomy while adopting epoch −657 for his astrological manual? — only later finally expanding back c.600^{E} from his time to Nab 1 for PH's f-table, which *effectively went back c.1200$^{\mathrm{y}}$ to c.−1350* for early eclipses: www.dioi.org/thr.htm#rbkv.

simply by subtracting 11^{y} of motion. (Thus replacing the awful EH solst, $-157/6/28$ 1/4, in future editions of his horoscopic publications, such as the material used by the *P.Fouad* 267A astrologer.) Moreover, Tihon 2010 (p.2) found that (along with parallel columns for sidereal & "tropical" longitudes) *the papyrus' ephemeris retains a Kallippically-computed column of solar longitudes* (at quarter-century intervals) — startlingly consistent with Rawlins 1991W §K's proposals that [i] EH's mean solar motion was Kallippic and [ii] EH's foundation S.Solst was -157. Tihon discovered from the papyrus that its practical epoch[15] was $-657/2/4$ (Nab 90 Thoth 1), running 500^{E} (Egyptian years of 365^{d} each) and (like Ptolemy's *Handy Tables* & *Almajest* 6.3) at 25^{E} per line. Given the *Fouad*-astrologer's addition of mean solar motion for 21^{h} to an integral number of days from epoch, we know (since his horoscope is for 3 AM) his $-657/2/4$ epoch was 6^{h}.

M2 From these findings & his ultimate immortality (& *Fouad*'s citing "nativity" as its calculational purpose), we can guess Hipparchos had published an internationally popular, profitably-multiple-tradition astrological manual in -157, including a purely Kallippic table for mean solar longitude, eventually going 500^{E} into the past and perhaps 100^{E} more into the future: 600^{E} in all, possibly [vs §K1] the basis for Pliny 2.8.53's reference to Hipparchos' 600^{y} of calculations. The curious failure of the papyrus' (pre-*Almajest*) astrologer to cite any work later than -157 may indicate that Hipparchos' mature researches were more scientific than popular and were primarily intended for an astronomical not astrological audience. (Financed by selling horoscopes & manuals for? And-or gov't support?) When in -146 he realized how wrong the EH orbit's solstice was, he appended at least the column of mean solar longitudes based upon Metonic Y_{p} (eq.23). We may compute the Kallippic column's ϵ_{o} by working backwards from Kallippos' epoch (eq.1), when true solar longitude $\phi = 90°$ at $-329/6/28$ 1/4, which (by PH's $e\&A$) is when mean solar longitude $f = 90°59'$. Result: the papyrus' middle (Kallippic) column's mean-longitude-at-epoch for $-657/2/4$ 6^{h} (§M1) was solar $\epsilon_{\mathrm{o}} = 309°03'$ (vs actually $306°.7$).

M3 *Fouad* bears 3 columns of computed ϕ: [1] left,[16] [2] Kallippic or "mean" (middle), [3] Metonic "tropical" (right). The last is PH (but for eq.23's yearlength): we revolve back (again from $f = 90°59'$) for the $182767^{\mathrm{d}}1/2$ from $-157/6/26$ 3/4 to $-657/2/4$ 1/4, finding $\epsilon_{\mathrm{o}} = 308°56'$ ($49'$ for $14^{\mathrm{h}}3/4$ klima). (This & §M2 rounded to $\epsilon_{\mathrm{o}} = 309°$ for computing?)

M4 Finally, in answer to this section's semi-facetious titular question: the "$-157/6/26$" solstice was truly[17] outdoor-observed by Hipparchos at $-146/6/26$ 1/2 & then — to replace his erroneous indoor epochal $-157/6/28$ 1/4 solstice — he Kallippically-*reconstituted*[18] it back at $-157/6/26$ 3/4, with but tiny concomitant error (§L5; Table 2) as he was fully aware. So, was Pliny 2.5.27 wrong in claiming that not even god can change the past?

N Statistical Impregnability of the $-157/6/28$ 1/4 Solstice's Adoption

N1 To understand what *DIO* has accomplished here regarding Hipparchan solar theory, let us catalog the ***FIVE types of fits simultaneously achieved*** at Rawlins 1991W §§K&M:

[15] *Almajest* 3.1 shows that Hipparchos' solar observations were dated according to the number of years after "the death of Alexander" or equivalently epoch Phil 1, the ascension of Philip III: §O3.

[16] The $365^{\mathrm{d}}1/4 + 1^{\mathrm{d}}/(102\ 2/3)$ yearlength was far closer to the real anomalistic year (remainder: $+1^{\mathrm{d}}/102$) than sidereal. Left-column yearlength is consistent with remainder $-7°1/2$ in ancients' key 345^{y} equation (Rawlins 1996C §C: implicit yearlength $\doteq 365^{\mathrm{d}} + 1^{\mathrm{d}}/100$), used in *Almajest* 4.2 to find ‡1 eq.4. *Fouad* not explicit whether left-column is for sidereal (Tihon 2010 p.6) or apsidal precession.

[17] The $-157/6/26$ 3/4 solstice is not at all a fabrication. Hipparchos knew that extrapolating the $-146/6/26$ 1/2 solstice to produce it would yield a datum differing but ordmag 1^{h} from the truth if his -146 observation was accurate. Hipparchos no more thought of extrapolation-reconstructing it as dishonest than he thought it a trick to find a solst by eq.5's interpolation. Neither resulting datum is a direct observation, but the procedure is scientifically proper and justifiable in both cases.

[18] One may hypothesize the reverse: indoor -146 solstice reconstituted from outdoor -157 solstice. But, aside from the question (§L4) of Hipparchos' crude instruments in -157: was getting-rich (fn 20) why he waited 12^{y} before adopting (fn 13) the $-145/6/26$ 18^{h} S.Solstice to found his PH tables?

[1] The -157 EH orbit fits the usual 3 cardinal pts, which are either attested (-157 A.Eqx) as Hipparchos' or extrapolatable (krikos -157 V.Eqx) from data cited by him, or taken exactly (-157 S.Solst) from a famous calendar repeatedly used by him (*Almajest* 3&5-7).
[2] A valid EH orbit must (within c.$1'$: *ibid* §K11) place the solar true longitude ϕ at $00°$ for [1]'s V.Eqx, at $90°$ for [1]'s S.Solst, at $180°$ for [1]'s A.Eqx *all at the* $1^{\mathrm{d}}/4$ *precision Hipparchos always uses* (*Almajest* 3.1). The PH orbit is already known (Neugebauer 1975 p.58) to satisfy such a like condition. See statistical exploitation of this point at §N3.
[3] For each trio, the proposed orbit must fit *Almajest* 4.11's true longitude ϕ-intervals:
The EH orbit (Rawlins 1991W §K) fits both Trio B intervals within ordmag $1'$.
The EH-PH-meld Frankenstein orbit (*ibid* §M) fits both Trio A intervals within ordmag $1'$.
[4] The solar mean motion must be reasonable, not conjured-up at convenience. Trios A&B both fit the already famous Kallippic solar motion. Additionally, thanks to Anne Tihon, we now have (for the 1st time) *direct* evidence that Hipparchos used Kallippic motion (it's right at *P.Fouad* 267A's middle column), as earlier 1st hypothesized at Rawlins 1991W §§K&M.
[5] Each ϵ_{o} must be convincing. For Kallippic solar speed, Trio B's strikingly integral Phil 1 ϵ_{o} ($228°00'$) exactly fits the Kallippic calendar's founding S.Solst, $-329/6/28$ 1/4. The Frankenstein orbit's $\epsilon_{\mathrm{o}} = 227°2/3$ exactly fits (via PH's $365^{\mathrm{d}}1/4 - 1^{\mathrm{d}}/300$: eq.32) the Nab 1 Thoth 1 $\epsilon_{\mathrm{o}} = 330°3/4$ of the standard *Almajest* 3.2 solar mean motion table built upon the PH orbit which supplies the *constants*-half of the hybrid (§L3) Frankenstein orbit's elements ($A = 65°$ & $\epsilon_{\mathrm{o}} = 227°2/3$), the other half being the *tabular* elements of the EH orbit ($e = 3^{\mathrm{p}}1/4$ & $Y_{\mathrm{K}} = 365^{\mathrm{d}}1/4$), a dichotomy explained here at §L3 & ‡3 §E3.

N2 In the effectively unrefereed[19] *JHA*, one of its board members volunteers to be the sole critic publicly rejecting the EH orbit, contending (fn 10) that *DIO*'s analysis has not established unique multi-fits. *He can prove his contention anytime by coming up with alternate orbits (distinct from ours) which also neatly satisfy §N1's five conditions.*
[As of 2018, our challenge has not been met. And never will be.]

N3 A Funny Thing Happened on the Way to the Dumpster. Our cynosure Muffia's wisdom has decreed (‡3 fn 8) that anything gotten from the allegedly loose *Almajest* 4.11 eclipse trio-intervals is worthless. When crazy Rawlins 1991W claimed *previously unheard-of* Hipparchan use of [1] a -157 S.Solst & [2] Kallippic solar motion, could be extracted from the eclipse intervals, all right-thinking JHADists knew better: just throw out all that *DIO* junk. But then, over 10^{y} later, a funny thing happened: the miraculous, 1-in-a-million finding of an ancient papyrus on the subject. And — (*don't tell anybody*) you know what it said? *It testified that Hipparchos had used [1] a -157 Summer Solst & [2] Kallippic solar motion.*

N4 Even funnier: an unbigoted community wouldn't need papyrus-confirmation. Check the several speed-bumps that should've slowed *JHA* ere reaching the Orwellian dumpster it would burn vital research in: [a] The coincidence that *both* Worthless trios happened to mutually confirm Hipparchos' use of Kallippic motion. [b] The *a priori* improbability of so many fits *accidentally* flowing from Rawlins 1991W §K's spare premises is obvious. [c] Contra *JHA*'s careless contempt (§N2) for scrupulous and multi-expert-checked (fn 9) research, *ibid* fn 205 showed in *JHA*-uncited detail that *Almajest* 4.11's intervals set narrow limits on elements. [d] If *JHA* is right, then the fact, that our EH orbit (mathematically-consistent with *Almajest* 4.11's data) fits on-the-nose all 3 quarter-day cardinal-points (§N2 condition [2]), ***is just pure-coincidental luck***. How lucky? Of Rawlins 1991W's three EH cardinal points (Hipparchan orbit-math avoided W.Solstices: fn 13), all occur within $\pm 20^{\mathrm{m}}$ (solar motion under $1'$) of a quarter-day marker (00^{h}, 06^{h}, 12^{h}, or 18^{h}): $\phi = 00°$ at $-157/3/24$ 11:41, $90°$ at $-157/6/28$ 6:06, $180°$ at $-157/9/27$ 11:42. The chance probability of 3 hits within 20^{m} is 1 in nearly 8000. Note: the universally-accepted *Almajest* PH orbit fits much less well (than EH): V.Eqx at $-145/3/24$ 5:13; or 5:26, for $A = 65°$.

[19] See www.dioi.org/jha.htm#pdfr. *JHA* Pb (!) paper Duke 2008W displays not only the mistake just analysed but 3 others equally obvious to nonzombie refereeing: 2 at §B4 & 1 more at fn 10's final line.

N5 Until -145 Hipparchos was demonstrably (§L3) using the EH orbit and thus the co-foundational Kallippic 6/28 1/4 S.Solstice consistent with it — ***but not at all with 6/26 3/4***. The historic $1^{\rm st}$ PH calculation, narrowly datable (§O3) to the 4 weeks between -145/3/24 V.Eqx & -145/4/21 eclipse, presaged the centuries-durable PH orbit through altering (vs the EH orbit) V.Eqx by $-1^{\rm d}/4$, S.Solst by $-1^{\rm d}1/2$, A.Eqx by $-1^{\rm d}/4$.

O Recovering Hipparchos' Lost -146 Solstice

O1 From -145 to -134, Hipparchos' mainstay solar orbit was PH, later appropriated (essentially unaltered) by Ptolemy (*Almajest* 3.2&6), standard among astrologers for centuries, cited as "perfect" by Julian the Apostate (1:429), $500^{\rm y}$ later, though by then differing from reality by $2^{\rm d}$ or $2°$! Rawlins 1991H §§C-D showed that in -134 Hipparchos abandoned PH and adopted the superior UH orbit. But the question that has never previously been answered (or even asked) is: whence came the S.Solst needed for the PH orbit?
O2 The only 2 years *Almajest* 3.1's Hipparchos cardinal-point data lists both equinoxes: -146 AE to -145 AE & -142 VE to -141 VE, the latter barred by its A.Eqx's discord with the PH orbit. His $1^{\rm st}$ outdoor-observed S.Solst cannot be -145/6/26 3/4 since Hipparchos used (Rawlins 2009E §B5) the PH orbit months earlier to place the mid-eclipsed Moon (-145/4/21), so the -146/6/26 1/2 S.Solst was his $1^{\rm st}$ Rhodos sky-record.
O3 *Almajest* 3.1's collection of Hipparchan cardinal-point observations cites only Autumn[20] Equinoxes before his -145/3/24 V.Eqx capped capture (§O2) of his $3^{\rm rd}$ Rhodos solar cardinal-pt data, of the three needed to compute (like Neugebauer 1975 pp.58-60) his PH orbit, just in time to figure mid-eclipse for his -145/4/21 measure of Spica's place (*Almajest* 3.1). [Added 2018/2/10. The timing suggests: did he move to Rhodos for its good weather just before the -146 S.Solst, partly to ensure that he wouldn't miss measuring the -145 eclipse?] *Almajest*'s PH orbit (epoch Phil 1 Thoth 1 = -323/11/12 Alex App Noon; elements at Rawlins 1991W §K10) gives solar true longitude $\phi_{\rm AE}$ for his -145/9/27 1/4 A.Eqx, only $2^{\rm d}1/4$ before the regnal epoch Ptolemy Physkon 1 Thoth 1 (Toomer 1984 pp.11&133), with PH mean anomaly $g = 116°2/3$ (*Almajest* 3.7; Neugebauer 1975 p.59):

$$\phi_{\rm AE} = 227°2/3 + \frac{360° \cdot 64967^{\rm d}3/4}{Y_{\rm H}} - \arctan\frac{\sin 116°2/3}{24 + \cos 116°2/3} = 180°00'00'' \quad (26)$$

It appears[21] that the PH solar mean-longitude-at-epoch (same as Ptolemy's at *Almajest* 3.2) $\epsilon_{\rm o} = 227°2/3$ was set by Hipparchos to ensure the exactitude of eq.26, consistent with the PH orbit's launch upon -145 V.Eqx's capture. So we have traced the A.Eqx-origin of Ptolemy's hitherto-unexplained Nabonassar 1 Thoth 1 $\epsilon_{\rm o} = 330°45'$ (*Almajest* 3.2&7), $424^{\rm E}$ prior to Hipparchos' Phil 1 epoch. Hipparchos thus gave calendaric priority to the A.Eqx (fn 20). Anyway, it's obvious that -146 SS to -145 AE (§O2) was the period of the

[20] Or did Hipparchos have an unusual calendaric interest in the Autumn Equinox, since it was near the Egyptian calendar's start (Thoth 1) in his era? During the year of his UH-founding -134 S.Solst (eq.4), his -134/9/24 A.Eqx occurred smack-on Thoth 1 (Rawlins 1991H fn 14); and his UH solar mean longitude $180°$ occurred at $10^{\rm h}$ on Thoth 1 during the UH orbit's -127/9/24 epoch-day (*ibid* eq.28). During the $11^{\rm y}$ gap 'twixt Hipparchos' -157 & -146 observations, did astrological tables' (Tihon 2010) sales make him rich enough to return to creativity (Rachmaninov [www.dioi.org/rar.htm] parallel 1917-1926)? — moving to clear-skied Rhodos, to facilitate fulfilling a dream of founding astronomy empirically.

[21] Note: -145 S.Solst proposed for ultimate PH orbit on thin evidence as early as Rawlins 1985H. And see Rawlins 1991W §M6, where it is also noted that -145 was ***a regnal year***, Ptolemy VII Physkon's. See Rawlins 1991H fn 7 for Physkon 1 Thoth 1, which usefully clinches -145 as PH's epoch, crucially since eq.26 adjusted for other nearby years would be nearly as well-fitting for A.Eqx; -145's V.Eqx is $+1'.9$ off & S.Solst off $+0'.7$. (Rawlins 1991W §M4's best Frankenstein-orbit fit was for $A = 65°$, but $A = 65°1/2$ fits Trio A's data nearly as well & it's the Hipparchan apogee preserved even centuries later at *Almajest* 3.7: Neugebauer 1975 pp.58f. The superb analysis of van Dalen 1994 showed that the *Almajest* 3.6 anomaly table's numbers were actually generated from $A = 66°$.)

3 observations used to found the PH orbit (Rawlins 1991H §C8 or Rawlins 1991W §K10); which allows us to reconstruct the previously unknown real outdoor Hipparchos S.Solst that co-launched PH (and, as already seen at §M1, retro-created the papyrus' -157 solst):

$$-157/6/26\ 3/4 + 11 \cdot (365^{\rm d}1/4) = -146/6/26\ 1/2 \quad (27)$$

Adding this new find to those cited at §C2, we have four genuine outdoor ancient solstices. Again: none's hour is unambiguously cited in extant material. All are *DIO* reconstructions.
O4 There is no proof in Table 3 that observational error exceeded even a fraction of an hour, since rounding to $1^{\rm d}/4$ precision could account for most of the O$-$C error.[22]
O5 We have already (§D) considered how well previously-known real Greek solstices support *DIO*'s steady (from *DIO 1.1* ‡1 fn 24, 1991) contention that ancient science was far more empirical and competent than longtime orthodoxy has recognized. The -146 S.Solst just adds further confirmation to what in a sane scholarly community would have long since been incorporated — to its intellectual (and reputational) profit.

P PH Yearlength's Origin? Hipparchos' Ingenious Great-Year Cycle

P1 From his -145 V.Eqx, S.Solst, & A.Eqx, Hipparchos computed (method: *Almajest* 3.4; Rawlins 1991H §C) three of his final PH orbit's elements: $\epsilon_{\rm o}$, e, & A. But the $4^{\rm th}$ of the required 4 elements, the mean motion F, must depend in part upon earlier astronomers' observations. Except for hist.astron's dearest archons, scientific historians know how ancients estimated year-lengths: by comparing solstices centuries apart.
P2 In order to gauge ancient solstices' and year-lengths' accuracies, we need to know the actual values at that time. For Hipparchos' era, the true mean tropical[23] year was (Rawlins 1999 §C10) about $365^{\rm d}.2425$:

$$\text{Actual Hipparchos-Era Tropical Year-Length} \doteq 365^{\rm d}.2425 = 365^{\rm d}1/4 - 3^{\rm d}/400 \quad (28)$$

The foregoing rounding happens to be equal to Jesuit Christopher Clavius' Gregorian-rule year-length, established 17 centuries later (when the year was nearer $365^{\rm d}.2423$), and which we live-by today. (See puzzle at *DIO 4.2* p.2, instantly solved by K.Pickering & R.Freitag.)
P3 And there is an extra factor which is oft-forgot, namely (fn 23): each of the four cardinal-points has its own year-length — generally differing from the others by a few ten-thousandths of a day. Both their relative proportions and their absolute lengths vary secularly. For the present discussion, we should know the Hipparchos-era S.Solst value:

$$\text{Actual Hipparchos-Era S.Solst Year-Length} = 365^{\rm d}.2419 \quad (29)$$

To measure empirical error, compare ancient figures to eq.29; to measure vs mean year-length (which ancients thought they were determining), compare to eq.28.
P4 As soon as his -146 S.Solst measurement was in hand, Hipparchos returned to his earlier dabbling with Meton, which had led (§K5) to a yearlength tantalizingly close to compatibility with Meton's definition from his ratio (still used for modern Easter):

$$\text{Metonic Year } Y_{\rm M} = 235 \text{ months}/19 \quad (30)$$

which (via ‡1 eq.4) requires

$$Y_{\rm HM} = (235/19) \cdot 29^{\rm d}31'50''08'''20'''' \doteq 365^{\rm d}1/4 - 1^{\rm d}/314.7 \doteq 365^{\rm d}.24682 \quad (31)$$

[22] To consider an extreme case: if a S.Solst that occurred at 14:00 were measured by the observer as having occurred at 15:01, which he accurately rounded to traditional $1^{\rm d}/4$ precision (i.e., to $18^{\rm h}$), an O$-$C error of merely $1^{\rm h}$ would effectively quadruple, appearing to us to be a $4^{\rm h}$ O$-$C error. See fn 8.

[23] Technically, what has long been called a "tropical year" is a misnomer, since it refers to the sidereal year minus the effect of precession. But that standard figure — eq.28 — was not (§P3) the same as either of the two solstitial years: i.e., the mean Sun's returns to the Summer Tropic & Winter Tropic. Nor the same as the years measuring the mean Sun's returns to the Vernal & Autumnal Equinoxes. (You'll have to ask the esteamed *Journal for the History of Astronomy* about the Winter Equinoctial Year: §B1.) Note that in antiquity the average of the years of the S.Solst&W.Solst virtually equalled eq.28, as did the average of the V.Eqx&A.Eqx years.

Comparing his $-146/6/26\ 1/2$ solst with his hugely erroneous $-431/6/27\ 1/4$ dawn-version of Meton's solst ($-1^{\mathrm{d}}.2$ off: eq.4), $104095^{\mathrm{d}}1/4$ earlier, he found (for best *Almajest* 2.6 Athens-Rhodos klima $14^{\mathrm{h}}1/2$) PH-vs-Meton remainder $= -1^{\mathrm{d}}/300.66$; trivially rounding:

$$Y_{\text{P-M}} = \frac{104095^{\mathrm{d}} + [14^{\mathrm{h}}1/2]/2}{431 - 146} \doteq 365^{\mathrm{d}}1/4 - 1^{\mathrm{d}}/300 \doteq 365^{\mathrm{d}}.24667 = Y_{\mathrm{H}} \qquad (32)$$

This is the 1st time a modern has empirically justified by calculation astronomer Hipparchos' famous yearlength Y_{H}, adopted by Ptolemy and used for centuries thereafter. From here, Hipparchos devised his ***astonishing Great Year vision with its 5-stage geometrically embedded integral-return cycles*** ($304^{\mathrm{y}}1/4$, $608^{\mathrm{y}}1/2$, 1217^{y}, 2434^{y}, 4868^{y}), fully unfurled at Rawlins 2002A fnn 14, 16, 17. This Great Year fixed his long-view yearlength Y_{G}:

$$Y_{\mathrm{G}} = 365^{\mathrm{d}}1/4 - \frac{1^{\mathrm{d}}}{304\ 1/4} = Y_{\mathrm{K}} - \frac{16^{\mathrm{d}}}{4868} \doteq 365^{\mathrm{d}}.24671 \qquad (33)$$

We note that period 304^{y} (which is exactly 4 Kallippic cycles and 16 Metonic cycles) is clearly attested for Hipparchos by Censorinus (Heath 1913 p.297); for 4868^{y}, see fn 10 [4].

P5 But then, 12^{y} later, along came Hipparchos' $-134/6/26\ 1/4$ S.Solst, 5^{h} earlier than predicted by the PH orbit. (For potential effect, compare eq.35 to eq.32!) So did Hipparchos switch to a new year-length value? No — he instead (like the conservatism of §M1) also chose (*Alm* 3.1) a *near-equally* PH-discordant (Rawlins 1991H §B5) prior S.Solst, that of Aristarchos of Samos (eq.2; Table 1; Rawlins 1991H eq.8), $-279/6/26\ 1/2$, such that the new equation paralleling eq.32 gives near-enough the same PH mean motion F, but now from finding that seasonal $52960^{\mathrm{d}}3/4$ in the 145^{y} Aristarchos-Hipparchos gap yields:

$$Y_{\text{H-A}} = \frac{52961^{\mathrm{d}} - [14^{\mathrm{h}}1/2]/2}{279 - 134} \doteq Y_{\mathrm{K}} - 1^{\mathrm{d}}/263 \doteq 365^{\mathrm{d}}.24619 \qquad (34)$$

(Remainder was $-1^{\mathrm{d}}/290$, if Hipparchos computed $Y_{\text{H-A}}$ without accounting for seasonal hours.) The intent to somehow roughly justify preserving his original PH orbit's year-length (eq.32) is obvious. This evaded (see similarly at ‡3 fn 6) recalculating-replacing his PH solar mean motion tables, based on eq.32. That he adopted $Y_{\mathrm{H}} = 365^{\mathrm{d}}1/4 - 1^{\mathrm{d}}/300$ is clear from his own words (quoted at *Almajest* 3.1). Y_{H} exactly underlies PH' ϵ_{o} & table of mean solar motion F (§O3; *Almajest* 3.2).

P6 But an admirably independent party dissented from locked-in Metonic (eq.30) rigidity, as we know from Babylonian cuneiform text BM55555 (ACT 210, c.100 BC), the 1st Babylonian record provably based (Rawlins 1991H) upon Greek astronomers' work. (A text also containing the "Babylonian" month, likewise based on Greek research: Rawlins 2002A & Rawlins 2002U.) BM55555 bears a schismatic year-length (eq.3 & Rawlins 1991H §§A1-A2), which was anciently found (eq.4) by comparing Hipparchos' -134 solstice to Meton's solstice (instead of Aristarchos'), a 108478^{d} interval over 297^{y} (eq.3):

$$Y_{\text{U-M}} = \frac{108478^{\mathrm{d}}}{431 - 134} = Y_{\mathrm{K}} - \frac{1^{\mathrm{d}}1/4}{297} = Y_{\mathrm{K}} - \frac{1^{\mathrm{d}}}{237\ 3/5} \doteq 365^{\mathrm{d}}.2458 \qquad (35)$$

— the most accurate (see §P7) of the poor (truncation-corrupted) anciently-adopted tropical year values that have come down to us. (Actual mean year-length then was $365^{\mathrm{d}}.2425$: eq.28.) It is possible that Hipparchos flirted with using $Y_{\text{U-M}}$ (or published it in one of his many lost works without using it any orbit that we have), but all extant records indicate that Hipparchos stuck with eq.32's remainder (or a close approximation thereto).

P7 After our extensive discussion (§§F-J) of how ancients found solstices, it is disappointing that [a] so few accurate ones survived for us, and [b] the calendaric intent was so consistently vitiated by truncation, which neatly (§E3) led to apparent repeated-confirmation of the delusion that Meton's seriously-inaccurate effective-equating of the tropical year with 235/19 months (eq.30), was valid. We now list the above three ancient year-length-estimates' errors vs the actual mean year (eq.28): Y_{HM} $+6^{\mathrm{m}}.2$, Y_{H} $+6^{\mathrm{m}}.0$,[24] $Y_{\text{U-M}}$ $+4^{\mathrm{m}}.7$. And vs S.Solst yr (eq.29): Y_{HM} $+7^{\mathrm{m}}.1$, Y_{H} $+6^{\mathrm{m}}.9$, $Y_{\text{U-M}}$ $+5^{\mathrm{m}}.6$.

[24] The $+6^{\mathrm{m}}/1^{\mathrm{y}}$ excess of Hipparchos' Metonic year over reality (eq.28) produced solar mean longitude error $-1^{\circ}.1$ by Ptolemy's time, thus (Thurston 1998A §S2) revealing Ptolemy's faked "observations".

P8 BM55555's yearlength $Y_{\text{U-M}}$ (eq.3), while not close to the mark, is the best of a poor lot; but no known ancient year-length was within 4^{m} of being correct. This, while Aristarchos' sidereal year-length was correct to ordmag 10 timesecs [Rawlins 2002A fn 15; Rawlins 1999 §§C8-C9] since the sidereal year is of no public interest [*ibid* §D3], so: no danger of an astronomical disaster like Meton's lunar&solar priesthood-peacepact (Rawlins 1991H fn 1), $-431/6/27\ 3/4$ kicking off the year *containing* the start of Greece's Great War $-430/4/4$-$403/4/25$. (Correlation un-noted in history-of-astronomy literature?) Meton's ploy launched a tradition: "innocently guilty" of preconception, observing scientists Aristarchos & mature Hipparchos were (Rawlins 1985S) cascadingly attracted to prior data that seemed to reconfirm eq.30's longago pax-Meton, which lay in wait for 600^{y} ere undoing non-observing non-scientist C.Ptolemy, who instead just computed his 140 S.Solstice from Hipparchos' Metonic calendar (as young astrologer Hipparchos had computed his original -157 SS from Kallippos').

Q Preconception's Wages: Hipparchos Neglects Kallippos' Solstice

Q1 The contention of Rawlins 1999 §D4 was that the tropical year-length estimates we have from antiquity (with the exception of eq.35) flock quite unrandomly around the artificial Metonic value of eq.31. These results vindicate Tobias Mayer's solution (modernly rediscovered by R.Mercier, K.Moesgaard, N.Swerdlow, & DR) of the source of the systematic error in the Hipparchos-Ptolemy solar tables, namely, the Hipparchos year mimicked the Metonic luni-solar yearlength: eq.31. So preconception from (evidently) near-universal belief in eq.31 caused Hipparchos to *miss the opportunity to acquire the 1st accurate tropical year-length.* Survey his career-long search for a trustworthy ancient-to-him solstitial anchor: [a] In -157, he uses Kallippos' -329 Summer Solst to anchor EH. [b] While 12^{y} later adopting Meton's eq.30, he observes the -145 S.Solst but finds it won't work Metonically (eq.31) with Meton's own -431 S.Solst unless (§C1) Meton's "start of day" is (falsely: eq.1) taken to mean dawn, thus his -12^{h}-fudged -431 S.Solst anchors PH. [c] When, 11^{y} later, his new -135 S.Solst observation jars vis-à-vis the previous -145 one, he shifts anchor from Meton's -431 solst to Aristarchos' -279 solst for UH, in order to maintain (§Q1) his year-remainder at c.$-1^{\mathrm{d}}/300$. [d] But for his ultimate anchor, Hipparchos never goes back to the only accurate solst of the now-known lot: Kallippos' *where he started* (item [a] above; or §K1). This takes us into the plainest proof of Metonic preconception's grip (§§P6&P8), & an obvious, previously-unasked question: ***why did Hipparchos never compare either of his outdoor solstices to Kallippos'***, whose S.Solst offered longer baselines than Aristarchos'. Had he done so for his 1st empirical solstice (-146), he'd have found (interval 183^{y}), treating seasonal hours naïvely:

$$Y_{\text{H1-K}} = \frac{66839^{\mathrm{d}}1/4}{329 - 146} = Y_{\mathrm{K}} - 1^{\mathrm{d}}/122 \doteq 365^{\mathrm{d}}.2418 \doteq 365^{\mathrm{d}}1/4 - 1^{\mathrm{d}}/122 \qquad (36)$$

and, for his 2nd empirical solstice (-134), using an interval of 195^{y}:

$$Y_{\text{H2-K}} = \frac{71222^{\mathrm{d}}}{329 - 134} = Y_{\mathrm{K}} - 7^{\mathrm{d}}/780 \doteq 365^{\mathrm{d}}.2410 \doteq 365^{\mathrm{d}}1/4 - 1^{\mathrm{d}}/111 \qquad (37)$$

These 2 potential (historically-unrealized) yearlengths' errors vs the real mean year (eq.28): $Y_{\text{H1-K}}$ $-1^{\mathrm{m}}.0$, $Y_{\text{H2-K}}$ $-2^{\mathrm{m}}.1$. And vs S.Solst yr (eq.29): $Y_{\text{H1-K}}$ $-0^{\mathrm{m}}.1$, $Y_{\text{H2-K}}$ $-1^{\mathrm{m}}.3$.

Q2 Despite solstices' failure to yield an accurate tropical year (due to truncations, prejudice for eq.31, & not choosing Kallippos' solstice as earlier anchor), solstices nonetheless contributed to gradual improvement of the solar orbit, being (§G1) the most reliable of the 4 cardinal points. Whatever the quality of the calendaric uses made of them, the 4 recoverable outdoor ancient solstices (Table 3) were so conscientiously accomplished by the methods we discussed at the outset (culminating in §J), that all four are accurate within their quarter-day rounding — rounding which (§O4) has made it impossible to tell whether the pre-rounded values were more than trivially in error. As noted at §O5, this is yet another vindication for the high level of ancient Greek science, and for those who've defended it.

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